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On Bc-Lindelof spaces and nearly Bc-Lindelof spaces

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Abstract

In this paper, we have discussed a new class of ωBc-open sets and ωBc-regular open sets. Throughout this work, new concepts have been illustrated including an Bc-Lindelof spaces and nearly Bc-Lindelof spaces and the behavior of these invariant under kinds of functions.

Mathematics Subject Classification: 54XX

1.Introduction

The concept of Bc-open set in topological spaces was introduced in 2013 by Hariwan Z [2]. This set was also considered in [3].

This paper consist of three section. In section one, we give similar definition by using of Bc-open sets and also we proof some properties about it. In section two we introduce a new generalization of ωBc-open set, ωBc-regular open and investigate some properties of this set. In section three we obtain new a characterization and preserving theorems of Bc-Lindelof space and nearly Bc-Lindelof space.

Definition(1.1)[1]:

Let X be a space and $A \subseteq X$. Then A is called b-open set in X if $A \subseteq \overline{A^\circ} \cup \overline{A}^\circ$. The family of all b-open subset of a topological space (X, τ) is denoted by $BO(X, \tau)$ or (Briefly $BO(X)$).

Definition(1.2)[2]:

Let X be a space and $A \subset X$. Then A is called Bc-open set in X if for each $x \in A \in BO(X,\tau)$, there exists a closed set F such that $x \in F \subset A$. The family of all Bc-open subset of a topological space (X,τ) is denoted by $BcO(X,\tau)$ or (Briefly $BcO(X)$), A is Bc-closed set if A^c is Bc-open set. The family of all Bc-closed subset of a topological space (X,τ) is denoted by $BcC(X,\tau)$ or (Briefly $BcC(X)$).

Example(1.3):

It is clear from the definition that every Bc-open set is b-open, but the converse is not true in general.

Let $X = \{1,2,3\}$, $\tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$. Then the closed set are: $X, \phi, \{2,3\}, \{1,3\}, \{3\}$. Hence $BO(X) = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$ and $BcO(X) = \{\phi, X, \{1,3\}, \{2,3\}\}$. Then $\{1\}$ is b-open but $\{1\}$ is not Bc-open.

Definition(1.4)[2]:

Let X be a space and $A \subset X$. Then A is called θ-open set in X if for each $x \in A$, there exists an open set G such that $x \in G \subset \bar{G} \subset A$. The family of all θ-open subset of a topological space (X,τ) is denoted by $\theta O(X,\tau)$ or (Briefly $\theta O(X)$).

Remark(1.5)[2]:

1) Every θ-open is Bc-open.

2) Every θ-closed is Bc-closed.

Example(1.6):

The intersection of two Bc-open sets is not Bc-open in general.

Let $X = \{1,2,3\}$, $\tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$. Then $\{1,3\}, \{2,3\}$ is Bc-open set, where as $\{1,3\} \cap \{2,3\} = \{3\}$ is not Bc-open set.

Remark(1.7)[4]:

The intersection of an b-open set and an open set is b-open set.

Remark(1.8):

Let X be a space and $A, B \subset X$. If A is Bc-open set and B is an θ-open set, then $A \cap B$ is Bc-open set.

Proof:

Let A be a Bc-open set and B be an θ-open set, then A is b-open set and B is an open set since every θ-open is open. Then $A \cap B$ is b-open set by (Remark(1.7)). Now, let $x \in A \cap B$, $x \in A$ and $x \in B$. If $x \in A$, then there exists a closed set F such that $x \in F \subset A$ and if $x \in B$, then there exists an open set E such that $x \in E \subset \bar{E} \subset B$. Therefore, $F \cap \bar{E}$ is closed since the intersection of closed sets is closed. Thus $x \in F \cap \bar{E} \subset A \cap B$. Then $A \cap B$ is Bc-open set.

Proposition(1.9)[2]:

Let X be a space and $A \subset X$. Then A is Bc-open set if and only if A is b-open set and it is a union of closed sets. That is $A = \cup F_\alpha$ where A is b-open set and F_α is closed subsets for each α.

Proposition(1.10)[2]:

Let $\{A_\alpha : \alpha \in \Lambda\}$ be a collection of Bc-open sets in a topological space X. Then $\cup\{A_\alpha : \alpha \in \Lambda\}$ is Bc-open.

Definition(1.11)[2]:

Let X be a space and $A \subset X$. A point $x \in X$ is said to Bc-interior point of A, if there exist a Bc-open set U such that $x \in U \subset A$. The set of all Bc-interior points of A is called Bc-interior of A and is denoted by $A^{\circ Bc}$.

Theorem(1.12)[2]:

Let X be a space and $A, B \subset X$, then the following statements are true.

1) $A^{\circ Bc}$ is the union of all Bc-open set which are contained in A.
2) $A^{\circ Bc}$ is Bc-open set in X.
3) A is Bc-open if and only if $A = A^{\circ Bc}$.
4) $A^{\circ Bc} \subset A$.
5) $(A^{\circ Bc})^{\circ Bc} = A^{\circ Bc}$.
6) If $A \subset B$, then $A^{\circ Bc} \subset B^{\circ Bc}$.
7) $A^{\circ Bc} \cup B^{\circ Bc} \subset (A \cup B)^{\circ Bc}$.
8) $(A \cap B)^{\circ Bc} \subset A^{\circ Bc} \cap B^{\circ Bc}$.

Definition(1.13)[2]:

Let X be a space and $A \subset X$. The Bc-closure of A is defined by the intersection of all Bc-closed sets in X containing A, and is denoted by $\bar{A}^{Bc}$.

Theorem(1.14)[2]:

Let X be a space and $A, B \subset X$. Then the following statements are true.

1) $\bar{A}^{Bc}$ is the intersection of all Bc-closed sets containing A.

2) $A \subset \bar{A}^{Bc}$.

3) $\bar{A}^{Bc}$is Bc-closed set in X.

4) A is Bc-closed set if and only if $A = \bar{A}^{Bc}$.

5) $\overline{(\bar{A}^{Bc})}^{Bc} = \bar{A}^{Bc}$.

6) If $A \subset B$. then $\bar{A}^{Bc} \subset \bar{B}^{Bc}$.

7) $\bar{A}^{Bc} \cup \bar{B}^{Bc} \subset \overline{(A \cup B)}^{Bc}$.

8) $\overline{(A \cap B)}^{Bc} \subset \bar{A}^{Bc} \cap \bar{B}^{Bc}$.

