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An Approach to Compute the Multi-Objective Programming Problem Under Triangular Fuzzy Numbers

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ABSTRACT

The most important one advantages of the operation research (OR) theory is displayed of this paper for tackling multi-objective function programming problems (MOFPP). In this study, we present an approach for multi-objective fractional programming problems (MOFrPP) using fuzzy numbers (FN) coefficients in the objective functions (OF). The objective functions' (OF) parameters are all considered to be fuzzy triangular fuzzy numbers (FTrFN). This problem's multi-objective function (MOF) case was resolved by transforming the MOF to a linear programming problem (LnPP) that could be resolved using the simplex method (SM). The problem is being solved using algorithms. Then the value is compared with a result, which obtained by simplex method (SM). These techniques are illustrated with numerical examples (NE). This study shows the reliability of our methods and their utility to this specific set of multi-objective functions (MOF).

MSC..

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1. Introduction

The method of multi-objective function (MOF) is useful for analyzing the best trade-off solutions that balance various criteria. It is a little challenging to maximize two or more goals at once, and the challenge increases if the goals are incompatible. It was intended to analyze the potential for coming up with a compromise solution that achieves all the goals at once. Several techniques have been introduced for solving linear problems (LP). Some of them are extensions of the fractional linear problems (FrLP) and others are based on different idea. Mediya B. Mrakhan et al. [1] a new approach to the scaling-based multi-objective linear programming problem (MOLrPP). Ronak M. Abdullah and Snoor O.Abdalla [2] studied solutions of MOLrPP by Advanced Transformation Technique. Samsun Nahar and Md. Abdul Alim [3] presented a new technique to solve a MOLnFrPP by using new geometric averaging method and used some other techniques such as arithmetic averaging, geometric averaging. Farhana Akond Pramy and Md. Ainul Islam [4] the multi-objective linear fractional programming problem (MOLnFrP) problem was given a single efficient solution, and finds the effective solutions of (MOLnFrPP). S. F. Tantawy [5]

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Described a feasible direction be used to locate all effective extreme points for MOLnFrPP when all denominators are equal. Moslem Ganji and Mansour Sara [6] showed the relationship between its Robust Counterpart (RC) formulations and a Multi Objective Linear Fractional Programming (MOLrFrP) challenge with uncertain data in the objective function (OF). R. Suganya and U. Afrin [7] provided a new statistical averaging method (New Quadratic mean) for MOLrPP. [8-16] A number of fuzzy numbers have been used to solve MOFrPP by many researchers.

In order to improve this work, we established a MOLrFrPP, offered an algorithm to resolve the fractional programming problem for the multi-objective function, and proposed a creative approach to resolve the issue in order to produce the best optimal solution. By resolving a numerical example, the computer application of our approach has also been explained. Finally, we presented the findings and their comparability. Some basic definitions have been proposed, which will be important in this paper.

Definition 2.1: Fuzzy Number (FN) [17]

If the membership function is satisfied, the FN $\tilde{E}$ is fuzzy

(i) A fuzzy set of the discourse G is convex

(ii) $\tilde{E}$ is normal if $\exists\ g_i \in G$, $M_{\tilde{E}}(g_i) = 1$

(iii) $and\ M_{\tilde{E}}(b)$ is piecewise incessant.

Definition 2.2: Triangular Fuzzy Number (TrFN) [18]

A TrFN is defined as $\tilde{J} = (\text{ŧ}, \text{ƞ}, \text{ε})$ for $\text{ŧ}, \text{ƞ}, \text{ε} \in \text{Ŕ}$ and its membership is given by

$$M_{\tilde{J}}\text{ħ} = \begin{cases} \frac{\text{ħ}-\text{ŧ}}{\text{ƞ}-\text{ŧ}} & \text{ŧ} \le \dot{g} \le \text{ƞ} \\ \frac{\text{ε}-\text{ħ}}{\text{ε}-\text{ƞ}} & \text{ƞ} \le \dot{g} \le \text{ε} \\ 0 & otherwise \end{cases}$$

2. Linear Programming Problem (LnPP) [19]

A LnPP of the form

Maximize $\text{ƈ}^t \text{ϱ}$

Subject to

$\text{Ơ}\text{ϱ} = \text{Ƃ}$

And

$\text{ϱ} \ge 0$

Where $\text{ƈ} \in \text{Ŷ}^n$, $\text{Ƃ} \in \text{Ŷ}^n$ and $\text{Ơ} \in \text{Ŷ}^m\, \text{Ų}^n$.

