Journal of Al-Qadisiyah for Computer Science and Mathematics Vol. 14(4) 2022 , pp Math. 66–73

University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Solving Linear Programming Problems Involving Hexagonal and Octagonal Fuzzy Numbers via Two Ranking Functions

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A R T I C L E I N F O

Article history:
Received: 30 /10/2022
Rrevised form: 01 /12/2022
Accepted : 04 /12/2022
Available online: 31 /12/2022

Keywords:

Fuzzy set (FS), Linear programming (LP), Simplex method (SM), Hexagonal fuzzy number (HFN), Octagonal fuzzy number (OFN).

A B S T R A C T

Many types of ambiguous numbers have been studied in many mathematical fields. Two of these types, the obscure number hexagonal and octagonal are widely used, especially in mathematical programming. A linear fuzzy number (LFN), which generalizes these two types, is presented in this paper. Based on the definition of the order function by the total value, two ranking function for this linear fuzzy number is used. Arithmetic operations with their properties are entered on ambiguous numbers as a special case. It also turns out that the operations proposed on LFN are acceptable generalizations of conventional arithmetic operations on real numbers.

MSC..

https://doi.org/10.29304/jqcm.2022.14.4.1117

1. Introduction

In 1965, Zadeh, Los Angeles clarifies ambiguous groups [1] by giving an approach to address issues when ambiguity and inaccuracy arise. Many applications have emerged in all different fields, for example, expert system, decision-making, artificial intelligence, etc.

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Communicated by 'sub etitor'

S.Sandhiya, K.Selvakumari [2] applied the concept of hexagonal mysterious membership matrix in a medical diagnosis. M.Prabhavathi et al. [3] Introduced new operations the fuzzy hexadecimal number on the basis of truncated alpha sets of ambiguous numbers. A. Thiruppathi, C. K. Kirubhashankar [4] used a hexadecimal ambiguous number assignment problem with the application of the ordering technique.

P. Rajarajeswari, A.Sahaya Sudha [5] described the method of arranging fuzzy numbers based on the area, mode, prevalence, and weights of generalized (unusual) hexagonal fuzzy numbers. S. JOHNSON SAVARIMUTHU, N. KARTHIKA [6] used fuzzy hex numbers and fuzzy matrices to choose the best one among the group in the field of decision-making is the most an application in fuzzy group theory. A. THIRUPPATHI, C. K. KIRUBHASHANKAR [7] Introduced a new method based on the hexagonal shape an ambiguous number using the centroid of a triangle and a rectangle the hexadecimal fuzzy number is converted to a clear number using ordering methods. M. S. Annie Christi, Malini. D. [8] worked the better candidate method for solving FTP. Centroid order is used to get the solution. N.Rameshan and D.Stephen Dinagar, [9] Suggest a new method to find Fuzzy critical path with intuitive octagonal fuzzy number activity time. M.Venkatachalapathy et al. [10] discussed transportation problems of the mysterious octagonal were discussed under the misty environment. In the fuzzy octagonal transportation problem in which cost, supply, and demand are all fuzzy octagonal number.

Many authors [11-14] discussed the set of solutions to the fuzzy programming (FP) problem using different algorithms. In this paper, we present the problem of linear programming that depends on the fuzzy hexagonal and octagonal number and in it. The hexagonal and octagonal fuzzy number is transformed into a crisp number using two ordering functions, and the problem of the crisp number is solved in the simple way, and then compared.

The paragraphs of the paper appear in the following sequence, in Section 2, we address some important concepts of Fuzzy, in Section 3,the general formula for a (LP) problem is presented, in section 4, the ordinal function is explained, and in section 5, It included the formulation of a hexagonal and octagonal fuzzy number for the ordinal function, and in the section 6, we defined a procedure to solve the fuzzy linear programming problem followed by a numerical example for clarification in Section 7, and finally the paper is completed in Section 8, and includes the discussion.

2. Preliminaries: [15-17]

This section recalls some basic definitions of FS, hexagonal fuzzy number and octagonal fuzzy number.

Definition 2.1: (Fuzzy Set FS): The fuzzy group is a subset of the universe of discourse $\mathcal{L}$, where each point in the fuzzy group has degrees of affiliation that are specifically characterized from $\mathcal{L}$ to [0 , 1].

Definition 2.2: (HFN): A fuzzy group $\tilde{A}_{hex}$ is a HFN [16] denoted by $\tilde{A}_{hex} = (\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5, \hat{a}_6)$. where $\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5$ *and* $\hat{a}_6$ are actual numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\frac{(x - \hat{a}_1)}{(\hat{a}_2 - \hat{a}_1)} & for \quad \hat{a}_1 \le x \le \hat{a}_2 \\ \frac{1}{2}+\frac{1}{2}\frac{(x-\hat{a}_2)}{(\hat{a}_3 - \hat{a}_2)} & for \quad \hat{a}_2 \le x \le \hat{a}_3 \\ 1 & for \quad \hat{a}_3 \le x \le \hat{a}_4 \\ 1-\frac{1}{2}\frac{(x - \hat{a}_4)}{(\hat{a}_5 - \hat{a}_4)} & for \quad \hat{a}_4 \le x \le \hat{a}_5 \\ \frac{1}{2}\frac{(\hat{a}_6 - x)}{(\hat{a}_6 - \hat{a}_5)} & for \quad \hat{a}_5 \le x \le \hat{a}_6 \\ 0 & otherwise \end{cases}$$

Where $0 < ķ < 1$.

