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Subclass of Univalent Functions with Positive Coefficients

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Abstract. In this paper, we consider the class $H(\gamma,\alpha)$ consisting of analytic function with positive coefficients. We obtain some geometric properties, like, arithmetic mean, some distortion theorems and Hadmard product in the class $H(\gamma,\alpha)$.

Keywords: Univalent function, Distortion theorem, Integral operator, Hadamard product, δ-Neighborhood.

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1. Introduction and Definitions.

Let T denote the class of functions:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \ , \qquad (1.1)$$

which are analytic andunivalent in $U = \{z: z \in \mathbb{C} \ and \ |z| < 1\}$.

Let H denote the subclass of T consisting of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (a_n \geq 0, n \in \mathbb{N} = \{1,2,\dots\}) \qquad (1.2)$$

A function $f \in T$ is called univalent starlike of order β $(0 \leq \beta < 1)$ if f is satisfies the condition

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \beta \ , \quad (z \in U).$$

Also a function $f \in T$ is called univalent convex of order β $(0 \leq \beta < 1)$ if f is satisfies the condition

$$Re\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \beta\,, \quad (z \in U).$$

Definition 1.1[5]. The Gaussian hypergeometric function defined by $_2F_1$ and is defined by

$$_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n\, n!} z^n\,,$$

where $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$, $c > b > 0$ and $c > a + b$ .It is well known (see[2]) that under the conditions $c > b > 0$ and $c > a + b$, we have

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n\, n!} z^n = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}.$$

Definition 1.2. Let $f \in H$ be of the form (2.1). Then the Hohlov operator $F(a,b,c)$ is defined by means of Hadamard product below

$$F(a,b,c)f(z) = z\ {}_2F_1(a,b;c;z) * f(z)$$

$$= z + e^{i\vartheta} \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}\,(n-1)!} a_n z^n\,, \qquad (1.3)$$

$$(a,b,c \in \mathbb{N}_0\,, c \neq \mathbb{Z}_0^- = \{0,-1,-2,\dots\}\,; z \in U).$$

The same operator have been studied by Atshan [4] on a class of univalent functions.

Definition 1.3. The Hadamard product of the two functions $F(a,b,c)f$ given by (1.3) and

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \ (1.4)$$

is defined by

$$\big(g * F(a,b,c)(f)\big)(z) = z + \sum_{n=2}^{\infty} \Gamma_n\, a_n\, b_n z^n\,,$$

where $\Gamma_n = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}$. (1.5)

Definition 1.4. A function $f \in H$ is said to be in the class $H(\gamma,\alpha)$ if satisfies the condition

$$Re\left\{\frac{\big(g*F(a,b,c)(f)\big)'(z) + \gamma\, z\,\big(g*F(a,b,c)(f)\big)''(z)}{\gamma\,\big(g*F(a,b,c)(f)\big)'(z) + (1-\gamma)}\right\} > \alpha\,, \qquad (1.6)$$

where $0 \leq \alpha < 1\,, 0 < \gamma < 1$.

Lemma 1.1.[3] If $\alpha \geq 0$, then $Re\, w > \alpha$ if and only if $|w-(1+\alpha)| < |w+(1-\alpha)|$, where w be any complex number.

2. Coefficient Bounds.

We obtain here a necessary and sufficient condition to be the function $f(z)$ in the class $H(\gamma,\alpha)$.

Theorem 2.1. Let $f \in T$. Then $f \in H(\gamma,\alpha)$ if and only if

$$\sum_{n=2}^{\infty} \Gamma_{\mathrm{n}}\, n\,[\gamma(n-\alpha-1)+1]a_n b_n \leq 1-\alpha\,, \qquad (2.1)$$

where Γ_{n} is defined by (1.5) and $0 \leq \alpha < 1$, $0 < \gamma < 1$, $z \in U$. The result is sharp for the function

$$f(z) = z + \frac{1-\alpha}{\Gamma_{\mathrm{n}}\, n\,[\gamma(n-\alpha-1)+1]b_n} z^n\,.$$