Proposition(1.15)[2]:

Let X be a space and $A \subset X$, then the following statements are true.

1) $(\bar{A}^{Bc})^c = (A^c)^{\circ Bc}$.

2) $(A^{\circ Bc})^c = \overline{(A^c)}^{Bc}$.

3) $\bar{A}^{Bc} = \left(A^{c \circ Bc}\right)^c$.

4) $A^{\circ Bc} = \left(\overline{A^c}^{Bc}\right)^c$.

Definition(1.16):

Let X be a space and $A \subset X$. Then A is called Bc-regular open set in X iff $A = \bar{A}^{Bc \circ Bc}$. The complement of Bc-regular open set is called Bc-regular closed.

Remark(1.17):

Let X be a space and $A \subset X$. A is Bc-regular closed set iff $A = \overline{A^{\circ Bc}}^{Bc}$.

Proof:

Let A be a Bc-regular closed set, then A^c is a Bc-regular open set $A = (A^c)^c = \left(\overline{A^c}^{Bc\circ Bc}\right)^c = \left(\overline{A^{c^c\circ Bc^{c^c}}}^{Bc^c}\right)^c = \overline{A^{\circ Bc}}^{Bc^{c^c}} = \overline{A^{\circ Bc}}^{Bc}$. Then $A = \overline{A^{\circ Bc}}^{Bc}$.

Conversely, let $A = \overline{A^{\circ Bc}}^{Bc}$. To prove A is a Bc-regular closed set we must prove that A^c is a Bc-regular open set. $A^c = \left(\overline{A^{\circ Bc}}^{Bc}\right)^c = \left(\overline{A^c}^{Bc^{c^c\circ Bc^c}}\right)^c = \overline{A^c}^{Bc\circ Bc^{c^c}} = \overline{A^c}^{Bc\circ Bc}$. Then A^c is a Bc-regular open set. Therefore A is Bc-regular closed set.

Remark(1.18):

Let X be a space and $A \subset X$. A is a Bc-regular open set, then $\bar{A}^{Bc^{\circ Bc}}$ is a Bc-regular open set.

Proof:

To prove $\bar{A}^{Bc^{\circ Bc}}$ is a Bc-regular open we must prove that $\bar{A}^{Bc^{\circ Bc}} = \overline{\bar{A}^{Bc^{\circ Bc}}}^{Bc^{\circ Bc}}$, since $A \subset \bar{A}^{Bc}$, then $A^{\circ Bc} \subset \bar{A}^{Bc^{\circ Bc}}$ and since A is a Bc-open set, hence $A \subset \bar{A}^{Bc^{\circ Bc}}$

$\bar{A}^{Bc^{\circ Bc}} \subset \overline{\bar{A}^{Bc^{\circ Bc}}}^{Bc^{\circ Bc}}$ … (1) Since $\bar{A}^{Bc^{\circ Bc}} \subset \bar{A}^{Bc}$, then $\overline{\bar{A}^{Bc^{\circ Bc}}}^{Bc} \subset \bar{A}^{Bc^{\circ Bc}} = \bar{A}^{Bc}$, hence $\overline{\bar{A}^{Bc^{\circ Bc}}}^{Bc^{\circ Bc}} \subset \bar{A}^{Bc^{\circ Bc}}$ … (2) From (1) and (2) we get $\bar{A}^{Bc^{\circ Bc}} = \overline{\bar{A}^{Bc^{\circ Bc}}}^{Bc^{\circ Bc}}$. Hence $\bar{A}^{Bc^{\circ Bc}}$ is a Bc-regular open.

Diagram I shows the relations among BcO(X) , BO(X), θO(X) and O(X)

θO(X)	→	O(X)
↓		↓
BcO(X)	→	BO(X)

Diagram I

2. ωBc-open sets and ωBc-regular open sets

Definition(2.1)[5]:

Let X be a space and $A \subset X$. Then A is said to be ωb-open if for every $x \in A$, there exists a b-open subset $U_x \subseteq X$ containing x such that $U_x - A$ is countable. The complement of an ωb-open subset is said to be ωb-closed.

Lemma(2.2)[5]:

Let X be a space and $A \subset X$. A is said to be ωb-open if and only if for every $x \in A$, there exists a b-open subset U containing x and a countable subset D such that $U - D \subseteq A$.

Definition(2.3):

Let X be a space and $A \subset X$. Then A is said to to be ωBc-open if for each $x \in A$, there exists a Bc-open subset $U_x \subseteq X$ containing x such that $U_x - A$ is countable. The complement of an ωBc-open subset is said to be ωBc-closed.

Lemma(2.4):

Every ωBc-open is ωb-open.

Proof:

Let A be an ωBc-open, then for each $x \in A$, there exists Bc-open U_x subset containing x such that $U_x - A$ is countable set. Since every Bc-open set is b-open, then A is ωb-open.

Lemma(2.5):

Let X be a space and $A \subset X$. A is said to be ωBc-open if and only if for every $x \in A$, there exists a Bc-open subset U containing x and a countable subset D such that $U - D \subseteq A$.

Proof:

Let A be an ωBc-open and $x \in A$, then there exists a Bc-open subset U_x containing x such that $|U_x - A|$ is countable. Let $D = U_x - A = U_x \cap A^c$, then $U_x - D \subseteq A$. Conversely, let $x \in A$, Then there exists a Bc-open subset U_x containing x and a countable subset D such that $U_x - D \subseteq A$. Thus $U_x - A \subseteq D$ and $U_x - A$ is countable set.

Theorem(2.6):

Let X be a space and $D \subseteq X$. If D is ωBc-closed, then $D \subseteq K \cup B$ for some Bc-closed subset K and a countable subset B.

Proof:

If D is ωBc-closed, then D^c is ωBc-open and hence for every $x \in D^c$, there exists a Bc-open set U containing x and a countable set B such that $U - B \subseteq D^c$. Thus $D \subseteq (U - B)^c = (U \cap B^c)^c = U^c \cup B$. Let $K = U^c$, then K is Bc-closed such that $D \subseteq K \cup B$.

Proposition(2.7):

The union of any family of ωBc-open is ωBc-open.

Proof:

If $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of ωBc-open subset of X, then for every $x \in \bigcup_{\alpha\in\Lambda} A_\alpha$, $x \in A_\beta$ for some $\beta \in \Lambda$. Hence there exists a Bc-open subset U of X containing x such that $U - A_\beta$ is countable. Now as $U - \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq U - A_\beta$ and thus $U - (\bigcup_{\alpha\in\Lambda} A_\alpha)$ is countable. Therefore $\bigcup_{\alpha\in\Lambda} A_\alpha$ is ωBc-open set.

