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New random version of stability via fixed point method

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ABSTRACT

We studied the stability of the cubic functional equation:

3 ß(ķ +3 ý)-ß(3 ķ + ý)=12 [ß(ķ + ý)+ß(ķ - ý)]+80 ß(ý)-48 ß(ķ). (1.1)

via fixed point method in random normed space (ȒṄ –space).

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1. Introduction

Mathematicians use the random norm where the usual norm is not suitable. Therefore, popularization the normed space to random normed space is of specific significance. The topic of stability, which was inquiry from S.M.Ulam [10] in1940 expanded hugely in numerous spaces and using different equations. Mathematicians scientists have accomplished success in various ways, including fixed point method [1,2,6,7,8,9,11,12]. The stability of many other functional equations where also investigated in random normed spaces[ȒṄ –space].

In this article, we discuss the stability of the cubic functional equation(1.1) in a random normed space reach successful outcomes through the fixed point method.

2. 2. Preliminaries

Definition (2.1)[3]: "A function Ⱥ:[0, 1]2→ [0, 1] is said a triangular norm, if Ⱥsatisfies the following four axioms :"

(1) Ⱥ(ϰ , ý) = Ⱥ(ý, ϰ);

(2) Ⱥ (ϰ, Ⱥ(ý, ȥ)= Ⱥ (Ⱥ(ϰ, ý), ȥ) ;

(3) Ⱥ (ϰ, 1)=ϰ , ∀ϰ, ý∈ [0, 1];

(4) Ⱥ (ϰ, ý) ≤ Ⱥ (ȥ, Ŝ) when ϰ ≤ ȥ where ý ≤ Ŝ, ∀ ϰ, ý, ȥ, Ŝ ∈ [0, 1].

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Definition (2.2)[3]:" A random normed space (briefly, R̀Ṅ −space) is a triple (ℵ, ℘, A̶), where ℵ is a vector space. ℘ is a continuous t−norm and ℘ is a mapping from ℵ into D+ satisfying the following conditions:"

(1) $\wp_{\varkappa}$ (ƅ) = ε_0 (ƅ) ∀ƅ > 0 ⟺ ϰ = 0, ∀ ϰ ∈ ℵ;

(2) $\wp_{\lambda\varkappa}(b) = \wp_{\varkappa}\left(\frac{b}{|\lambda|}\right)$ ∀ ϰ ∈ ℵ, and λ ≠ 0;

(3) $\wp_{\varkappa+y}(b + \mathcal{P}) \geq$ A̶ $(\wp_{\varkappa}(b), \wp_{y}(\mathcal{P}))$ ∀ ϰ, y ∈ ℵ and ƅ, $\mathcal{P} \geq 0$.

Definition (2.3)[3] " Let (ℵ, ℘, A̶) be an R̀Ṅ −space

(1) A sequence $\{A_n\}$ in ℵ is said to be convergent to a point A ∈ ℵ if, for any ε > 0 and λ > 0, there exists a positive integer N such that $\wp_{A_n-A}(\varepsilon) > 1-\lambda$, ∀n > N.

(2) A sequence $\{A_n\}$ in ℵis called a Cauchy sequence if, for any ε > 0 and λ > 0, there exists a positive integer N such that $\wp_{A_n-A_m}(\varepsilon) > 1-\lambda$, ∀ n ≥ m ≥ N.

(3) An RN−space (ℵ, ℘, A̶) is said to be complete, if every Cauchy sequence in ℵ is convergent to a point in A."

Definition(2.4)[5];" Let ℵ be a set. A mapping Ɇ: ℵ × ℵ →[0,∞]is called a generalized metric on ℵ if and only if Ɇ satisfies the following conditions:

(1) Ɇ (ϰ , y) = 0 ⟺ ϰ = y;

(2) Ɇ (ϰ , y)= Ɇ (y, ϰ) ∀ ϰ, y ∈ ℵ;

(3) Ɇ (ϰ, ɀ) ≤Ɇ (ϰ, y)+ Ɇ (y, ɀ) ∀ ϰ, y, ɀ ∈ ℵ. "

3. Theorem Luxemburg–Jung's [4]:

Let (ℵ, Ɇ) be a complete generalized

metric space and B : ℵ→ℵ be a strict contraction with the Lipschitz constant ß ∈

(0, 1) such that Ɇ (x0, B (x0)) <+∞ for some x0 ∈ ℵ. Then we have the following:

(a) B has a unique fixed point x⊛ in the set {Ɇ (x0, B (x0)) <+∞ for some x0 ∈ ℵ}.