Definition 2.3: (OFN): A fuzzy group $\bar{A}_{oct}$ is a natural Octagonal Fuzzy Number (NOFN) represented by $\tilde{A}_{oct} = (é_1, é_2, é_3, é_4, é_5, é_6, é_7, é_8)$. Where $é_1, é_2, é_3, é_4, é_5, é_6, é_7$ *and* $é_8$ are fact numbers and its membership function as following

$$\mu\bar{A}_o(j) = \begin{cases} 0 & for \quad j < é_1 \\ ķ\left(\frac{j-é_1}{é_2-é_1}\right) & for \quad é_1 \le j \le é_2 \\ ķ & for \quad é_2 \le j \le é_3 \\ ķ + (1-k)\left(\frac{j-é_3}{é_4-é_3}\right) & for \quad é_3 \le j \le é_4 \\ 1 & for \quad é_4 \le j \le é_5 \\ ķ + (1-k)\left(\frac{é_6-j}{é_6-é_5}\right) & for \quad é_5 \le j \le é_6 \\ ķ & for \quad é_6 \le j \le é_7 \\ ķ\left(\frac{é_8-j}{é_8-é_7}\right) & for \quad é_7 \le j \le é_8 \\ 0 & for \quad j > é_8 \end{cases}$$

Where $0 < ķ < 1$.

3. Linear Programming (LP) Model [18]

The general formula for the LP problem with:

$$Max \ or \ Min \ Q = \sum_{j=1}^{n} \ell_j \ \mathfrak{z}_j$$

subject to

$$\sum_{j=1}^{n} \ell_{ij} \ \mathfrak{z}_j \le u_i \quad i = 1,2, \ldots . . m$$
$$\mathfrak{z}_j \ge 0$$

Where $\ell_j \in \mathcal{W}^n, u_i \in \mathcal{W}^m, w_{ij} \in \mathcal{W}^{n*m}$.

Parameters in the above model are fragile. And if some or all of the parameters are fuzzy groups, then the LP model becomes a fuzzy model.

4. Ranking Function [19]

A good method used to compare fuzzy groups is to use the classification function $\mathcal{A}(\mathcal{P})\colon \mathfrak{D} \to \mathfrak{D}$,

where $\mathcal{A}(\mathcal{P})$ is a set of fuzzy groups in the collection of actual groups, which you plot for each The actual line number, for example, if

$(i)\ \tilde{\mathfrak{F}} > \tilde{\mathfrak{P}}\ if\ and\ only\ if\ Ŋ(\tilde{\mathfrak{F}}) > Ŋ(\tilde{\mathfrak{P}})$

$(ii)\ \tilde{\mathfrak{F}} < \tilde{\mathfrak{P}}\ if\ and\ only\ if\ Ŋ(\tilde{\mathfrak{F}}) < Ŋ(\tilde{\mathfrak{P}})$

$(iii)\ \tilde{\mathfrak{F}} = \tilde{\mathfrak{P}}\ if\ and\ only\ if\ Ŋ(\tilde{\mathfrak{F}}) = Ŋ(\tilde{\mathfrak{P}})$

5. Formulation of hexagonal and octagonal Fuzzy Number for Ranking

Sort function methods for FN comparison, especially in fuzzy decision model, are more benefit than other methods.

In the current section, we present a method for arranging ambiguous numbers. M. S. Annie Christi, Mrs. Malini. D. [20] and Mitlif R J. [21] studied the methods of ordering using hexagonal and octagonal fuzzy numbers. Here, we present the methods of arrangement, simplex and arithmetic operations using a hexagonal and octagonal ambiguous number to find an optimal solution.

$\tilde{A}_{hex}$ is a collation function of a HFN defined by [20]

$$Œ(\tilde{Ŋ}) = \frac{3\sqrt{3}}{4}[(â_1 + â_3 + â_6) \times ķ + (â_2 + â_4 + â_5) \times (1 - ķ)] \qquad \ldots\ldots (1)$$

$Where\ ķ = 0.5.$

And

$\tilde{A}_{oct}$ is a collation function of an octagonal fuzzy number defined by [21]

$$Œ(\tilde{Ŋ}) = \frac{1}{4}[(é_1 + é_2 + é_7 + é_8) \times ķ + (é_3 + é_4 + é_5 + é_6) \times (1 - ķ)] \qquad \ldots\ldots (2)$$

$Where\ ķ = 0.5.$

6. Procedure for Solving Fuzzy linear programming Problem

Step 1: We choose the LP and solve it using a SM in a program WinQSB.

Step 2: We formulate the chosen problem in the following hexagonal fuzzy LP problem.