Definition(2.8):

Let X be a space and $A \subset X$. Then A is said to be ωBc*-open if for every $x \in A$, there exists a Bc-open subset $U_x \subseteq X$ containing x such that $U_x - A$ is finite. The complement of an ωBc*-open subset is said to be ωBc*-closed.

Lemma(2.9):

Let X be a space and $A \subset X$. A is ωBc*-open if and only if for every $x \in A$, there exists a Bc-open subset U containing x and a finite subset D such that $U - D \subseteq A$.

Proof:

Let A be an ωBc*-open and $x \in A$, then there exists a Bc-open subset U_x containing x such that $U_x - A$ is finite. Let $D = U_x - A = U_x \cap (A)^c$. Then $U_x - D \subseteq A$. Conversely, let $x \in A$, then there exists a Bc-open subset U_x containing x and a finite subset D such that $U_x - D \subseteq A$, thus $U_x - A \subseteq D$ and $U_x - A$ is finite set.

Theorem(2.10):

Let X be a space and $D \subseteq X$ if D is ωBc*-closed, then $D \subseteq K \cup B$ for some Bc-closed subset K and a finite subset B.

Proof:

If D is ωBc*-closed, then D^c is ωBc*-open and hence for every $x \in D^c$, there exists a Bc-open set U containing x and a finite set B such that $U - B \subseteq D^c$, thus $D \subseteq (U - B)^c = (U \cap (B)^c)^c = U^c \cup B$. Let $K = U^c$. Then K is Bc-closed such that $D \subseteq K \cup B$.

Proposition(2.11):

The union of any family of ωBc*-open sets is ωBc*-open.

Proof:

If $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of ωBc*-open subset of X, then for every $x \in \bigcup_{\alpha\in\Lambda} A_\alpha$, $x \in A_\beta$ for some $\beta \in \Lambda$. Hence there exists Bc-open subset U of X containing x such that $U - A_\beta$ is finite. Now as $U - \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq U - A_\beta$ and thus $U - (\bigcup_{\alpha\in\Lambda} A_\alpha)$ is finite. Therefore, $\bigcup_{\alpha\in\Lambda} A_\alpha$ is ωBc*-open set.

The following diagram shows the implication for properties of subsets

ωBc*-open → ωBc-open
↑ ↗
Bc-open

Diagram II

Lemma(2.12):

Every ωBc*-open is ωBc-open.

Proof:

Let A be an ωBc*-open, then for each $x \in A$ there exists Bc-open subset $U_x \subseteq X$ containing x such that $U_x - A$ is finite. Since every finite is countable, then $U_x - A$ is countable. Therefore, A is a ωBc-open.

Lemma(2.13):

Every Bc-open is ωBc-open and ωBc*-open

Proof:

1)

Let A be a Bc-open, then for each $x \in A$ there exists Bc-open set $U_x = A$ containing x such that $U_x - A = \phi$, then $U_x - A$ is countable. Therefore, A is a ωBc-open.

2)

Let A be a Bc-open, then for each $x \in A$ there exists Bc-open set $U_x = A$ containing x such that $U_x - A = \phi$, then $U_x - A$ is finite. Therefore, A is a ωBc*-open.

Example(2.14):

Let $\mathbb{R}$ be the set of all real numbers with the usual topology and $\mathbb{Q}$ the set of all rational numbers. Then $A = \mathbb{R} - \mathbb{Q}$ is an ωBc-open set but it is not ωBc*-open.

Example(2.15):

Let $X = \{1,2,3\}, \tau = \{\phi, X, \{1\}, \{1,2\}, \{2,3\}\}$. Then $BcO(X) = \{\phi, X, \{1,3\}, \{2,3\}\}$. Therefore, $\{3\}$ is ωBc-open and ωBc*-open but it is not Bc-open.

Definition(2.16):

Let X be a space and $A \subset X$. Then A is ωBc-regular open if for each $x \in A$, there exists a Bc-regular open subset U_x containing x such that $U_x - A$ is a countable.

Lemma(2.17):

Every Bc-regular open is ωBc-regular open.

Proof:

Let A be a Bc-regular open, then for each $x \in A$ there exists Bc-regular open subset $U_x = A$ containing x such that $U_x - A = \phi$, then $U_x - A$ is countable. Therefore, A be a ωBc-regular open.

Lemma(2.18):

Let X be a space and $A \subset X$. A is ωBc-regular open if and only if for every $x \in A$, there exists a Bc-regular open subset U_x containing x and a countable subset D such that $U_x - D \subseteq A$.

Proof:

Let A be an ωBc-regular open and $x \in A$, then there exists a Bc-regular open subset U_x containing x such that $U_x - A$ is countable. Let $D = U_x - A = U_x \cap (A)^c$. Then $U_x - D \subseteq A$. Conversely, let $x \in A$. Then there exists a Bc-regular open subset U_x containing x and a countable subset D such that $U_x - D \subseteq A$. Thus $U_x - A \subseteq D$ and $U_x - A$ is countable.

Theorem(2.19):

Let X be a space and $F \subseteq X$. If F is ωBc-regular closed, then $F \subseteq K \cup D$ for some Bc-regular closed subset K and a countable subset F.

Proof:

If F is ωBc-regular closed, then F^c is ωBc-regular open and hence choose $x \in F^c$, there exists a Bc-regular open set U_x containing x and a countable set D_x such that $U_x - D_x \subseteq F^c$. Thus $F \subseteq (U_x - D_x)^c = (U_x \cap (D_x)^c)^c = {U_x}^c \cup D_x$. Let $K = {U_x}^c$. Then K is Bc-closed such that $F \subseteq K \cup D_x$.

Proposition(2.20):

The union of any family of ωBc-regular open is ωBc-regular open.

Proof:

If $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of ωBc-regular open subset of X, then for every $x \in \bigcup_{\alpha\in\Lambda} A_\alpha$, $x \in A_\beta$ for some $\beta \in \Lambda$. Hence there exists Bc-regular open subset U of X containing x such that $U - A_\beta$ is countable. Now as $U - \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq U - A_\beta$ and thus $U - (\bigcup_{\alpha\in\Lambda} A_\alpha)$ is countable. Therefore, $\bigcup_{\alpha\in\Lambda} A_\alpha$ is ωBc-regular open set.