3. Linear Fractional Programming Problem (LnFrPP) [20]

The general linear fractional programming problem is defined as follows:

$Maximize\ \check{Z} = \frac{Ʈ(ϱ)}{З(ϱ)}$

$Ʈ(ϱ) = c^t ϱ + \alpha$, $З(ϱ) = d^t ϱ + \beta$ are valued and continuous functions on Ụ and $d^t ϱ + \beta \neq 0$ $for\ all\ ϱ \in Ụ\ and\ Ụ = \{ϱ: Ơϱ = Ƃ, ϱ \geq 0\}, ϱ, c^t, d^t \in R^n$, $\alpha, \beta \in \hat{Y}$ $Ƃ \in R^m$, $Ơ \in \hat{Y}^{m*n}$.

4. Multi-Objective function Technique (MOFT) [21]

The mathematical Multi-Objective function Technique (MOFT) [21] is explained as:

Optimal Max $\check{Z}$ = [Max $Ƶ_1$, Max $Ƶ_2$, Max $Ƶ_3$, …]

Subject to:

$Ơϱ = Ƃ, ϱ \geq 0$

5. Complementary Multi-Objective Fractional Programming Formula (CMOFrPP)

The Complementary MOLnFrPP method have been used by the scientists [22-23] to solve the multi-objective (MO) optimization problems.

6. Ranking function (RF)

In a fuzzy numbers the ranking function is essential in the process for determining decisions. Ranking of FN is required to find the largest and smallest fuzzy numbers. The article [24] have proposed ranking function methods as the follows:

$$H(\tilde{V}) = \frac{ƹ + 14 * ƫ + ƅ}{16} \quad ..... \quad (1)$$

7. The New Algorithm for Solving multi-objective function programming problems (MOFPP)

An algorithm to explain MOFPP by SM which can be summarized as follows:

Step 1: We suggest two techniques for process of the converting fuzzy multi-objective fractional functions to linear fractional functions programming problems (FMOLrFrPP) into LPP.

Step2: At first consider the OF $Maximize\ Ƶ = \frac{Ʈ(ϱ)}{З(ϱ)}$ is FMOLrFPP.

Step3: FMOLrFPP is changed to the LrFPP by two techniques: the first technique is complementary fractional and the second technique is ranking function.

Step4: dividing the first fractional objective function (FrOF) into two linear functions (LnF), the first function serving as the numerator function (NF) and the second as the denominator function (DF). Maximum ($Ʈ(ϱ)$) for the numerator and minimize $З(ϱ)$ for the denominator functions are used to determine the objective function's value.

Step 5: An LrP problem is written as $Max\ \mathrm{Ƶ}^{*}$ subject to the original problem's restrictions, where $Max\ \mathrm{Ƶ}^{*}$ is the result of subtracting the denominator function $\mathrm{Ʒ}(\varrho)$ from the numerator function$\mathrm{Ʈ}(\varrho)$, or $Maximize\ \mathrm{Ƶ} = \mathrm{Ʈ}(\varrho) - \mathrm{Ʒ}(\varrho)$.

Step 6: The second, third, and fourth objective functions all follow the same process.

Step7: Find the optimal solution by using Simplex process of each objective function.

The algorithm is illustrated with the following example.

