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Some Properties of The Difference Quadruple Sequences Spaces of Fuzzy Complex Numbers Related to The Space $\left(\ell_{\wp}\right)^4$ Defined by Triple Maximal Orlicz Functions

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ABSTRACT

In this paper, we present some properties of the difference for quadruple sequences spaces of fuzzy complex numbers related to the space $\left(\ell_{\wp}\right)^4$ defined by triple maximal Orlicz functions and discuss some properties such as completeness, solidity, monotonicity, symmetric, convergence-freeness, and so on.

MSC..

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1. introduction

Sargent ([3],[4]) was the first to use the space m(φ). He looked at some of the properties of the space m(φ). Later, it was studied from the perspective of sequence space, and Rath and Tipathy [2],Tripathy and Sen ([7],[8],[9]), Tripathy and Mahanta [6], and others characterized some matrix classes with one member as m(φ).

In this paper , we introduce the class of the quadruple sequences spaces of fuzzy complex numbers related to the space $\left(\ell_{\wp}\right)^4$ defined by the triple maximal Orlicz function $m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4}$. Definitions and preliminaries which are needed in our work have been provided in Section two. In the third section , we look at some of the properties of the class .

P(Q) denotes the set of all permutations of the element of $(\mathfrak{Q}_{\mathfrak{nmtj}})$,, i.e. $\mathbb{P}(\mathfrak{Q}) =$

$\{(\mathfrak{Q}_{\pi(\mathfrak{n})\pi(\mathfrak{m})\pi(\mathfrak{t})\pi(\mathfrak{j})}): \pi$ be a permutation on $\mathbb{N}\}$, in which N be a set of natural numbers..

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Assume that $\mathfrak{Y}_{\mathfrak{srec}}$ is a class of all $\mathbb{N}$ subsets that do not have more than $\mathfrak{s}$ elements. For all $\mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$, $(\varphi_{\mathfrak{nmtj}})$ is a non-decreasing triple sequence of positive complex numbers with the form $\mathfrak{nmtj}\varphi_{(\mathfrak{n}+1)(\mathfrak{m}+1)(\mathfrak{t}+1)(\mathfrak{j}+1)} \preccurlyeq (\mathfrak{n}+1)(\mathfrak{m}+1)(\mathfrak{t}+1)(\mathfrak{j}+1)\varphi_{\mathfrak{nmtj}}$.

2. Definitions and Preliminaries

If $\mathfrak{W}_1 \prec \mathfrak{M}_2$ implies $\mathbb{G}(\mathfrak{M}_1) \preccurlyeq \mathbb{G}(\mathfrak{M}_2)$, $\forall\, \mathfrak{M}_1, \mathfrak{M}_2 \in \mathbb{R}$ then the map $\mathbb{G} : \mathbb{R} \to \mathbb{R}$ is called non-decreasing .

If $(\mathfrak{W}_1, \mathfrak{M}_2) \prec (\mathfrak{N}_1, \mathfrak{N}_2)$ implies $\mathbb{G}\,(\mathfrak{M}_1, \mathfrak{M}_2) \preccurlyeq \mathbb{G}\,(\mathfrak{N}_1, \mathfrak{N}_2)$, $\forall (\mathfrak{W}_1, \mathfrak{M}_2)\,, (\mathfrak{N}_1, \mathfrak{N}_2) \in \mathbb{R} \times \mathbb{R}$ then the map $\mathbb{G} : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ is said to be non-decreasing .

If the inequality $\mathbb{G}\left(\frac{\mathfrak{N}_1 + \mathfrak{N}_2}{2}\right) \preccurlyeq \frac{1}{2}\left(\mathbb{G}(\mathfrak{N}_1) + \mathbb{G}(\mathfrak{N}_2)\right), \forall\, \mathfrak{N}_1, \mathfrak{N}_2 \in \mathbb{R}$ then the real-valued function $\mathbb{G}(\mathfrak{N})$ of the real variable $\mathfrak{N}$ is called convex .

If $\forall\, \varepsilon \succ 0, \exists\, \varsigma \succ 0 \ni |\mathbb{G}(\mathfrak{N}) - \mathbb{G}(\mathbb{a})| \prec \varepsilon, \forall\, \mathfrak{N} \in (\mathbb{a}\,, \mathbb{a} + \varsigma)$ then the map $\mathbb{G} : \mathbb{R} \to \mathbb{R}$ is said to be a continuous from the right at $\mathbb{a}$.