(b) For all x ∈ W, the sequence { B n(x)} converges to the fixed point x⊛;

(c) Ɇ (x0, B (x0)) ≤ Ş implies ȡ (x⊛, x0) ≤ $\frac{Ş}{1-ß}$. "

4-Main Results

Theorem 4.1. consider ℵ as a real linear space, ß function from ℵ to a complete R̀Ṅ-space (y, ℘, A̶M) with ß (0) = 0 and the function ϕ: ℵ2→D+ where

D⁺ ={C:C∈$\triangle^+$, $\lim_{x\to\infty}$ C(x) =1}and $\triangle^+$ is a distance distribution function there is ὤ ∈ (0, 27) where

$\phi_{3\varkappa,3y}$ (ὤƅ) ≥ $\phi_{\varkappa,y}$ (ƅ) (1.2)

for each ϰ , y ϵ ℵ where ƅ > 0 . If

$\wp_{D_S ß(\varkappa,y)}$ (ƅ)≥ $\phi_{\varkappa,y}$ (ƅ) (1.3) for each ϗ , ƴ ϵ ℵ where ƅ > 0, where

D_Sß(ϗ , ƴ) =3 ß(ϗ +3 ƴ)-ß(3 ϗ + ƴ)-12 [ß(ϗ + ƴ)+ß(ϗ - ƴ)]-80 ß(ƴ)+48 ß(ϗ)

there is Single cubic function Ƶ:ℵ→ ƴ where

$\wp_{Ƶ(ϗ)-h(ϗ)}(ƅ) \geq \phi_{ϗ,0}$ (Ḱ ƅ) (1.4)

for each ϗ ϵ Ḱ where ƅ > 0, and Ḱ = (27− ῶ).

where $Ƶ(ϗ) = \lim_{j\to\infty}\left\{\frac{ß(3^j ϗ)}{3^{3j}}\right\}$.

Proof By setting ƴ = 0 in (1.3), we get

$\wp_{ß\,(3ϗ)-27ß(ϗ)}(ƅ) \geq \phi_{ϗ,0}(ƅ)$

for each ϗ ϵ ℵ where ƅ > 0 and hence

$\wp_{\frac{1}{27}ß\,(3ϗ)-ß\,(ϗ)}(ƅ) = \wp_{\frac{1}{27}(ß\,(3ϗ)-27ß\,(ϗ))}(ƅ) = \wp_{(ß\,(3ϗ)-27ß\,(ϗ))}(27ƅ) \geq \phi_{x,0}(27ƅ)$

for each ϗ ϵ ℵ where ƅ > 0. Consider C(ϗ, ƅ) := $\phi_{ϗ,0}$ (27 ƅ).

let Ĥ = { Ƶ : ℵ→ ƴ : Ƶ (0) = 0}where the function Ɇ G indicated by Ĥ × Ĥ by

ɆC(Ƶ, Ţ) = inf{n ∈ R+: $\wp_{Ƶ(ϗ)-Ţ(ϗ)}(nƅ) \geq$ C(ϗ, ƅ), ∀ ϗ ∈ ℵ, ƅ > 0}.

Then (Ĥ, ɆG) is a complete generalized metric space. Suppose the linear

function Ł:Ĥ →Ĥ indicated by Ł(Ƶ(ϗ)) := $\frac{1}{27}$ Ƶ (3 ϗ)

prove the Ł is a strictly contractive function of Ĥ by Lipschitz

constant m = $\frac{ῶ}{27}$. Correct, let Ƶ, Ţ ∈ Ĥ be the function So that ɆC(Ƶ,h) < ε.