Definition(2.21):

Let X be a space and $A \subset X$. Then A is is said to be ωBc*-regular open if for each $x \in A$, there exists a Bc-regular open subset U_x containing x such that $U_x - A$ is a finite set.

Lemma(2.22):

Let X be a space and $A \subset X$. A is ωBc*-regular open if and only if for every $x \in A$, there exist a Bc-regular open subset U containing x and a finite subset D such that $U - A$.

Proof:

Let A be an ωBc*-open and $x \in A$, then there exists a Bc- regular open subset U containing x such that $U - A$ is finite. Let $D = U - A = U \cap (A)^c$. Then $U - D \subseteq A$. Conversely, let $x \in A$, then there exists a Bc- regular open subset U containing x and a finite subset D such that $U - D \subseteq A$, thus $U - A \subseteq D$ and $U - A$ is finite set.

Theorem(2.23):

Let X be a space and $F \subseteq X$. If F is ωBc*-regular closed, then $F \subseteq K \cup D$ for some Bc-regular closed subset K and a finite subset F.

Proof:

If F is ωBc*-regular closed, then F^c is ωBc*-regular open and hence choose $x \in F^c$, there exists a Bc-regular open set U_x containing x and a finite set D_x such that $U_x - D_x \subseteq F^c$. Thus $F \subseteq (U_x - D_x)^c = (U_x \cap (D_x)^c)^c = {U_x}^c \cup D_x$. Let $K = {U_x}^c$. Then K is Bc-closed such that $D \subseteq K \cup D_x$.

Proposition(2.24):

The union of any family of ωBc*-regular open sets is ωBc*-regular open.

Proof:

If $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of ωBc*-regular open subset of X, then for every $x \in \bigcup_{\alpha\in\Lambda} A_\alpha$, $x \in A_\beta$ for some $\beta \in \Lambda$. Hence there exists Bc-regular open subset U of X containing x such that $U - A_\beta$ is finite. Now as $U - \bigcup_{\alpha\in\Lambda} A_\alpha \subseteq U - A_\beta$ and thus $U - (\bigcup_{\alpha\in\Lambda} A_\alpha)$ is finite. Therefore, $\bigcup_{\alpha\in\Lambda} A_\alpha$ is ωBc*-regular open set.

Definition(2.25)[6]:

A covering of a space X is the family $\{A_\alpha : \alpha \in \Lambda\}$ of subsets such that $\bigcup_{\alpha\in\Lambda} A_\alpha = X$. If each $A_\alpha$is open, then $\{A_\alpha : \alpha \in \Lambda\}$ is called an open covering, and if each set $A_\alpha$is closed, then $\{A_\alpha : \alpha \in \Lambda\}$ is called a closed covering. A covering $\{B_\gamma : \gamma \in \Gamma\}$ is said to be refinement of a covering $\{A_\alpha : \alpha \in \Lambda\}$ if for each γ in Γ there exists some α in Λ such that $B_\gamma \subset A_\alpha$.

Definition(2.26):

Let $f: X \to Y$ be a function

1) f is called Bc-continuous function if $f^{-1}(A)$ is Bc-open subset of X for each θ-open subset A of Y.

2) f is called Bc*-continuous function if $f^{-1}(A)$ is Bc-open subset of X for each Bc-open subset A of Y.

3) f is called ωBc*-closed function if $f(A)$ is ωBc*-closed of Y for each Bc-closed set A of X.

4) f is called ωBc-continuous function if $f^{-1}(A)$ is ωBc-open subset of X for each θ-open subset A of Y.

5) f is called ωBc*-continuous function if $f^{-1}(A)$ is ωBc-open subset of X for each Bc-open subset A of Y.

6) f is called ωBc**-continuous function if $f^{-1}(A)$ is ωBc*-open subset of X for each θ-open subset A of Y.

Remark(2.27):

Every ωBc**-continuous is ωBc-continuous.

Proof:

Let $f: X \rightarrow Y$ be a function and let A be an θ-open of Y. Since f is an ωBc**-continuous function, then $f^{-1}(A)$ is an ωBc*-open of X. Since every ωBc*-open is ωBc-open, then $f^{-1}(A)$ is an ωBc-open of X. Thus f is an ωBc-continuous.

Definition(2.28):

Let $f: X \rightarrow Y$ be a function

1) f is called BcR-continuous function if $f^{-1}(A)$ is Bc-regular open subset of X for each θ-open subset A of Y.

2) f is called Bc*R-continuous function if $f^{-1}(A)$ is Bc-regular open subset of X for each Bc-open subset A of Y.

3) f is called ωBc*R-closed function if $f(A)$ is ωBc*-regular closed of Y for each Bc-closed set A of X.

4) f is called ωBcR-continuous function if $f^{-1}(A)$ is ωBc-regular open subset of X for each θ-open subset A of Y.

5) f is called ωBc*R-continuous function if $f^{-1}(A)$ is ωBc-regular open subset of X for each Bc-open subset A of Y.

6) f is called ωBc**R-continuous function if $f^{-1}(A)$ is ωBc*-regular open subset of X for each θ-open subset A of Y.

Remark(2.29):

Every ωBc**R-continuous is ωBcR-continuous.

Proof:

Let $f: X \rightarrow Y$ be a function and let A be an θ-open of Y. Since f is an ωBc**R-continuous function, then $f^{-1}(A)$ is an ωBc*-regular open of X. Since every ωBc*-regular open is ωBc-regular open, then $f^{-1}(A)$ is an ωBc-regular open of X. Thus f is an ωBcR-continuous.

3.Bc-Lindelof space and nearly Bc-Lindelof space

Definition(3.1):

A space X is said to be θ-Lindelof if every θ-open cover of X has a countable subcover.

Definition(3.2):

1) A space X is said to be Bc-Lindelof if every Bc-open cover of X has a countable subcover.

2) For a subset B of a space X is said to be Bc-Lindelof relative to X if every cover of B by Bc-open sets of X has a countable subcover.

Theorem(3.3):

For any space X, the following properties are equivalent:

1) X is Bc-Lindelof.

2) Every ωBc-open cover of X has a countable subcover.