Proof . Suppose that the inequalities (2.1) holds true and let $|z| = 1$, in view of (1.6), we need to prove that $Re(w) > \alpha$, where

$$w = \frac{\left(g * F(a,b,c)(f)\right)'(z) + \gamma\, z\left(g * F(a,b,c)(f)\right)''(z)}{\gamma\left(g * F(a,b,c)(f)\right)'(z) + (1-\gamma)}$$

$$= \frac{1 + \sum_{\mathrm{n}=2}^{\infty} \Gamma_n\, n[\gamma(\mathrm{n}-1)+1]a_n\, b_n z^{n-1}}{1 + \sum_{\mathrm{n}=2}^{\infty} \Gamma_n\, \gamma\, n\, a_n\, b_n z^{n-1}}$$

$$= \frac{A(z)}{B(z)}\,.$$

By Lemma 1.1 , it suffices to show that

$$|A(z) - (1+\alpha)B(z)| - |A(z) + (1-\alpha)B(z)| \leq 0.$$

Therefore , we obtain

$$|A(z) - (1+\alpha)B(z)| - |A(z) + (1-\alpha)B(z)|$$

$$\leq \alpha + \sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-1)+1-(1+\alpha)\gamma]a_n\, b_n |\mathrm{z}|^{n-1} - (2-\alpha)$$

$$+ \sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-1)+1+(1-\alpha)\gamma]a_n\, b_n |z|^{n-1}$$

$$= \alpha + \sum_{n=2}^{\infty} \Gamma_n \, n \, [\gamma(n-2-\alpha)+1] a_n \, b_n - (2-\alpha)$$

$$+ \sum_{n=2}^{\infty} \Gamma_n \, n \, [\gamma(n-\alpha)+1] a_n \, b_n$$

$$= 2\alpha - 2 + \sum_{n=2}^{\infty} \Gamma_n \, n \, [2\gamma n - 2\gamma\alpha - 2\gamma + 2] a_n \, b_n$$

$$= \sum_{n=2}^{\infty} \Gamma_n \, n \, [\gamma(n-\alpha-1)+1] a_n \, b_n - (1-\alpha) \leq 0 \ ,$$

by hypothesis. Then by maximum modulus Theorem, we have $f \in H(\gamma, \alpha)$. Conversely , assume that

$$Re\left\{\frac{\left(g * F(a,b,c)(f)\right)'(z) + \gamma \, z \left(g * F(a,b,c)(f)\right)''(z)}{\gamma \left(g * F(a,b,c)(f)\right)'(z) + (1-\gamma)}\right\}$$

$$= Re\left\{\frac{1 + \sum\limits_{n=2}^{\infty} \Gamma_n \, n[\gamma(n-1)+1] a_n \, b_n z^{n-1}}{1 + \sum\limits_{n=2}^{\infty} \Gamma_n \, \gamma \, n \, a_n \, b_n z^{n-1}}\right\} > \alpha \, , \qquad (2.3)$$

we can choose the value of z on the real axis and let $z \longrightarrow 1^-$, through real values, so we can write (2.3) as

$$\sum_{n=2}^{\infty} \Gamma_n \, n \, [\gamma(n-\alpha-1)+1] a_n \, b_n \leq (1-\alpha) \ .$$

Finally , sharpness follows if we take

$$f(z) = z + \frac{1-\alpha}{\Gamma_n \, n \, [\gamma(n-\alpha-1)+1] b_n} z^n \ , n = 2,3, \dots \ . \qquad (2.4)$$

Corollary 2.1. If $f(z) \in H(\gamma, \alpha)$. Then

$$a_n \leq \frac{1-\alpha}{\Gamma_n \, n \, [\gamma(n-\alpha-1)+1] b_n} \ , n = 2,3, \dots \ .$$

Theorem 2.2. Let the function $f(z)$ defined by (1.1) be in the class $H(\gamma, \alpha)$. Then

$$r - \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2} r^2 \leq |f(z)| \leq r + \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2} r^2 . \ (2.5)$$

The result is sharp for the function $f(z)$ given by

$$f(z) = z + \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2} z^2 \ .$$

Proof. Let$f(z) \in H(\gamma, \alpha)$. Then by Theorem (2.1), we have

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2}.$$

Hence

$$|f(z)| \le |z| + \sum_{n=2}^{\infty} a_n |z|^n \quad ,(|z| = r < 1)$$

$$\le r + r^2 \sum_{n=2}^{\infty} a_n$$

$$\le r + \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2} r^2 \qquad (2.6)$$

Similarly, we obtain

$$|f(z)| \ge |z| - \sum_{n=2}^{\infty} a_n |z|^n$$

$$\ge r - r^2 \sum_{n=2}^{\infty} a_n$$

$$\ge r - \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 b_2} r^2 (2.7)$$

From bounds (2.6) and (2.7), we get (2.5).