An Orlicz function is a function $\mathcal{M}: [0, \infty) \to [0, \infty)$, which is a continuous, non-decreasing , and convex with $\mathcal{M}(0) = 0, \mathcal{M}(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\mathcal{M}(\mathfrak{A}) \to \infty$ as $\mathfrak{A} \to \infty$.

A maximal Orlicz function is a function $\mathcal{H}: [0, \infty) \to [0, \infty) \ni \mathcal{H}(\mathfrak{A}) = \mathfrak{A}^2 \mathcal{M}(\mathfrak{A})$ and $\mathcal{M}$ is Orlicz function , which is a continuous , non-decreasing and convex with $\mathcal{H}(0) = 0, \mathcal{H}(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\mathcal{H}(\mathfrak{A}) \to \infty$ as $\mathfrak{A} \to \infty$.

A triple maximal Orlicz function is a function $\mathbb{M}:[0,\infty)\times[0,\infty)\times[0,\infty) \to [0,\infty)\times[0,\infty)\times[0,\infty) \ni \mathbb{M}(\mathfrak{A}\,, \mathfrak{S}\,, \mathfrak{R}) = \left(\mathbb{M}_1(\mathfrak{A}), \mathbb{M}_2(\mathfrak{S}), \mathbb{M}_3(\mathfrak{R})\right)$, where $\mathbb{M}_1 : [0, \infty) \to [0, \infty) \ni \mathbb{M}_1(\mathfrak{A}) = \mathfrak{A}^2 \mathcal{M}_1(\mathfrak{A})$ and $\mathbb{M}_2 : [0, \infty) \to [0, \infty) \ni \mathbb{M}_2(\mathfrak{S}) = \mathfrak{S}^2 \mathcal{M}_2(\mathfrak{S})$and $\mathbb{M}_3 : [0, \infty) \to [0, \infty) \ni \mathbb{M}_3(\mathfrak{R}) = \mathfrak{R}^2 \mathcal{M}_3(\mathfrak{R})$ .These functions are non-decreasing, continuous, even, convex , that hold the following conditions :

i) $\mathbb{M}_1(0) = 0, \mathbb{M}_2(0) = 0, \mathbb{M}_3(0) = 0 \Longrightarrow \mathbb{M}(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) = \left(\mathbb{M}_1(0), \mathbb{M}_2(0), \mathbb{M}_3(0)\right) = (0,0,0)$.

ii) $\mathbb{M}_1(\mathfrak{A}) \succ 0, \mathbb{M}_2(\mathfrak{S}) \succ 0, \mathbb{M}_3(\mathfrak{R}) \succ 0 \Longrightarrow \mathbb{M}(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) = \left(\mathbb{M}_1(\mathfrak{A}), \mathbb{M}_2(\mathfrak{S}), \mathbb{M}_3(\mathfrak{R})\right) \succ (0,0,0)$, for $\mathfrak{A} \succ 0, \mathfrak{S} \succ 0, \mathfrak{R} \succ 0$,by which we say $(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) \succ (0,0,0)$ that $\mathbb{M}_1(\mathfrak{A}) \succ 0, \mathbb{M}_2(\mathfrak{S}) \succ 0, \mathbb{M}_3(\mathfrak{R}) \succ 0$.

iii) $\mathbb{M}_1(\mathfrak{A}) \to \infty, \mathbb{M}_2(\mathfrak{S}) \to \infty, \mathbb{M}_3(\mathfrak{R}) \to \infty$ as $\mathfrak{A} \to \infty, \mathfrak{S} \to \infty, \mathfrak{R} \to \infty \Longrightarrow \mathbb{M}(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) = \left(\mathbb{M}_1(\mathfrak{A}), \mathbb{M}_2(\mathfrak{S}), \mathbb{M}_3(\mathfrak{R})\right) \to (\infty, \infty, \infty)$ as $(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) \to (\infty, \infty, \infty)$ by which we say $\mathbb{M}(\mathfrak{A}, \mathfrak{S}, \mathfrak{R}) \to (\infty, \infty, \infty)$ as $\mathbb{M}_1(\mathfrak{A}) \to \infty, \mathbb{M}_2(\mathfrak{S}) \to \infty, \mathbb{M}_3(\mathfrak{R}) \to \infty$.