Proof:

1→2

Let $\{G_\alpha: \alpha \in \Lambda\}$ be any ωBc-open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in G_{\alpha(x)}$. Since $G_{\alpha(x)}$ is ωBc-open, there exists Bc-open set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} - G_{\alpha(x)}$ is a countable. The family $\{V_{\alpha(x)}: x \in X\}$ is a Bc-open cover of X. Since X is Bc-Lindelof, then there exists a countable subset, say $\alpha(x_1), \dots, \alpha(x_n), \dots$ such that $X = \bigcup\{V_{\alpha(xi)}: i \in N\}$. Now, we have

$$X = \bigcup_{i\in N}\{(V_{\alpha(xi)} - G_{\alpha(xi)}) \cup G_{\alpha(xi)}\}$$
$$= (\bigcup_{i\in N}(V_{\alpha(xi)} - G_{\alpha(xi)})) \cup (\bigcup_{i\in N} G_{\alpha(xi)}).$$

For each $\alpha(xi), V_{\alpha(xi)} - G_{\alpha(xi)}$ is a countable set and there exists a countable subset $\Lambda_{\alpha(xi)}$ of Λ such that $V_{\alpha(xi)} - G_{\alpha(xi)} \subseteq \bigcup\{G_\alpha: \alpha \in \Lambda_{\alpha(xi)}\}$. Therefore, we have $X \subseteq (\bigcup_{i\in N}(\bigcup\{G_\alpha: \alpha \in \Lambda_{\alpha(xi)}\})) \cup (\bigcup_{i\in N} G_{\alpha(xi)})$.

2→1

Let $\{G_\alpha: \alpha \in \Lambda\}$ be any Bc-open cover of X. To prove X is Bc-Lindelof, since every Bc-open is ωBc-open by lemma(2.13). By(2), then $\{G_\alpha: \alpha \in \Lambda\}$ is ωBc-open cover of X has a countable subcover. Therefore, X is Bc-Lindelof.

Proposition(3.4):

For any space X, the following properties are equivalent:

1) X is Bc-Lindelof.

2) Every ωBc*-open cover of X has a countable subcover.

Proof:

1→2

Let $\{G_\alpha : \alpha \in \Lambda\}$ be any ωBc*-open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in G_{\alpha(x)}$. Since $G_{\alpha(x)}$ is ωBc*-open, there exists Bc-open set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} - G_{\alpha(x)}$ is a finite. Since every finite is countable, then $V_{\alpha(x)} - G_{\alpha(x)}$ is a countable. The family $\{V_{\alpha(x)} : x \in X\}$ is a Bc-open cover of X and since X is Bc-Lindelof, then there exists a countable subset, say $\alpha(x_1), \dots, \alpha(x_n), \dots$. such that $X = \bigcup\{V_{\alpha(xi)} : i \in N\}$. Now, we have

$$X = \bigcup_{i \in N}\{(V_{\alpha(xi)} - G_{\alpha(xi)}) \cup G_{\alpha(xi)}\}$$
$$= \left(\bigcup_{i \in N}(V_{\alpha(xi)} - G_{\alpha(xi)})\right) \cup \left(\bigcup_{i \in N} G_{\alpha(xi)}\right).$$

For each $\alpha(xi)$, $V_{\alpha(xi)} - G_{\alpha(xi)}$ is a countable set and there exists a countable subset $\Lambda_{\alpha(xi)}$ of Λ such that $V_{\alpha(xi)} - G_{\alpha(xi)} \subseteq \bigcup\{G_\alpha : \alpha \in \Lambda_{\alpha(xi)}\}$. Therefore, we have $X \subseteq \left(\bigcup_{i \in N}\left(\bigcup\{G_\alpha : \alpha \in \Lambda_{\alpha(xi)}\}\right)\right) \cup \left(\bigcup_{i \in N} G_{\alpha(xi)}\right)$.

2→1

Let $\{G_\alpha : \alpha \in \Lambda\}$ be any Bc-open cover of X. To prove X is Bc-Lindelof, since every Bc-open is ωBc-open, then $\{G_\alpha : \alpha \in \Lambda\}$ is ωBc-open cover of X has a countable subcover by theorem(3.3). Therefore, X is Bc-Lindelof.

Proposition(3.5):

If X is a space such that every Bc-open subset of X is a Bc-Lindelof relative to X, then every subset is Bc-Lindelof relative to X.

Proof:

Let B be an arbitrary subset of X and let $\{U_i : i \in \mathrm{I}\}$ be a cover of B by Bc-open set. Then the family $\{U_i : i \in \mathrm{I}\}$ is a Bc-open cover of the Bc-open set $\bigcup\{U_i : i \in \mathrm{I}\}$ by proposition(1.10). Hence by assumption there is a countable subfamily $\{U_{ij} : j \in \mathrm{I}\}$ which covers $\bigcup\{U_i : i \in \mathrm{I}\}$. This subfamily is also a cover of the set B.

Proposition(3.6):

For any space X, the following statements are equivalent:

1) X is Bc-Lindelof.

2) Every family of ωBc-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$, then there exists a countable subset $\Lambda_\circ \subseteq \Lambda$ such that $\bigcap_{\alpha\in \Lambda_\circ} F_\alpha = \phi$.

3) Every family of ωBc*-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$, then there exists a countable subset $\Lambda_\circ \subseteq \Lambda$ such that $\bigcap_{\alpha\in \Lambda_\circ} F_\alpha = \phi$.

4) Every family of Bc-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$, then there exists a countable subset $\Lambda_\circ \subseteq \Lambda$ such that $\bigcap_{\alpha\in \Lambda_\circ} F_\alpha = \phi$.

Proof:

1→2

Suppose that X is Bc-Lindelof. Let $\{F_\alpha : \alpha \in \Lambda\}$ be a family of ωBc-closed subsets of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$. Then the family $\{F_\alpha{}^c : \alpha \in \Lambda\}$ is ωBc-open cover of Bc-Lindelof space X, there exists a countable subset $\Lambda_\circ$ of Λ such that $X = \bigcup\{F_\alpha{}^c : \alpha \in \Lambda_\circ\}$.Then $\phi = (\bigcup\{F_\alpha{}^c : \alpha \in \Lambda_\circ\})^c = \bigcap\{(F_\alpha{}^c)^c : \alpha \in \Lambda_\circ\} = \bigcap\{F_\alpha : \alpha \in \Lambda_\circ\}$

2→3

It is clear since every ωBc*-closed is ωBc-closed.

3→4

It is clear since every Bc-closed is ωBc*-closed.