Step 3: First convert the LP values which are all hexagonal fuzzy in an objective function (OF) and constraint to clear values using the order scale above eq. (1).

Step 4 - We solve the LP problem with crisp values using the SM to get the optimal solution (OS).

Step 5: Second, convert the LP values that are all octagonal in an objective function and constraint into explicit values using the order scale above. (2).

Step 6: We solve the LP problem with crisp values using the SM to get the OS.

Step 7: Compare the results of the solution.

7. **Numerical Example** :

In this section example are given in order to illustrate the proposed method.

$Max\ Ģ = 80\ đ_1 + 90\ đ_2$

s. to

$410\ đ_1 + 140\ đ_2 \leq 950$

$160\ đ_1 + 135\ đ_2 \leq 980$

$đ_1\ ,\ đ_2 \geq 0.$

$Max\ Ģ = 610\ ,\ đ_1 = 0\ ,\ đ_2 = 6.785$

We formulate the chosen problem in the following hexagonal fuzzy linear programming problem:

$Max\ Ģ = (55,65,75,85,95,105)\ đ_1 + (65,75,85,95,105,115)\ đ_2$

s. to

$(385,395,405,415,425,435)\ đ_1 + (65,75,85,95,105,115)\ đ_2 \leq (935,945,955,965,975,985)$

$(135,145,155,165,175,185)\ đ_1 + (110,120,130,140,150,160)\ đ_2 \leq (965,975,985,995,1005,1015)$

$đ_1\ ,\ đ_2 \geq 0.$

Using the order function (1) we get

$$Œ(\tilde{Ŋ}) = \frac{3\sqrt{3}}{4}[(\hat{a}_1 + \hat{a}_3 + \hat{a}_6) \times ķ + (\hat{a}_2 + \hat{a}_4 + \hat{a}_5) \times (1 - ķ)] \quad \ldots\ldots (1)$$

$Max\ Ģ = 312\ đ_1 + 351\ đ_2$

s. to

$1599\ đ_1 + 351\ đ_2 \leq 3744$

$624\ đ_1 + 526.500\ đ_2 \leq 3861$

$đ_1\ ,\ đ_2 \geq 0.$

We solve the LP problem using the SM by win.QSB program, we get the solution

$Max\ Ģ = 2574\ ,\ đ_1 = 0\ ,\ đ_2 = 7.333$

We formulate the chosen problem in the following octagonal fuzzy LP problem:

$Max\ Ģ = (45,55,65,75,85,95,105,115)\ đ_1 + (55,65,75,85,95,105,115,125)\ đ_2$

s. to
$(375,385,395,405,415,425,435,445)\ đ_1 + (55,65,75,85,95,105,115,125)\ đ_2 \leq (925,935,945,955,965,975,985,995)$
$(125,135,145,155,165,175,185,195)\ đ_1 + (100,110,120,130,140,150,160,170)\ đ_2 \leq (955,965,975,985,995,1005,1015,1025)$
$đ_1\ ,\ đ_2 \geq 0.$

Using the order function (2) we get

$$Œ(\tilde{Ŋ}) = \frac{1}{4}[(é_1 + é_2 + é_7 + é_8) \times ķ + (é_3 + é_4 + é_5 + é_6) \times (1 - ķ)] \quad \ldots\ldots (2)$$

$Max\ Ģ = 80\ đ_1 + 90\ đ_2$

s. to
$410\ đ_1 + 140\ đ_2 \leq 950$
$160\ đ_1 + 135\ đ_2 \leq 980$
$đ_1\ ,\ đ_2 \geq 0.$

We solve the LP problem using the SM by win.QSB program, we get the solution.

$Max\ Ģ = 610\ ,\ đ_1 = 0\ ,\ đ_2 = 6.785$.

To compare the methods presented in this paper, the results of LP problems

$Max\ Ģ = 610\ ,\ đ_1 = 0\ ,\ đ_2 = 6.785$

and LP problems involving HFN

, obtained using the ordinal function in eq. (1).

The OS to the problem of LP was found after converting it from HFN to a fuzzy number and solving by SM $Max\ Ģ = 2574\ ,\ đ_1 = 0\ ,\ đ_2 = 7.333$

and LP problems involving OFN , obtained using the ordinal function in eq. (2).

The OS to the LP problem was found after converting it from an OFN to a crisp number and the solution by SM $Max\ Ģ = 610\ ,\ đ_1 = 0\ ,\ đ_2 = 6.785$

When comparing the solution, we notice that the results of the LP problem are equal to the results of the LP problem after converting it from OFN to a crisp number and the solution by SM. As for the results of the LP problem after converting it from HFN to a fragile number and solving by SM, they are higher and better than the previous results

8. **Conclusion**:

In this paper presents a solution to the problem of LP in a fuzzy environment where the objective function and constraint are described with fuzzy numbers. The fuzzy LP problem is a kind of optimization problem associated with our daily work. The perfect solution was found to show the effectiveness of this method. In this, the problem is solved using order method and simple method for hexagonal and octagonal ambiguous numbers and their comparison. An example is provided to illustrate this method.

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