If $(\ltimes_{\ell k j i}\, \mathfrak{A}_{\ell k j i}) \in \mathbb{E}^4$ whenever $(\mathfrak{A}_{\ell k j i}) \in \mathbb{E}^4$ for every quadruple sequence $(\ltimes_{\ell k i})$ of scalars with $\left|\ltimes_{\ell k j i}\right| \preccurlyeq 1, \forall\, \ell, k, j, i \in \mathbb{N}$ then the quadruple sequence spaces $\mathbb{E}^4$ is a solid .

Assume that $\mathbb{K} = \{(\ell_n, k_n, j_n, i_n), \forall\, n \in \mathbb{N}, \ell_1 \prec \ell_2 \prec \ell_3 \prec \ \dots, k_1 \prec k_2 \prec k_3$ and $j_1 \prec j_2 \prec j_3, i_1 \prec i_2 \prec i_3, \dots\} \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and let $\mathbb{E}^4$ be a quadruple sequences space . A $\mathbb{K}$-step space of $\mathbb{E}^4$ be a quadruple sequences space

$\xi_{\mathbb{K}}^{\mathbb{E}^4} = \left\{\left(\mathfrak{A}_{\ell k j i}\right) \in \mathbb{W}^4 :\ (\mathfrak{A}_{\ell k j i}) \in \mathbb{E}^4\right\}$, where $\mathbb{W}^4 = \left\{\left(\mathfrak{A}_{\ell k j i}\right) :\ \mathfrak{A}_{\ell k j i} \in \mathbb{R} \text{ or } \mathbb{C}\right\}$.

A quadruple sequence $(\mathfrak{A}_{\ell k j i})$ in $\mathbb{E}^4$ has a canonical pre-image that is a quadruple sequence $(\mathfrak{U}_{\ell k j i})$ is defined by :

$$\mathfrak{U}_{\ell k j i} = \begin{cases} \mathfrak{A}_{\ell k j i} & \text{if} \quad (\ell, k, j, i) \in \mathbb{K} \\ \bar{0} & \text{otherwise} \end{cases} .$$

A set of canonical pre-images of all elements in $\zeta_{\mathbb{K}}^{\mathbb{E}^4}$ be a canonical pre-image of a step space $\zeta_{\mathbb{K}}^{\mathbb{E}^4}$.

If a quadruple sequence spaces $\mathbb{E}^4$ contains the canonical pre-image of all its step spaces then it is a monotone.

If $(\mathfrak{A}_{\pi(\ell)\pi(k)\pi(j)\pi(i)}) \in \mathbb{E}^4$ whenever $(\mathfrak{A}_{\ell k j i}) \in \mathbb{E}^4$, then a quadruple sequence spaces $\mathbb{E}^4$ is a symmetric .

If $(\mathfrak{U}_{\ell \mathscr{k} j i}) \in \mathbb{E}^4$ whenever $(\mathfrak{A}_{\ell \mathscr{k} j i}) \in \mathbb{E}^4$ and $\mathfrak{A}_{\ell \mathscr{k} j i} = 0$ implies $\mathfrak{U}_{\ell \mathscr{k} j i} = 0$, then a quadruple sequence spaces $\mathbb{E}^4$ is a convergent-free .

A fuzzy real number $\mathbb{F}$ is a fuzzy subset of the real line $\mathbb{R}$, i.e. a mapping $\mathbb{F} : \mathbb{R} \to [0,1]$ associating each real number $\mathbb{r}$ with its grade of membership $\mathbb{F}(\mathbb{r})$, satisfies the following conditions [34] :

1. $\mathbb{F}$ is a convex if for each $\mathbb{F}(\mathbb{r}_2) \succcurlyeq \mathbb{F}(\mathbb{r}_1) \wedge \mathbb{F}(\mathbb{r}_3) = \min\{ \mathbb{F}(\mathbb{r}_1), \mathbb{F}(\mathbb{r}_3)\}$, $\forall \mathbb{r}_1 \prec \mathbb{r}_2 \prec \mathbb{r}_3$, $\forall\, \mathbb{r}_1, \mathbb{r}_2, \mathbb{r}_3 \in \mathbb{R}$.
2. $\mathbb{F}$ is normal if there is a $\mathbb{r}_0 \in \mathbb{R}$ and $\mathbb{F}(\mathbb{r}_0) = 1$.
3. $\mathbb{F}$ is upper-semi-continuous $\forall\, \mathbb{a} \in \mathbb{I}$, $\forall\, \varepsilon \succ 0$ and $\mathbb{F}^{-1}([0, \mathbb{a} + \varepsilon))$ is open in the usual topology of $\mathbb{R}$
4. $\mathbb{F}$ is a non-negative fuzzy number $\forall\, \mathbb{r} \prec 0$ implies $\mathbb{F}(\mathbb{r}) = 0$.