4→1

Let $\{G_\alpha : \alpha \in \Lambda\}$ be any a Bc-open cover of X. Then $\{G_\alpha{}^c : \alpha \in \Lambda\}$ is a family of Bc-closed subset of X with $\bigcap\{G_\alpha{}^c : \alpha \in \Lambda\} = \phi$. By assumption, there exists a countable subset $\Lambda_\circ$ of Λ , then $\bigcap\{G_\alpha{}^c : \alpha \in \Lambda_\circ\} = \phi$. So that $X = (\bigcap\{G_\alpha{}^c : \alpha \in \Lambda_\circ\})^c = \bigcup\{G_\alpha : \alpha \in \Lambda_\circ\}$. Hence X is Bc-Lindelof.

Theorem(3.7):

A space X is Bc-Lindelof if and only if for every collection of Bc-closed sets with countable intersection property $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$.

Proof:

Let X is Bc-Lindelof and $\{F_\alpha: \alpha \in \Lambda\}$ be a collection of Bc-closed sets with countable intersection property, $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Suppose that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$. Then $X = \bigcup_{\alpha\in\Lambda} {F_\alpha}^c$ where ${F_\alpha}^c$ is Bc-open set for each $\alpha \in \Lambda$. Therefore, $\{{F_\alpha}^c: \alpha \in \Lambda\}$ is Bc-open cover of X which is a Bc-Lindelof, there exist countable many members $\alpha_1, \dots, \alpha_n, \dots$ such that $X = \bigcup_{i\in N} {F_{\alpha i}}^c = (\bigcap_{i\in N} F_{\alpha i})^c$

$\bigcap_{i\in N} F_{\alpha i} = F_{\alpha 1} \cap \dots \cap F_{\alpha n} \cap \dots = \phi$

which is a contradiction with our assumption that $\{F_\alpha: \alpha \in \Lambda\}$ has a countable intersection property. Hence $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Conversely, let every collection of Bc-closed subset of X with the countable intersection property, $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Suppose that X is not Bc-Lindelof, then there exist Bc-open cover $\{G_\alpha: \alpha \in \Lambda\}$ of X has no countable subcover $\{G_{\alpha 1}, \dots, G_{\alpha n}, \dots\}$ i.e $X = G_{\alpha 1} \cup \dots \cup G_{\alpha n} \cup \dots$ .Then $(\bigcup_{i\in N} G_{\alpha i})^c = \bigcap_{i\in N} {G_{\alpha i}}^c \neq \phi$. But $\{{G_\alpha}^c: \alpha \in \Lambda\}$ be a collection of Bc- closed of X with countable intersection property by assumption. Then $\bigcap_{\alpha\in\Lambda} {G_\alpha}^c \neq \phi$, $(\bigcup_{\alpha\in\Lambda} G_\alpha)^c \neq \phi$ which is a contradiction that G is Bc-open cover of X. Thus must have countable subcover. Hence X is Bc-Lindelof.

Theorem(3.8):

Every ωBc-closed subset of a Bc-Lindelof space of X is Bc-Lindelof relative to X.

Proof:

Let A be an ωBc-closed subset of X. Let $\{G_\alpha: \alpha \in \Lambda\}$ be a cover of A by Bc-open set of X. Now, for each $x \in A^c$, there is a Bc-open set V_x such that $V_x \cap A$ is a countable. Since X is Bc-Lindelof and the collection $\{G_\alpha: \alpha \in \Lambda\} \cup \{V_x: x \in A^c\}$ is a Bc-open cover of X, there exists a countable subcover $\{G_{\alpha i}: i \in N\} \cup \{V_{xi}: i \in N\}$. Since $\bigcup_{i\in N}(V_{xi} \cap A)$ is countable, so for each $x_j \in \bigcup(V_{xi} \cap A)$, there is $G_{\alpha(xj)} \in \{G_\alpha: \alpha \in \Lambda\}$ such that $x_j \in G_{\alpha(xj)}$ and $j \in N$. Hence $\{G_{\alpha i}: i \in N\} \cup \{G_{\alpha(xj)}: j \in N\}$ is a countable subcover of $\{G_\alpha: \alpha \in \Lambda\}$ and it covers A. Therefore, A is Bc-Lindelof relative to X.

Theorem(3.9):

Every Bc-closed subset of a Bc-Lindelof space of X is Bc-Lindelof relative to X.

Proof:

Let A be an Bc-closed subset of X. Then A^c is Bc-open ,since every Bc-open is ωBc-open. Therefore, A is ωBc-closed by theorem(3.8), then A is Bc-Lindelof relative to X.

Proposition(3.10):

Every ωBc*-closed subset of a Bc-Lindelof space of X is Bc-Lindelof relative to X.

Proof:

Let A be an ωBc*-closed subset of X. Then A^c is ωBc*-open ,since every ωBc*-open is ωBc-open. Therefore, A is ωBc-closed by theorem(3.8), then A is Bc-Lindelof relative to X.

Theorem(3.11):

Let $f: X \to Y$ be a Bc-continuous and onto function. If X is a Bc-Lindelof, then Y is an θ-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be an θ-open cover of Y and since f is Bc-continuous function, then $\{f^{-1}(G_\alpha): \alpha \in \Lambda\}$ is Bc-open cover of X. Since X is Bc-Lindelof, then X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \dots, f^{-1}(G_{\alpha n}), \dots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \dots, G_{\alpha n}, \dots\}$ is a countable subcover of Y. Hence Y is an θ-Lindelof.

Theorem(3.12):

Let $f: X \to Y$ be a Bc*-continuous and onto function. If X is a Bc-Lindelof, then Y is a Bc-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a Bc-open cover of Y and since f is Bc*-continuous function, then $\{f^{-1}(G_\alpha): \alpha \in \Lambda\}$ is Bc-open cover of X. Since X is Bc-Lindelof, then X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \dots, f^{-1}(G_{\alpha n}), \dots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \dots, G_{\alpha n}, \dots\}$ is a countable subcover of Y. Hence Y is a Bc-Lindelof.

Theorem(3.13):

Let $f: X \to Y$ be a ωBc-continuous and onto function. If X is a Bc-Lindelof, then Y is an θ-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be an θ-open cover of Y and since f is ωBc-continuous function, then $\{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is ωBc-open cover of X. Since X is Bc-Lindelof, then by theorem(3.4), X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \dots, f^{-1}(G_{\alpha n}), \dots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \dots, G_{\alpha n}, \dots\}$ is a countable subcover of Y. Hence Y is an θ-Lindelof.