Theorem 2.3. Let the function $f(z)$ defined by (1.1) be in the class $H(\gamma, \alpha)$. Then

$$1 - \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2} r \le |f'(z)| \le 1 + \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2} r. (2.8)$$

The result is sharp for the function $f(z)$ given by

$$f(z) = z + \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2} z^2 .$$

Proof. Let $f(z) \in H(\gamma, \alpha)$. Then by Theorem (2.1), we have

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2}.$$

Hence

$$|f'(z)| \le |1| + \sum_{n=2}^{\infty} a_n \, n \, |z|^{n-1}$$

$$\le 1 + \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2} r \qquad (2.9)$$

Similarly, we obtain

$$\left|f'(z)\right| \geq |1| - \sum_{n=2}^{\infty} a_n \; n\,|z|^{n-1}$$

$$\geq 1 - \frac{1-\alpha}{[\gamma(1-\alpha)+1]\Gamma_2 b_2} r \qquad (2.10)$$

From bounds (2.9) and (2.10), we get (2.8).

Theorem 2.4. Let the function f_k defined by

$$f_k(z) = z + \sum_{n=2}^{\infty} a_{n,k}\, z^n$$

be in the class $H(\gamma,\alpha)$ for every $k = 1,2,3,\dots,\ell$. Then the function

$$w(z) = z + \sum_{n=2}^{\infty} e_n\, z^n \quad ,(e_n \geq 0, n \in \mathbb{N}),$$

also belongs to the class $H(\gamma,\alpha)$, where

$$e_n = \frac{1}{\ell}\sum_{k=1}^{\ell} a_{n,k}.$$

Proof. Since $f_k \in H(\gamma,\alpha)$, if follows from Theorem (2.1), that

$$\sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-\alpha-1)+1]a_{n,k}\, b_n \leq (1-\alpha) \;,$$

for every $k = 1,\dots,\ell$. Hence

$$\sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-\alpha-1)+1]e_n\, b_n = \sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-\alpha-1)+1]\; b_n \left(\frac{1}{\ell}\sum_{k=1}^{\ell} a_{n,k}\right)$$

$$\leq \frac{1}{\ell}\sum_{k=1}^{\ell}(1-\alpha) = (1-\alpha),$$

which shows that $w(z) \in H(\gamma,\alpha)$.

Theorem 2.5. Let the function $f_j(z),(j = 1,2)$ defined by

$$f_j(z) = z + \sum_{n=2}^{\infty} a_{n,j}\, z^n \,, \qquad (j = 1,2)$$

be in the class $H(\gamma,\alpha)$. Then the function

$$T(z) = z + \sum_{n=2}^{\infty}\left(a_{n,1}^2 + a_{n,2}^2\right) z^n \,,$$

also belong to the class $H(\gamma,\epsilon)$, where

$$\epsilon \geq \frac{2(1-\alpha)^2[\gamma(n-1)+1]-n\Gamma_n\ b_n[\gamma(n-\alpha-1)+1]^2}{2\gamma(1-\alpha)^2+n\Gamma_n\ b_n[\gamma(n-\alpha-1)+1]^2}.$$

Proof. We must find the largest ϵ such that

$$\frac{\sum_{n=2}^{\infty}\ \Gamma_n\, n[\gamma(n-\epsilon-1)+1]\, b_n}{1-\epsilon}\left(a_{n,1}^2+a_{n,2}^2\right)\leq 1.$$

Since $f_j(z), (j=1,2)$ belong to the class $H(\gamma,\alpha)$, we have

$$\frac{\sum_{n=2}^{\infty}\ \Gamma_n^2\, n^2[\gamma(n-\alpha-1)+1]^2 b_n^2}{(1-\alpha)^2}a_{n,1}^2 \leq \left(\frac{\sum_{n=2}^{\infty}\ \Gamma_n\, n[\gamma(n-\alpha-1)+1]\, b_n}{1-\alpha}\ a_{n,1}\right)^2 \leq 1$$

and

$$\frac{\sum_{n=2}^{\infty}\ \Gamma_n^2\, n^2[\gamma(n-\alpha-1)+1]^2 b_n^2}{(1-\alpha)^2}a_{n,2}^2 \leq \left(\frac{\sum_{n=2}^{\infty}\ \Gamma_n\, n[\gamma(n-\alpha-1)+1]\, b_n}{1-\alpha}\ a_{n,2}\right)^2 \leq 1.$$