The set of all non-negative fuzzy numbers of $\mathbb{R}(\mathbb{I})$ denoted by $\mathbb{R}^*(\mathbb{I})$. Let $\mathbb{R}(\mathbb{I})$ denote the set of all fuzzy numbers which are upper-semi continuous , normal and have compact support, i.e. if $\mathbb{H} \in \mathbb{R}(\mathbb{I})$ then $\mathbb{H}^{\propto}$ is compact, for any $\propto \in [0,1]$, where

$$\mathbb{H}^{\propto} = \{\mathbb{r} \in \mathbb{R} : \mathbb{H}(\mathbb{r}) \succcurlyeq \propto , \text{if } \propto \in [0,1] \} .$$

$$\mathbb{H}^{0} = \text{closure of } (\{\mathbb{r} \in \mathbb{R} : \mathbb{H}(\mathbb{r}) \succ 0 , \text{if } \propto = 0\}) .$$

The class of the quadruple sequences space $\mathcal{m}(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^4$ is introduced as follows:

$$\mathcal{m}(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^4 = \Big\{ \mathfrak{Q}_{\mathfrak{nmtj}} = \big((\mathfrak{Q}_1)_{\mathfrak{nmtj}} , (\mathfrak{Q}_2)_{\mathfrak{nmtj}} , (\mathfrak{Q}_3)_{\mathfrak{nmtj}}\big) :$$

$$\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \succcurlyeq 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\varphi_{\mathfrak{srec}}} \sum_{\mathfrak{n} \in \sigma} \sum_{\mathfrak{m} \in \sigma} \sum_{\mathfrak{t} \in \sigma} \sum_{\mathfrak{j} \in \sigma} \Bigg[\Bigg(\mathbb{M}_1 \bigg(\Big(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}} , \bar{0})}{\rho} \Big) \bigg) \curlyvee \mathbb{M}_2 \bigg(\Big(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}} , \bar{0})}{\rho} \Big) \bigg) \curlyvee$$

$$\mathbb{M}_3 \bigg(\Big(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}} , \bar{0})}{\rho} \Big) \bigg) \Bigg) \Bigg]^{\wp} \prec \infty, \text{for some } \rho \succ 0 \Big\}, \forall\, 0 \prec \wp \prec \infty \text{ where } \mathbb{M} = (\mathbb{M}_1 , \mathbb{M}_2 , \mathbb{M}_3).$$

3. Main results

<u>Theorem 3.1</u> :

$\forall\, 0 \prec \wp \prec \infty$, $\mathcal{m}(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^4$ does not solid .

Proof :

Suppose $\mathfrak{a} = 2, \mathfrak{b} = 3, \wp = 2$. Let $(\mathfrak{Q}_{\mathfrak{nmtj}}) = ((\mathfrak{Q}_1)_{\mathfrak{nmtj}}, (\mathfrak{Q}_2)_{\mathfrak{nmtj}}, (\mathfrak{Q}_3)_{\mathfrak{nmtj}}) = (\overline{\mathfrak{nmtj}}, \overline{\mathfrak{nmtj}}, \overline{\mathfrak{nmtj}})$, $\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$ and $\varphi_{\mathfrak{srec}} = \mathfrak{srec}, \forall\, \mathfrak{s}, \mathfrak{r}, \mathfrak{e}, \mathfrak{c} \in \mathbb{N}$. Assume that $\mathbb{M}(x_1, x_2, x_3) = (|x_1|, |x_2|, |x_3|)$, $\forall\, x_1, x_2, x_3 \in [0, \infty)$. Then $\bar{d}(\triangle_3^2\, \mathfrak{Q}_{\mathfrak{nmtj}}, \bar{0}) = 0$, $\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$. Then we have

$$\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \geqslant 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\mathfrak{srec}} \sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left[\left(\mathbb{M}_1\left(\frac{\bar{\mathbb{d}}(\Delta_3^2(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{\mathbb{d}}(\Delta_3^2(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_3\left(\frac{\bar{d}(\Delta_3^2(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right)\right)\right]^2 \prec \infty$$

, for some $\rho \succ 0$. This tends to , $(\mathfrak{Q}_{\mathfrak{nmtj}}) = (\mathfrak{Q}_1)_{\mathfrak{nmtj}}, (\mathfrak{Q}_2)_{\mathfrak{nmtj}}, (\mathfrak{Q}_3)_{\mathfrak{nmtj}} \in m(\mathbb{M}, \mathfrak{sre}, \Delta_3^2, 2)_{\mathbb{F}}^4$.