Theorem(3.14):

Let $f: X \to Y$ be a ωBc*-continuous and onto function. If X is a Bc-Lindelof, then Y is a Bc-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a Bc-open cover of Y and since f is ωBc*-continuous function, then $\{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is ωBc-open cover of X. Since X is Bc-Lindelof, then by theorem(3.4), X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \dots, f^{-1}(G_{\alpha n}), \dots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \dots, G_{\alpha n}, \dots\}$ is a countable subcover of Y. Hence Y is a Bc-Lindelof.

Theorem(3.15):

Let $f: X \to Y$ be an ωBc**-continuous and onto function. If X is a Bc-Lindelof, then Y is a Bc-Lindelof.

Proof:

It is clear since every ωBc**-continuous is ωBc-continuous.

Proposition(3.16):

If $f: X \to Y$ be an ωBc*-closed onto such that $f^{-1}(y)$ is Bc-Lindelof relative to X and Y is a Bc-Lindelof for each $y \in Y$, then X is a Bc-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be an Bc-open cover of X. For each $y \in Y$, $f^{-1}(y)$ is Bc-Lindelof relative to X and there exists a countable subset $\Lambda_1(y)$ of Λ such that $f^{-1}(y) \subset \cup\{G_\alpha : \alpha \in \Lambda_1(y)\}$. Now we put $G(y) = \{G_\alpha : \alpha \in \Lambda_1(y)\}$ and $V(y) = Y - f(U(y)^c)$. Then, since f is an ωBc*-closed, $V(y)$ is an ωBc*-open set in Y containing y such that $f^{-1}(V(y)) \subset U(y)$. Since $V(y)$ is an ωBc*-open, there exists a Bc-open set $W(y)$ containing y such that $W(y) - V(y)$ is a countable set. For each $y \in Y$, we have $W(y) \subset \big(W(y) - V(y)\big) \cup V(y)$ and hence

$$f^{-1}\big(W(y)\big) \subset \big[f^{-1}\big(W(y) - V(y)\big)\big] \cup f^{-1}\big(V(y)\big)$$
$$\subset f^{-1}\big(W(y) - V(y)\big) \cup G(y)$$

since $W(y) - V(y)$ is a countable set and $f^{-1}(y)$ is Bc-Lindelof relative to X , there exists a countable set $\Lambda_2(y)$ of Λ such that

$$f^{-1}\big(W(y) - V(y)\big) \subset \cup\{G_\alpha : \alpha \in \Lambda_2(y)\}$$

and hence $f^{-1}\big(W(y)\big) \subset [\cup\{G_\alpha : \alpha \in \Lambda_2(y)\}] \cup [G(y)]$. Since $\{W(y) : y \in Y\}$ is Bc-open cover of the Bc-Lindelof space Y, there exists a countable points of Y, say $y_1, \dots, y_n, \dots$ such that $Y = \cup\{W(yi) : i \in N\}$. Therefore, we obtain

$$X = \cup_{i\in N} f^{-1}\big(W(yi)\big) = \cup_{i\in N}\big[\cup_{\alpha\in\Lambda_2(yi)} G_\alpha\big] \cup \big(\cup_{\alpha\in\Lambda_1(yi)} G_\alpha\big)$$
$$= \cup\{G_\alpha : \alpha \in \Lambda_1(yi) \cup \Lambda_2(yi), i \in N\}.$$

Hence X is Bc-Lindelof.

Definition(3.17):

1) A space X is said to be nearly Bc-Lindelof if every Bc-regular open cover of X has a countable subcover.

2) For a subset B of a space X is said to be nearly Bc-Lindelof relative to X if every cover of B by Bc-regular open sets of X has a countable subcover of B.

Theorem(3.18):

For any space X, the following properties are equivalent:

1) X is nearly Bc-Lindelof.

2) Every ωBc-regular open cover of X has a countable subcover.

Proof:

1→2

Let $\{G_\alpha : \alpha \in \Lambda\}$ be any ωBc-regular open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in G_{\alpha(x)}$. Since $G_{\alpha(x)}$ is ωBc-regular open, there exists Bc-regular open set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} - G_{\alpha(x)}$ is a countable. The family $\{V_{\alpha(x)} : x \in X\}$ is a Bc-regular open cover of X. Since X is nearly Bc-Lindelof, then there exists a countable subset, say $\alpha(x_1), \dots, \alpha(x_n), \dots$ such that $X = \bigcup\{V_{\alpha(xi)} : i \in N\}$. Now, we have

$$X = \bigcup_{i\in N}\{(V_{\alpha(xi)} - G_{\alpha(xi)}) \cup G_{\alpha(xi)}\}$$
$$= \left(\bigcup_{i\in N}(V_{\alpha(xi)} - G_{\alpha(xi)})\right) \cup \left(\bigcup_{i\in N} G_{\alpha(xi)}\right).$$

For each $\alpha(xi)$, $V_{\alpha(xi)} - G_{\alpha(xi)}$ is a countable set and there exists a countable subset $\Lambda_{\alpha(xi)}$ of Λ such that $V_{\alpha(xi)} - G_{\alpha(xi)} \subseteq \bigcup\{G_\alpha : \alpha \in \Lambda_{\alpha(xi)}\}$.Therefore, we have $X \subseteq \left(\bigcup_{i\in N}\left(\bigcup\{G_\alpha : \alpha \in \Lambda_{\alpha(xi)}\}\right)\right) \cup \left(\bigcup_{i\in N} G_{\alpha(xi)}\right)$.

2→1

Let $\{G_\alpha : \alpha \in \Lambda\}$ be any Bc-regular open cover of X. To prove X is nearly Bc-Lindelof, since every Bc-regular open is ωBc-regular open by (2) ,then $\{G_\alpha : \alpha \in \Lambda\}$ is ωBc-regular open cover of X has a countable subcover. Therefore, X is nearly Bc-Lindelof.

Proposition(3.19):

A space X is nearly Bc-Lindelof if and only if every family $\{F_\alpha : \alpha \in \Lambda\}$ of ωBc-regular closed sets has countable intersection property $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$.