Hence , we have

$$\sum_{n=2}^{\infty}\frac{1}{2}\left(\frac{\Gamma_n^2\, n^2[\gamma(n-\alpha-1)+1]^2 b_n^2}{(1-\alpha)^2}\right)\left(a_{n,1}^2+a_{n,2}^2\right)\leq 1.$$

$T(z)\in H(\gamma,\epsilon)$ if and only if

$$\sum_{n=2}^{\infty}\frac{\Gamma_n\, n[\gamma(n-\epsilon-1)+1]\, b_n}{1-\epsilon}\left(a_{n,1}^2+a_{n,2}^2\right)\leq 1.$$

Therefore , we need to find the largest ϵ such that

$$\frac{\Gamma_n\, n[\gamma(n-\epsilon-1)+1]\, b_n}{1-\epsilon}\leq\frac{\Gamma_n^2\, n^2[\gamma(n-\alpha-1)+1]^2 b_n^2}{(1-\alpha)^2}\ . \qquad (2.11)$$

From (2.11) , we have

$$\epsilon \geq \frac{2(1-\alpha)^2[\gamma(n-1)+1]-n\Gamma_n\ b_n[\gamma(n-\alpha-1)+1]^2}{2\gamma(1-\alpha)^2+n\Gamma_n\ b_n[\gamma(n-\alpha-1)+1]^2}.$$

The concept of neighborhood of analytic functions was first introduced by Goodman [5] and Ruscheweyh [7] investigated this concept for the elements of several famous subclasses of analytic functions and Altintas and Owa [1] considered for a certain family of analytic functions with negative coefficients.

Now, we define the (n,δ) –neighborhood of a function $f \in H$ by

$$N_{n,\delta}(f) = \{k \in H: k(z) = z + \sum_{n=2}^{\infty} c_n z^n \ and \ \sum_{n=2}^{\infty} n\,|a_n - c_n| \le \delta\,, 0 \le \delta < 1\}\,, \quad (2.12)$$

for the identity function $e(z) = z$, we have

$$N_{n,\delta}(e) = \{k \in H: k(z) = z + \sum_{n=2}^{\infty} c_n z^n \ and \ \sum_{n=2}^{\infty} n\,|c_n| \le \delta\,\}\,.$$

Definition 1.5. A function $f \in H$ is said to be in the class $H(\gamma,\alpha)$ if there exists a function $k \in H(\gamma,\alpha)$ such that

$$\left|\frac{f(z)}{k(z)} - 1\right| < 1 - \beta \qquad (z \in U\,, 0 \le \beta < 1).$$

Theorem 2.6. If $k \in H(\gamma,\alpha)$ and

$$\beta = 1 - \frac{\delta}{2}\frac{[2\gamma(1-\alpha)+2]\Gamma_2 c_2}{[2\gamma(1-\alpha)+2]\Gamma_2 c_2 - (1-\alpha)}\ . \quad (2.13)$$

Then $N_{n,\delta}(k) \subset H(\gamma,\alpha)$.

Proof. Let $f \in N_{n,\delta}(k)$. We want to find from (2.12) that

$$\sum_{n=2}^{\infty} n\,|a_n - c_n| \le \delta\,,$$

which readily implies the following coefficient inequality

$$\sum_{n=2}^{\infty} |a_n - c_n| \le \frac{\delta}{2}.$$

Next , since $k \in H(\gamma,\alpha)$ in view of Theorem (2.1) such that

$$\sum_{n=2}^{\infty} c_n \le \frac{1-\alpha}{[2\gamma(1-\alpha)+2]\Gamma_2 c_2}\,.$$

So that

$$\left|\frac{f(z)}{k(z)} - 1\right| \le \frac{\sum_{n=2}^{\infty} |a_n - c_n|}{1 - \sum_{n=2}^{\infty} c_n}$$

$$\le \frac{\delta}{2}\frac{[2\gamma(1-\alpha)+2]\Gamma_2 c_2 - (1-\alpha)}{[2\gamma(1-\alpha)+2]\Gamma_2 c_2} = 1 - \beta\,.$$

Thus by Definition (1.5) , $f \in H(\gamma,\alpha)$ for β given by (2.13).