8. Numerical Example :

The following example is illustrated to show the proposed methodology for MOFFr the following MOFFr is formulated as

$$\text{Max } \mathrm{Ƶ}_1 = \frac{(6,7,8)\, ɣ_1 + (4,5,6)\, ɣ_2}{(4,5,6)\, ɣ_1 + (3,4,5)\, ɣ_2}$$

$$\text{Max } \mathrm{Ƶ}_2 = \frac{(7,8,9)\, ɣ_1 + (4,5,6)\, ɣ_2}{(6,7,8)\, ɣ_1 + (3,4,5)\, ɣ_2}$$

$$\text{Max } \mathrm{Ƶ}_3 = \frac{(8,9,10)\, ɣ_1 + (5,6,7)\, ɣ_2}{(5,6,7)\, ɣ_1 + (2,3,4)\, ɣ_2}$$

subject to

$(4,5,6)\, ɣ_1 + (6,7,8)\, ɣ_2 \leq (16,17,18)$

$(5,6,7)\, ɣ_1 + ((8,9,10)\, ɣ_2 \leq (20,21,22)$

$(6,7,8)\, ɣ_1 + (3,4,5)\, ɣ_2 \leq (11,12,13)$

$ɣ_1 ,\ ƫ_2 \geq 0.$

1. First technique: The above problem can be converted with all the three maximization fuzzy multi-objective fractional functions to linear fractional functions as detailed below:

$$\text{Max } \mathrm{Ƶ}_1 = \frac{8\, ɣ_1 + 6\, ɣ_2}{6\, ɣ_1 + 5\, ɣ_2}$$

$$\text{Max } \mathrm{Ƶ}_2 = \frac{9\, ɣ_1 + 6\, ɣ_2}{8\, ɣ_1 + 5\, ɣ_2}$$

$$\text{Max } \mathrm{Ƶ}_3 = \frac{10\, ɣ_1 + 7\, ɣ_2}{7\, ɣ_1 + 4\, ɣ_2}$$

subject to

$6\, ɣ_1 + 8\, ɣ_2 \leq 18$

$7\, ɣ_1 + 10\, ɣ_2 \leq 22$

$8\, ɣ_1 + 5\, ɣ_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

2. Now the new objective function $Max\ Z^*$ is constructed as per the complementary fractional method.

$Max\ Z_1^* = 2\gamma_1 + \gamma_2$

$Max\ Z_2^* = \gamma_1 + \gamma_2$

$Max\ Z_3^* = 3\gamma_1 + 3\gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

We solve first linear objective function to obtain the optimal solution.

$Max\ Z_1^* = 2\gamma_1 + \gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \natural_2 \geq 0.$

Solving by Simplex Method using (Win QSB) Programming, we get

$Max\ Z_1^* = 3.250,\ \gamma_1 = 1.625 ,\ \gamma_2 = 0$

We solve second linear objective function to obtain the optimal solution.

$Max\ Z_2^* = \gamma_1 + \gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

Solving by Simplex Method using (Win QSB) Programming, we get

$Max\ Z_2^* = 2.333,\ \gamma_1 = 0.444 ,\ \gamma_2 = 1.888$

We solve third linear objective function to obtain the optimal solution.

$$Max\ \text{Ƶ}_3^* = 3\gamma_1 + 3\gamma_2$$

subject to

$$6\gamma_1 + 8\gamma_2 \le 18$$

$$7\gamma_1 + 10\gamma_2 \le 22$$

$$8\gamma_1 + 5\gamma_2 \le 13$$

$$\gamma_1 , \gamma_2 \ge 0.$$

Solving by Simplex Method using (Win QSB) Programming, we get

$$Max\ \text{Ƶ}_3^* = 7.000 ,\quad \gamma_1 = 0.444 ,\quad \gamma_2 = 1.888$$

Second technique: the ranking function of triangular fuzzy numbers $H(\tilde{V}) = \frac{\text{Ƹ}+14*\text{ƒ}+\text{ƀ}}{16}$, the above fuzzy multi-objective fractional functions (FMOFLFPP) can be converted to the following multi-objective fractional functions (MOLFPP):