Then $\wp_{Ƶ(ϗ)-Ţ(ϗ)}$ (εt) ≥ C(ϗ, ƅ) for each ϗ ϵ ℵ where ƅ > 0where

$\wp_{ŁƵ(ϗ)-ŁŢ(ϗ)})(\frac{ῶ}{27}εƅ) = \wp_{\frac{1}{27}(Ƶ(3ϗ)-Ţ(3ϗ))}\left(\frac{ῶ}{27}εƅ\right) = \wp_{(Ƶ(3ϗ)-Ţ(3ϗ))}(ῶεƅ)$

$\geq$ C(3ϗ, ῶƅ)

for each ϗ ϵ ℵ where ƅ > 0. Where C(3 ϗ,ῶƅ) ≥ C(ϗ, ƅ), we have

$\wp_{ŁƵ(ϗ)-ŁŢ(ϗ)})(\frac{ῶ}{27}εƅ) \geq$ C(ϗ, ƅ)

ɆC(Ƶ, Ţ) < ε ⇒ ɆC(ŁƵ, Ł Ţ) ≤ $\frac{ῶ}{27}$ ε

It follows ɆC(ŁƵ, Ł Ţ) ≤ $\frac{\alpha}{27}$ $Ɇ_C$(Ƶ, Ţ) ∀ Ƶ, Ţ in Ĥ.

Next, from $\wp_{ß\,(ϗ)-\frac{1}{27}ß\,(3ϗ)}(ƅ) \geq$ C(ϗ, ƅ) This means ɆC(ß, Łß) ≤ 1. By the theorem Luxemburg–Jung's, prove the presence of a fixed point Ł, the presence of a function Ƶ: ℵ →ƴ which that Ƶ (3 ϗ) = 27 Ƶ (ϗ) ∀ ϗ ∈ ℵ.,

for each ϗ ∈ ℵ where ƅ > 0, ɆC(u, v) < ε ⇒ ℘ (ƅ) ≥ C $\left(ϗ, \frac{b}{\varepsilon}\right)$

it follows Ɇ (Ł ʲ ß, Ƶ)⟶0 the $\lim_{j\to\infty}\left\{\frac{ß(3^J ꞓ)}{3^{3J}}\right\} = Ƶ(ꞓ)$, ∀ ꞓ ∈ ℵ. Also, from

ɆC(ß, Ƶ) ≤ $\frac{1}{1-ß}$ ɆC(ß,Jß) ⇒ ɆC(ß, Ƶ) ≤ $\frac{1}{1-\frac{ὤ}{27}}$ that means

$℘_{Ƶ(ꞓ)-ß(ꞓ)}\left(\frac{27}{27-ὤ}ƅ\right)$≥ C(ꞓ, ƅ) ∀ƅ > 0 where ꞓ ∈ ℵ. It follows that

$℘_{Ƶ(ꞓ)-ß(x)}$(ƅ)≥ C $\left(ꞓ,\frac{27-ὤ}{27}\right)$ for each ꞓ ϵ ℵ where ƅ > 0 that means

$℘_{Ƶ(ꞓ)-ß(ꞓ)}$(ƅ) ≥ϕ $_{ꞓ,0}$ ((27−ὤ)) ƅ for each ꞓ ϵ ℵ where ƅ > 0 .

at last, the singularity of Ł results from reality that Ƶ is Single fixed point of Ł which that there is S ∈ (0,∞) where

$℘_{Ƶ(ꞓ)-ß(ꞓ)}$ (S ƅ) ≥ C(ꞓ, t) ∀ ꞓ ∈ ℵ and t > 0.Now the proof is complete

Corollary 4.2. consider ℵ the real linear space, ß the function from ℵ to a complete ȒṄ-space (Ɏ, ℘, ₳M) and satisfying $℘_{D_S ß(ꞓ,Ɏ)}$ (ƅ) ≥$\frac{ƅ}{ƅ+\varepsilon\|ꞓ_0\|}$ ∀ ꞓ , Ɏ ϵ ℵ where ƅ > 0 .