Take the quadruple sequence for example $(\propto_{\mathfrak{nmtj}}) = ((\propto_1)_{\mathfrak{nmtj}}, (\propto_2)_{\mathfrak{nmtj}}, (\propto_3)_{\mathfrak{nmtj}})$ a collection of scalars specified by,

$$\propto_{\mathfrak{nmtj}} = \begin{cases} (1,1,1), & \text{in order to } \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \text{ is in addition} \\ (0,0,0), & \text{in any case} \end{cases}$$

Now, $\propto_{\mathfrak{nmtj}} \mathfrak{Q}_{\mathfrak{nmtj}} = \begin{cases} \overline{\mathfrak{nmtj}}, & \text{in order to } \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \text{ is in addition} \\ (0,0,0), & \text{in any case} \end{cases}$

This implies that,

$$\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \geqslant 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\mathfrak{srec}} \sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left[\left(\mathbb{M}_1\left(\frac{\bar{\mathbb{d}}(\Delta_3^2\, (\propto_1)_{\mathfrak{nmtj}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{\mathbb{d}}(\Delta_3^2\, (\propto_2)_{\mathfrak{nmtj}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_3\left(\frac{\bar{\mathbb{d}}(\Delta_3^2\, (\propto_3)_{\mathfrak{nmtj}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right)\right)\right]^2 = \infty$$

, for fixed $\rho \succ 0$.

This demonstrates that, $\propto_{\mathfrak{nmtj}} \mathfrak{Q}_{\mathfrak{nmtj}} \notin m(\mathbb{M}, \mathfrak{srec}, \Delta_3^2, 2)_{\mathbb{F}}^4$.

Consequently, $m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^4$ does not solid , for $0 \prec \wp \prec \infty$.

Proposition 3.2 :

$\forall\, 0 \prec \wp \prec \infty$, $m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^4$ does not symmetric.

Proof :

Assume $\mathfrak{a} = 1, \mathfrak{b} = 1, \wp = \frac{1}{2}$. Let $\mathbb{M}(x_1, x_2, x_3) = (x_1^2, x_2^2, x_3^2)$, $\forall\, x_1, x_2, x_3 \in [0, \infty)$.

Suppose $\varphi_{\mathfrak{srec}} = \mathfrak{srec}$, $\forall\, \mathfrak{s}, \mathfrak{r}, \mathfrak{e}, \mathfrak{c} \in \mathbb{N}$.

Let $(\mathfrak{Q}_{\mathfrak{nmtj}}) = ((\mathfrak{Q}_1)_{\mathfrak{nmtj}}, (\mathfrak{Q}_2)_{\mathfrak{nmtj}}, (\mathfrak{Q}_3)_{\mathfrak{nmtj}}) = (\overline{\mathfrak{nmtj}}, \overline{\mathfrak{nmtj}}, \overline{\mathfrak{nmtj}}), \forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$ .Then $\bar{\mathbb{d}}(\Delta\mathfrak{Q}_{\mathfrak{nmtj}}, \bar{0}) = 1$,$\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$. Consequently $(\mathfrak{Q}_{\mathfrak{nmtj}}) \in\ m(\mathbb{M}, \mathfrak{srec}, \Delta, \frac{1}{2})^4_{\mathbb{F}}$.

Assume $(\mathfrak{S}_{\mathfrak{nmtj}}) = ((\mathfrak{S}_1)_{\mathfrak{nmtj}}, (\mathfrak{S}_2)_{\mathfrak{nmtj}}, (\mathfrak{S}_3)_{\mathfrak{nmtj}})$ be a reorganization of $(\mathfrak{Q}_{\mathfrak{nmtj}}) \ni (\mathfrak{S}_{\mathfrak{nmtj}}) = (\mathfrak{Q}_{1111}, \mathfrak{Q}_{2222}, \mathfrak{Q}_{4444}, \mathfrak{Q}_{3333}, \mathfrak{Q}_{9999}, \mathfrak{Q}_{5555}, \mathfrak{Q}_{16161616}, \mathfrak{Q}_{6666}, \mathfrak{Q}_{25252525}, \ldots)$

Then , $\bar{\mathbb{d}}(\Delta\mathfrak{S}_{\mathfrak{nmtj}}, \bar{0}) = ((\mathfrak{nmtj}-1)^2 + (2\mathfrak{nmtj}-1), (\mathfrak{nmtj}-1)^2 + (2\mathfrak{nmtj}-1), (\mathfrak{nmtj}-1)^2 + (2\mathfrak{nmtj}-1)) = ((\mathfrak{nmtj})^2, (\mathfrak{nmtj})^2, (\mathfrak{nmtj})^2), \forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$.