$$\text{Max}\ \text{Ƶ}_1 = \frac{7\gamma_1 + 5\gamma_2}{5\gamma_1 + 4\gamma_2}$$

$$\text{Max}\ \text{Ƶ}_2 = \frac{8\gamma_1 + 5\gamma_2}{7\gamma_1 + 4\gamma_2}$$

$$\text{Max}\ \text{Ƶ}_3 = \frac{9\gamma_1 + 6\gamma_2}{6\gamma_1 + 3\gamma_2}$$

subject to

$$6\gamma_1 + 8\gamma_2 \le 18$$

$$7\gamma_1 + 10\gamma_2 \le 22$$

$$8\gamma_1 + 5\gamma_2 \le 13$$

$$\gamma_1 , \text{ţ}_2 \ge 0.$$

We solve three linear programming problems to obtain the optimal solution.

$$Max\ \text{Ƶ}_1^* = 2\gamma_1 + \gamma_2$$

$$Max\ \text{Ƶ}_2^* = \gamma_1 + \gamma_2$$

$$Max\ \text{Ƶ}_3^* = 3\gamma_1 + 3\gamma_2$$

subject to

$$6\gamma_1 + 8\gamma_2 \le 18$$

$$7\gamma_1 + 10\gamma_2 \le 22$$

$$8\gamma_1 + 5\gamma_2 \le 13$$

$\gamma_1 , \gamma_2 \geq 0.$

We solve first linear objective function to obtain the optimal solution.

$Max\ Z_1^* = 2\gamma_1 + \gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

Solving by Simplex Method using (Win QSB) Programming, we get

$Max\ Z_1^* = 3.250, \ \gamma_1 = 1.625 , \ \gamma_2 = 0$

We solve second linear objective function to obtain the optimal solution.

$Max\ Z_2^* = \gamma_1 + \gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

Solving by Simplex Method using (Win QSB) Programming, we get

$Max\ Z_2^* = 2.333, \ \gamma_1 = 0.444 , \ \gamma_2 = 1.888$

We solve third linear objective function to obtain the optimal solution.

$Max\ Z_3^* = 3\gamma_1 + 3\gamma_2$

subject to

$6\gamma_1 + 8\gamma_2 \leq 18$

$7\gamma_1 + 10\gamma_2 \leq 22$

$8\gamma_1 + 5\gamma_2 \leq 13$

$\gamma_1 , \gamma_2 \geq 0.$

Solving by Simplex Method using (Win QSB) Programming, we get

$Max\ Z_3^* = 7.000 , \ \gamma_1 = 0.444 , \ \gamma_2 = 1.888$

the results of above example when solved by complementary fractional method and ranking function method almost it same result are shown below in the table(1).

Table (1): Resulting multi-objective function programming problems (MOFPP):

3. No	4. Method	5. The optimal solution (OS)
6. 1	7. complementary fractional method	8. $Max\ Ƶ_1^* = 3.250$, $ɣ_1 = 1.625$, $ɣ_2 = 0$ 9. 10. $Max\ Ƶ_2^* = 2.333$, $ɣ_1 = 0.444$, $ɣ_2 = 1.888$ 11. 12. $Max\ Ƶ_3^* = 7.000$, $ɣ_1 = 0.444$, $ɣ_2 = 1.888$ 13.
14. 2	15. Ranking function	16. $Max\ Ƶ_1^* = 3.250$, $ɣ_1 = 1.625$, $ɣ_2 = 0$ 17. 18. $Max\ Ƶ_2^* = 2.333$, $ɣ_1 = 0.444$, $ɣ_2 = 1.888$ 19. 20. $Max\ Ƶ_3^* = 7.000$, $ɣ_1 = 0.444$, $ɣ_2 = 1.888$ 21.

9. Conclusion

A new TrFN algorithm to convert the MOLPP into a single LPP. Additionally, in order to compare the outcomes, we applied our strategy to the example using two distinct approaches to the problem. This comparison allowed us to see that two method produced results that were exactly the same value and its computer application employing programming in the mathematical language (Win QSB).

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