Theorem 2.7. Let r real number such that $r > -1$. If $f \in H(\gamma,\alpha)$, then the function F defined by

$$F(z) = \frac{r+1}{z^r}\int_0^z t^{r-1} f(t)dt\,,$$

also belong to $H(\gamma,\alpha)$.

Proof. Let

$$f(z) = z + \sum_{n=2}^{\infty} a_n\, z^n.$$

Then from the representation of F, it follows that

$$F(z) = z + \sum_{n=2}^{\infty} d_n\, z^n,$$

where $d_n = \frac{r+1}{r+n} a_n$. Therefore using Theorem (2.1) for the coefficients of F, we have

$$\sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-\alpha-1)+1] d_n\, b_n$$

$$= \sum_{n=2}^{\infty} \Gamma_n\, n\,[\gamma(n-\alpha-1)+1]\left(\frac{r+1}{r+n}\right) a_n\, b_n \le 1-\alpha$$

Since $\frac{r+1}{r+n} < 1$ and $f \in H(\gamma,\alpha)$. Hence $F \in H(\gamma,\alpha)$.

Theorem 2.8. Let $0 \le \alpha < 1, 0 < \gamma < 1$. Then $H(\gamma,\alpha) \subset H(\gamma,\tau)$ where $\tau = \frac{\alpha[\gamma(n-2)+1]}{\gamma(n-2)+1}$.

Proof. Let the function $f(z)$ given by (1.2) belong to the class $H(\gamma,\alpha)$. Then , by using Theorem 2.1. , we get

$$\frac{\sum_{n=2}^{\infty} n[\gamma(n-\alpha-1)+1]}{1-\alpha}\Gamma_n\, a_n\, b_n \le 1\,.$$

In order to prove that $f \in H(\gamma,\tau)$, we must have

$$\frac{\sum_{n=2}^{\infty} n[\gamma(n-\tau-1)+1]}{1-\tau}\Gamma_n\, a_n\, b_n \le 1\,. \qquad (2.14)$$

Note that (2.14) satisfies if

$$\frac{n[\gamma(n-\tau-1)+1]\Gamma_n}{1-\tau} \le \frac{n[\gamma(n-\alpha-1)+1]\Gamma_n}{1-\alpha}\,, \qquad (2.15)$$

from (2.15), we havewhere $\tau = \frac{\alpha[\gamma(n-2)+1]}{\gamma(n-2)+1}$.

Theorem 2.9. Let

$$f(z)=z+\sum_{n=2}^{\infty}a_n\,z^n\,,\; g(z)=z+\sum_{n=2}^{\infty}f_n\,z^n$$

belong to $H(\gamma,\alpha)$. Then the Hadamard product of f and g given by

$$(f*g)(z)=z+\sum_{n=2}^{\infty}a_n\,f_n\,z^n,$$

belong to $H(\gamma,\alpha)$.

Proof. Since f and $g\in H(\gamma,\alpha)$, we have

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\,f_n}{1-\alpha}\right]a_n\;\le 1$$

and

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\,a_n}{1-\alpha}\right]f_n\;\le 1$$

and by applying the Cauchy –Schwarz inequality , we have

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\sqrt{a_n\,f_n}}{1-\alpha}\right]\sqrt{a_n\,f_n}$$

$$\le\left(\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\,f_n}{1-\alpha}\right]a_n\right)^{1/2}\times\left(\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\,a_n}{1-\alpha}\right]f_n\right)^{1/2}$$

However , we obtain

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\sqrt{a_n\,f_n}}{1-\alpha}\right]\sqrt{a_n\,f_n}\le 1.$$

Now , we want to prove

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n}{1-\alpha}\right]a_n\,f_n\;\le 1.$$

Since

$$\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n}{1-\alpha}\right]a_n\,f_n\;=\sum_{n=2}^{\infty}\left[\frac{n[\gamma(n-\alpha-1)+1]\Gamma_n\sqrt{a_n\,f_n}}{1-\alpha}\right]\sqrt{a_n\,f_n}\,.$$

Hence , we get the required result.

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