This demonstrates that, $\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \geqslant 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\mathfrak{srec}} \sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \left[\left(\mathbb{M}_1\left(\frac{\bar{\mathbb{d}}(\Delta(\mathfrak{S}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{\mathbb{d}}(\Delta(\mathfrak{S}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_3\left(\frac{\bar{\mathbb{d}}(\Delta(\mathfrak{S}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right)\right)\right]^{\frac{1}{2}} = \infty$, for some $\rho \succ 0$.

Consequently $(\mathfrak{S}_{\mathfrak{nmtj}}) \notin\ m(\mathbb{M}, \mathfrak{srec}, \Delta, \frac{1}{2})^4_{\mathbb{F}}$.

This ,

$m(\mathbb{M}, \varphi, \triangle^{\mathfrak{a}}_{\mathfrak{b}}, p)^4_{\mathbb{F}}$ does not symmetric.

Proposition 3.3 :

$\forall\ 0 \prec p \prec \infty$, $m(\mathbb{M}, \varphi, \triangle^{\mathfrak{a}}_{\mathfrak{b}}, p)^4_{\mathbb{F}}$ does not convergence-free .

Proof :

Assume $\mathfrak{a} = 1, \mathfrak{b} = 4, p = \frac{1}{2}$, Let $\mathbb{M}(x_1, x_2, x_3) = (x_1^4, x_2^4, x_3^4)$, $\forall\, x_1, x_2, x_3 \in [0, \infty)$. Let take $\varphi_{\mathfrak{srec}} = \mathfrak{srec}$, $\forall\, \mathfrak{s}, \mathfrak{r}, \mathfrak{e}, \mathfrak{c} \in \mathbb{N}$. Consider the quadruple sequence $(\mathfrak{Q}_{\mathfrak{nmt}})= ((\mathfrak{Q}_1)_{\mathfrak{nmt}}, (\mathfrak{Q}_2)_{\mathfrak{nmt}}, (\mathfrak{Q}_3)_{\mathfrak{nmt}})$, which is described as:

$$\mathfrak{Q}_{\mathfrak{nmtj}}(\mathfrak{x}) = \begin{cases} (1+\mathfrak{nmtj}\mathfrak{x}, 1+\mathfrak{nmtj}\mathfrak{x}, 1+\mathfrak{nmtj}\mathfrak{x}), & \text{in order to} \quad \mathfrak{x} \in \left[\frac{-1}{\mathfrak{nmtj}}, 0\right], \\ (1-\mathfrak{nmtj}\mathfrak{x}, 1-\mathfrak{nmtj}\mathfrak{x}, 1-\mathfrak{nmtj}\mathfrak{x}), & \text{in order to} \quad \mathfrak{x} \in \left[0, \frac{1}{\mathfrak{nmtj}}\right], \\ (0,0,0) & \text{in any case} \end{cases}$$

Then, $\Delta_4\mathfrak{Q}_{\mathfrak{nmtj}}(\mathfrak{x}) =$

$$\begin{cases} \left(1+\frac{\mathfrak{nmtj}(\mathfrak{nmt}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}, 1+\frac{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}, 1+\frac{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}\right), & \text{in order to } \mathfrak{x}\in\left[\frac{-2\mathfrak{nmtj}+4}{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}, 0\right], \\ \left(1-\frac{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}, 1-\frac{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}, 1-\frac{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}{2\mathfrak{nmtj}+4}\mathfrak{x}\right), & \text{in order to } \mathfrak{x}\in\left[0, \frac{2\mathfrak{nmtj}+4}{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}\right], \\ (0,0,0) & \text{in any case} \end{cases}$$