there is Single cubic function Ƶ:ℵ⟶ Ɏ

Satisfies (1.1)where

$℘_{Ƶ(ꞓ)-Ʈ(ꞓ)}$(ƅ)≥ $\frac{ƅ}{ƅ+\frac{\varepsilon\|ꞓ_0\|}{(27-ὤ)}}$ ∀ꞓ ∈ ℵ where ƅ > 0.

where $Ƶ(ꞓ) = \lim_{j\to\infty}\left\{\frac{ß(3^J ꞓ)}{3^{3J}}\right\}$.

Proof. The outcome will be reached immediately, if put $ϕ_{ꞓ,y}$ (ƅ) = $\frac{ƅ}{ƅ+\varepsilon\|ꞓ_0\|}$

in Theorem (3.1). ∀ ꞓ , Ɏ ϵ ℵ and ƅ > 0 And choose 0< ὤ < 27.

corollary 4.3. consider ℵ the real linear space, ß the function from ℵ to a complete ȒṄ-space (Ɏ, ℘, ₳M) and satisfying $℘_{D_S ß(ꞓ,Ɏ)}$ (ƅ) ≥$\frac{ƅ}{ƅ+V(\|ꞓ\|^{Ꞧ}+\|Ɏ\|^{Ꞧ})}$, ∀ ꞓ, Ɏ ∈ ℵ where ƅ > 0, Ꞧ ∈ R.

there is Single cubic function Ƶ:ℵ⟶ Ɏ Satisfies (1.1)where

$℘_{Ƶ(ꞓ)-h(ꞓ)}$(ƅ) $\frac{ƅ}{ƅ+\frac{V(\|ꞓ\|^{Ꞧ})}{(27-ὤ)}}$ ∀ꞓ ∈ ℵ, V ≥ 0 where ƅ > 0.

where $Ƶ(ꞓ) = \lim_{j\to\infty}\left\{\frac{ß(3^J ꞓ}{3^{3J}}\right\}$

Proof. The outcome will be reached immediately, if put $ϕ_{ꞓ,y}$ (ƅ) = $\frac{ƅ}{ƅ+V(\|ꞓ\|^{Ꞧ}+\|Ɏ\|^{Ꞧ})}$

in Theorem (4.1). ∀ ꞓ , Ɏ ϵ ℵ, V ≥ 0 and ƅ > 0. And choose $3^{Ꞧ}$ ≤ α < 27, Ꞧ ∈ R.

Corollary 4.4.consder ℵ the real linear space, ß the function from ℵ to a complete ȒṄ-space (Ɏ, ℘, ₳M) and satisfying $℘_{D_S ß(ꞓ,Ɏ)}$ (ƅ) ≥1 − $\frac{\|ꞓ\|}{ƅ+\|ꞓ\|}$ ∀ ꞓ ∈ ℵ and ƅ > 0.

there is Single cubic function Ƶ:ℵ⟶ Ɏ Satisfies (1.1) where

$℘_{Ƶ(ꞓ)-h(ꞓ)}$(ƅ)≥ 1 − $\frac{\|ꞓ\|}{(27-ὤ)ƅ+\|ꞓ\|}$,∀ ꞓ ∈ ℵ and ƅ > 0.

where $Ƶ(ϗ) = \lim_{j\to\infty} \left\{\frac{ß(3^J ϗ}{3^{3J}}\right\}$

Proof. The outcome will be reached immediately, if put $\phi_{ϗ,ÿ}(Ϸ) = \frac{Ϸ}{Ϸ+\varepsilon\|ϗ_0\|}$

in Theorem (4.1). ∀ ϗ , ÿ ϵ ℵ and Ϸ > 0. And choose 3≤ ῷ < 27.

5. Conclusion

In this paper, the stability of the cubic functional equation (1.1) has been proved and satisfied in the range of the random normed space (ȒṄ −space) by using the fixed point method. Finally, we contributed our works to obtained good results by applying our model as mentioned in the above equation.

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