Proof:

Let X be a nearly Bc-Lindelof space and suppose that $\{F_\alpha : \alpha \in \Lambda\}$ be a family of ωBc-regular closed sets with countable intersection property, $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$. Let us consider the ωBc-regular open sets $G_\alpha = {F_\alpha}^c$,the family $\{G_\alpha : \alpha \in \Lambda\}$ is ωBc-regular open cover of X. Since X is nearly Bc-Lindelof, the cover $\{G_\alpha : \alpha \in \Lambda\}$ has a countable subcover $\{G_{\alpha i} : i \in N\}$. Therefore, $X = \bigcup\{G_{\alpha i} : i \in N\}$

$$= \bigcup\{{F_{\alpha i}}^c : i \in N\}$$
$$= (\bigcap\{F_{\alpha i} : i \in N\})^c.$$

Then $\bigcap\{F_{\alpha i} : i \in N\} = \phi$. Thus, if the family $\{F_\alpha : \alpha \in \Lambda\}$ of of ωBc-regular closed sets with countable intersection property, then $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Conversely, Let $\{G_\alpha : \alpha \in \Lambda\}$ be an ωBc-regular open cover of X and suppose that every family $\{F_\alpha : \alpha \in \Lambda\}$ of ωBc-regular closed sets has countable intersection property, $\bigcap_{\alpha\in\Lambda} F_\alpha \neq \phi$. Then $X = \bigcup\{G_\alpha: \alpha \in \Lambda\}$. Therefore, $\phi = X^c = \bigcap\{{G_\alpha}^c: \alpha \in \Lambda\}$ and $\{{G_\alpha}^c: \alpha \in \Lambda\}$ is a family of ωBc-regular closed sets with an empty intersection. By assumption, there exists a countable subset $\{{G_{\alpha i}}^c: i \in N\}$ such that $\bigcap\{{G_{\alpha i}}^c: i \in N\} = \phi$. Hence $(\bigcap\{{G_{\alpha i}}^c: i \in N\})^c = X = \bigcup\{G_{\alpha i}: i \in N\}$. Thus X is nearly Bc-Lindelof.

Theorem (3.20):

Every ωBc-regular closed subset of a nearly Bc-Lindelof space X is nearly Bc-Lindelof relative to X.

Proof:

Let A be an ωBc-regular closed subset of X. Let $\{G_\alpha: \alpha \in \Lambda\}$ be a cover by Bc-regular open sets of X. Now, for each $x \in A^c$, there is a Bc-regular open set V_x such that $V_x \cap A$ is a countable. Since X is nearly Bc-Lindelof and the collection $\{G_\alpha: \alpha \in \Lambda\} \cup \{V_x: x \in A^c\}$ is a Bc-regular open cover of X, there exists a countable subcover $\{G_{\alpha i}: i \in N\} \cup \{V_{xi}: i \in N\}$. Since $\bigcup_{i\in N}(V_{xi} \cap A)$ is a countable, so for each $x_j \in \bigcup(V_{xi} \cap A)$, there is $G_{\alpha(xj)} \in \{G_\alpha: \alpha \in \Lambda\}$ such that $x_j \in G_{\alpha(xj)}$ and $j \in N$. Hence $\{G_{\alpha i}: i \in N\} \cup \{G_{\alpha(xj)}: i \in N\}$ is a countable subcover of $\{G_\alpha: \alpha \in \Lambda\}$ and it covers A. Therefore, A is nearly Bc-Lindelof relative to X.

Theorem(3.21):

Let $f: X \to Y$ be an BcR-continuous and onto function. If X is a nearly Bc-Lindelof, then Y is an θ-Lindelof.

Proof:

Let $\{G_\alpha: \alpha \in \Lambda\}$ be an θ-open cover of Y and since f is BcR-continuous function, then $\{f^{-1}(G_\alpha): \alpha \in \Lambda\}$ is Bc-regular open cover of X. Since X is nearly Bc-Lindelof, then X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \ldots, f^{-1}(G_{\alpha n}), \ldots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \ldots, G_{\alpha n}, \ldots\}$ is a countable sub cover of Y. Hence Y is an θ-Lindelof.

Theorem(3.22):

Let $f: X \rightarrow Y$ be a Bc*R-continuous and onto function. If X is a nearly Bc-Lindelof, then Y is a Bc-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a Bc-open cover of Y and since f is Bc*R-continuous function, then $\{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is Bc-regular open cover of X. Since X is nearly Bc-Lindelof, then X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \ldots, f^{-1}(G_{\alpha n}), \ldots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \ldots, G_{\alpha n}, \ldots\}$ is a countable subcover of Y. Hence Y is a Bc-Lindelof.

Theorem(3.23):

Let $f: X \rightarrow Y$ be a ωBcR-continuous and onto function. If X is a nearly Bc-Lindelof, then Y is a θ-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a θ-open cover of Y and since f is ωBcR-continuous function, then $\{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is ωBc-regular open cover of X. Since X is Bc-nearly Bc-Lindelof, then by theorem(3.18), X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \ldots, f^{-1}(G_{\alpha n}), \ldots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \ldots, G_{\alpha n}, \ldots\}$ is a countable sub cover of Y. Hence Y is a θ-Lindelof.

Theorem(3.24):

Let $f: X \rightarrow Y$ be a ωBc*R-continuous and onto function. If X is a nearly Bc-Lindelof, then Y is a Bc-Lindelof.

Proof:

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a Bc-open cover of Y and since f is ωBc*R-continuous function, then $\{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is ωBc-regular open cover of X. Since X is nearly Bc-Lindelof, then by theorem(3.18), X has a countable subcover $\{f^{-1}(G_{\alpha 1}), \ldots, f^{-1}(G_{\alpha n}), \ldots\}$. Since f is onto, then $f(f^{-1}(G_\alpha)) = G_\alpha$ for each $\alpha \in \Lambda$. Therefore, $\{G_{\alpha 1}, \ldots, G_{\alpha n}, \ldots\}$ is a countable subcover of Y. Hence Y is a Bc-Lindelof.

Theorem(3.25):

Let $f: X \rightarrow Y$ be an ωBc**R-continuous and onto function. If X is a nearly Bc-Lindelof, then Y is a θ-Lindelof.

Proof:

It is clear since every ωBc**R-continuous is ωBcR-continuous.

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المستخلص

نناقش في هذا البحث صنف جديد من المجاميع المفتوحة من النمط- ωBc والمجاميع المفتوحة المنتظمة من النمط- ωBc . بالاضافة الى ذلك Bc واللندلوف تقريبا من النمط- Bc ظهرت خلال هذا العمل مفاهيم جديدة وتتضمن اللندلوف من النمط- درسنا سلوك هذه الصفات تحت تأثير بعض انواع معينه من الدوال.