As a result,

$$\bar{\mathbb{d}}\left(\Delta_4\mathfrak{Q}_{\mathfrak{nmtj}}, \bar{0}\right) = \left(\frac{2\mathfrak{nmtj}+4}{\mathfrak{nmtj}(\mathfrak{nmt}+4)}, \frac{2\mathfrak{nmtj}+4}{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}, \frac{2\mathfrak{nmtj}+4}{\mathfrak{nmtj}(\mathfrak{nmtj}+4)}\right) = \left(\frac{1}{\mathfrak{nmtj}}+\frac{1}{\mathfrak{nmtj}+4}, \frac{1}{\mathfrak{nmtj}}+\frac{1}{\mathfrak{nmtj}+4}, \frac{1}{\mathfrak{nmtj}}+\frac{1}{\mathfrak{nmtj}+4}\right).$$

We have get $\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c}\geqslant 1,\sigma\in\mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\mathfrak{srec}}\sum_{\mathfrak{n}\in\sigma}\sum_{\mathfrak{m}\in\sigma}\sum_{\mathfrak{t}\in\sigma}\sum_{\mathfrak{j}\in\sigma}\left[\left(\mathbb{M}_1\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_3\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right)\right)\right]^{\frac{1}{2}} \prec \infty$, for fixed $\rho \succ 0$.

Consequently, $(\mathfrak{Q}_{\mathfrak{nmtj}}) \in m(\mathbb{M}, \mathfrak{srec}, \Delta_4, \frac{1}{2})^4_{\mathbb{F}}$.

Let's try another quadruple sequence now $(\mathfrak{S}_{\mathfrak{nmtj}}) = \left((\mathfrak{S}_1)_{\mathfrak{nmtj}}, (\mathfrak{S}_2)_{\mathfrak{nmtj}}, (\mathfrak{S}_3)_{\mathfrak{nmtj}}\right) \ni \mathfrak{S}_{\mathfrak{nmt}}(\mathfrak{x}) =$

$$\begin{cases} \left(1+\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}, 1+\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}, 1+\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}\right), & \text{in order to } \mathfrak{x}\in[-(\mathfrak{nmtj})^2, 0], \\ \left(1-\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}, 1-\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}, 1-\frac{\mathfrak{x}}{(\mathfrak{nmtj})^2}\right), & \text{in order to } \mathfrak{x}\in[0, (\mathfrak{nmtj})^2], \\ \overline{(0,0,0)} & \text{in any case} \end{cases}$$

So that, $\Delta_4\mathfrak{S}_{\mathfrak{nmtj}}(\mathfrak{x}) =$

$$\begin{cases} \left(1+\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}, 1+\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}, 1+\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}\right), & \text{for } \mathfrak{x}\in[-(2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16), 0], \\ \left(1-\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}, 1-\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}, 1-\frac{\mathfrak{x}}{2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16}\right), & \text{for } \mathfrak{x}\in[0, (2(\mathfrak{nmtj})^2+8\mathfrak{nmtj}+16)], \\ (0,0,0) & \text{in any case} \end{cases}$$

Hence, $\bar{\mathbb{d}}\left(\Delta_4\mathfrak{S}_{\mathfrak{nmtj}}, \bar{0}\right) = (2(\mathfrak{nmtj})^2 + 8\mathfrak{nmtj} + 16)$, $\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$. Hence

$\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c}\geqslant 1,\sigma\in\mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\mathfrak{srec}}\sum_{\mathfrak{n}\in\sigma}\sum_{\mathfrak{m}\in\sigma}\sum_{\mathfrak{t}\in\sigma}\sum_{\mathfrak{j}\in\sigma}\left[\left(\mathbb{M}_1\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{S}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{S}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_3\left(\frac{\bar{\mathbb{d}}(\Delta_4(\mathfrak{S}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho}\right)\right)\right]^{\frac{1}{2}} = \infty$. Therefore $(\mathfrak{S}_{\mathfrak{nmtj}}) \notin m(\mathbb{M}, \mathfrak{srec}, \Delta_4, \frac{1}{2})^4_{\mathbb{F}}$. This,

$m(\mathbb{M}, \varphi, \Delta^{\mathfrak{a}}_{\mathfrak{b}}, p)^4_{\mathbb{F}}$ does not convergence-free .

Proposition 3.4 :

$$\forall\, 1 \preccurlyeq \wp \prec \infty, m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4} \subseteq m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4}.$$

Proof :

Assume $\mathfrak{Q} = (\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_2) \in m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4}$. Then we have,

$$\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \succcurlyeq 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\varphi_{\mathfrak{srec}}} \sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left[\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right] = \mathbb{K}\ (\prec \infty),$$

for fixed exists $\rho \succ 0$ .Then, we have get , $\forall$ fixed $\mathfrak{s}, \mathfrak{r}, \mathfrak{e}, \mathfrak{c}$

$$\sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left[\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right] \preccurlyeq \mathbb{K}\varphi_{\mathfrak{srec}},$$

$\forall\, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}$.

$$\Rightarrow \left[\sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right)^{\wp} \right]^{\frac{1}{\wp}} \preccurlyeq \mathbb{K}\varphi_{\mathfrak{srec}}, \forall\, \sigma \in \mathfrak{Y}_{\mathfrak{sre}}.$$

$$\Rightarrow \sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \succcurlyeq 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\varphi_{\mathfrak{srec}}} \left[\left(\sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right)^{\wp} \right)^{\frac{1}{\wp}} \right] \preccurlyeq \mathbb{K}.$$

i.e. $\sup_{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \succcurlyeq 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\varphi_{\mathfrak{srec}}} \left[\left(\sum_{\mathfrak{n}\in\sigma} \sum_{\mathfrak{m}\in\sigma} \sum_{\mathfrak{t}\in\sigma} \sum_{\mathfrak{j}\in\sigma} \left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right)^{\wp} \right)^{\frac{1}{\wp}} \right] \prec \infty$ .Therefore $\mathfrak{Q} \in m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4}$, for $1 \preccurlyeq \wp \prec \infty$.

Proposition 3.5 :

$$\forall\, 1 \preccurlyeq \wp \prec \infty, \ell_{\wp}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4} \subseteq m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4} \subseteq \ell_{\infty}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4}.$$

Proof :

Assume $\mathbb{M}(x_1, x_2, x_3) = \left(x_1^{\wp}, x_2^{\wp}, x_3^{\wp}\right)$, $\forall\, x_1, x_2, x_3 \in [0, \infty)$ and $1 \preccurlyeq \wp \prec \infty$ and $\varphi_{\mathfrak{nmtj}} = (1,1,1)$, $\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{t}, \mathfrak{j} \in \mathbb{N}$, Let take $m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4} = \ell_{\wp}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4}$ is obtained. As a consequence, the first inclusion is self-evident. Assume that,

$(\mathfrak{Q}_{\mathfrak{nmtj}}) = ((\mathfrak{Q}_1)_{\mathfrak{nmtj}}, (\mathfrak{Q}_2)_{\mathfrak{nmtj}}, (\mathfrak{Q}_3)_{\mathfrak{nmtj}}) \in m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4}$.

$$\sup_{\mathfrak{s,r,e,c} \succcurlyeq 1, \sigma \in \mathfrak{Y}_{\mathfrak{srec}}} \frac{1}{\varphi_{\mathfrak{srec}}} \left[\sum_{\mathfrak{n} \in \sigma} \sum_{\mathfrak{m} \in \sigma} \sum_{\mathfrak{t} \in \sigma} \sum_{\mathfrak{t} \in \sigma} \left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right)^{\wp} \right]^{\frac{1}{\wp}} = \mathbb{K}(\prec \infty). \forall\, \mathfrak{s}, \mathfrak{r}, \mathfrak{e}, \mathfrak{c} = 1 ,$$

$$\left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right) \preccurlyeq \mathbb{K}\varphi_{1111}$$

, which suggests that,

$$\sup_{\mathfrak{s,r,e,c} \succcurlyeq 1} \left(\mathbb{M}_1 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_1)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_2 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_2)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \curlyvee \mathbb{M}_3 \left(\frac{\bar{\mathbb{d}}(\triangle_{\mathfrak{b}}^{\mathfrak{a}}(\mathfrak{Q}_3)_{\mathfrak{nmtj}}, \bar{0})}{\rho} \right) \right) \prec \infty,$$

which indicate that $(\mathfrak{Q}_{\mathfrak{nt}}) \in \ell_{\infty}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4}$.

Consequently $\ell_{\wp}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4} \subseteq m(\mathbb{M}, \varphi, \triangle_{\mathfrak{b}}^{\mathfrak{a}}, \wp)_{\mathbb{F}}^{4} \subseteq \ell_{\infty}(\mathbb{M}, \triangle_{\mathfrak{b}}^{\mathfrak{a}})_{\mathbb{F}}^{4}$.

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