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Generalization Approximation Spaces Using Combined Edges Systems of a finite number of undirected graphs

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ABSTRACT

In this paper we will study approximate coefficients. New based on a finite family of Lower approximation and Upper approximation and we present a generalization of some concepts and definitions and boundary and we will also study the accuracy factor for this family

MSC..

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1. Introduction and preliminaries

Graph theory is closely related with set theory and matrix theory. Mathematics' topological graph theory has a wide range of theoretical and practical applications [1, 2, 3, 4, 5, 8, and 9]. We predicted that topological graph structure will play a crucial role in bridging the gap between topology and applications. For all concepts and notation relating to graph theory, we cite Harary [6], and for all terms and notation relating to topology, we cite Moller [7]. These graph theory essential concepts are listed in [10]. A und. g. or graph is pair $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ where $\mho(\Omega)$ is a non-empty set whose elements are called points or vertices (called vertex set) and $\mathcal{E}(\Omega)$ is the set of unordered pairs of elements of $\mho(\Omega)$ (called edge set). An edge of a graph that joins a vertex to itself is called a loop. If two edges of a graph are joined by an vertex then these edges are called the edges ϱ incident with the edges ϱ_1. The set of ϱ is $\{\varrho_1 \in \mathcal{E}(\Omega)\colon \varrho_1 \text{incident with } \varrho\}$ and the edges ϱ non incident with the edges ϱ_1. The set of ϱ is $\{\varrho_1 \in \mathcal{E}(\Omega)\colon \varrho_1 \text{nonincident with } \varrho\}$. A sub graph of a graph Ω is a graph each of whose vertices belong to $\mho(\Omega)$ and each of

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whose edges belong to $\mathcal{E}(\Omega)$. An empty graph if the vertices set and edge set is empty. A degree of a vertex $\mathfrak{V}$ in a graph Ω is the number of edges of Ω incident with $\mathfrak{V}$. Let $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ be und. g. and a edge $\varrho \in \mathcal{E}(\Omega)$.The incident edges set of ϱ is denoted by $\mathrm{I}\mathcal{E}(\varrho)$ and defined by $\mathrm{I}\mathcal{E}(\varrho) = \{\varrho_1 \in \mathcal{E}(\Omega): \varrho_1 \text{incident with } \varrho\}$ and The non-incident edges set of ϱ is denoted by $\mathrm{NI}\mathcal{E}(\varrho)$ and defined by $\mathrm{NI}\mathcal{E}(\varrho) = \{\varrho_1 \in \mathcal{E}(\Omega): \varrho_1 \text{nonincident with } \varrho\}$. The incident edges system (resp. non incident edges system) of an edge $\varrho \in \mathcal{E}(\Omega)$ is denoted by $\mathrm{I}\mathcal{E}S(\varrho)$ (resp. $NI\mathcal{E}S(\varrho)$) and defined by : $\mathrm{I}\mathcal{E}S(\varrho) = \{\mathrm{I}\mathcal{E}(\varrho)\}$ (resp. $NI\mathcal{E}S(\varrho) = \{\mathrm{NI}\mathcal{E}(\varrho)\}$). The Combined edges System of a edge $\varrho \in \mathcal{E}(\Omega)$ is denoted by $C\mathcal{E}S(\varrho)$ and defined by $C\mathcal{E}S(\varrho) = \{\mathrm{I}\mathcal{E}S(\varrho), NI\mathcal{E}S(\varrho)\}$. A edge $\varrho \in \mathcal{E}(\Omega)$ is called isolated edge if $\{\varrho \in \mathcal{E}(\Omega);\ \exists C\mathcal{E}S(\varrho) \cap (\mathcal{E}(\text{ƕ}) - \{\varrho\}) = \phi\}$. lower approximations of ƕ using Combined edges systems is denoted by $L_{c_i}(\mathcal{E}(\text{ƕ})), i = 1,2,3, \dots, n$ and defined by $\{\varrho \in \mathcal{E}(\Omega); \exists\, C_i\mathcal{E}(\varrho) \subseteq \mathcal{E}(\text{ƕ}), i = 1,2,3, \dots, n\}$ and upper lower approximations of ƕ using Combined edges systems is denoted by $U_{c_i}(\mathcal{E}(\text{ƕ})), i = 1,2,3, \dots, n$ and defined by $\{\varrho \in \mathcal{E}(\Omega); \forall\, C_i\mathcal{E}(\varrho) \cap \mathcal{E}(\text{ƕ}) \neq \phi, i = 1,2,3, \dots, n\}$.

2. New Approximation Operators Based On a Finite Family Of und.g. Using Combined Edges Systems.

This section studies some of their definitions and hypotheses regarding novel approximation operators on a finite family of und. g. using Combined edges systems, and provides instances of qualities that are not generally true.

Definition 2.1. Let $\Omega = \{\Omega_i; \boldsymbol{i = 1, 2, 3, \dots, n}\}$ be a finite family of arbitrary non-empty finite und. g. The n- lower and n- upper approximations of ƕ $\subseteq \Omega$ according to Ω are denoted by $\boldsymbol{L}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ}))$ and $\boldsymbol{U}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ}))$ and defined by:

$$\boldsymbol{L}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) = \cup_i^{\mathbf{n}} \boldsymbol{L}_{\boldsymbol{c_i}}(\boldsymbol{\mathcal{E}}(\text{ƕ})),$$

$$\boldsymbol{U}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) = \cap_i^{\mathbf{n}} \boldsymbol{U}_{\boldsymbol{c_i}}(\boldsymbol{\mathcal{E}}(\text{ƕ})),$$

Definition 2.2. Let $\Omega = \{\Omega_i; \boldsymbol{i = 1, 2, 3, \dots, n}\}$ be a finite family of arbitrary non-empty finite und. g. The n-boundary n-positive and n-negative approximations of ƕ $\subseteq \Omega$ according to Ω are denoted by $\boldsymbol{Bd}_{\mathbf{n}}(\mathcal{E}(\text{ƕ})), \boldsymbol{POS}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ}))$ and $\boldsymbol{NEG}_{\mathbf{n}}(\mathcal{E}(\text{ƕ}))$ and defined by:

$$\boldsymbol{Bd}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) = \boldsymbol{U}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) - \boldsymbol{L}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})),$$

$$\boldsymbol{POS}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) = \boldsymbol{L}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})),$$

$$\boldsymbol{NEG}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})) = \boldsymbol{\mathcal{E}}(\Omega) - \boldsymbol{U}_{\mathbf{n}}(\boldsymbol{\mathcal{E}}(\text{ƕ})),$$

Definition 2.3: Let $\Omega = \{\Omega_i; i = 1,2,3, \dots, \mathrm{n}\}$ be a finite family of arbitrary non-empty finite und. g. The n- accuracy measure of approximations of ƕ $\subseteq \Omega$ is denoted by $\zeta_{\mathrm{n}}(\mathcal{E}(\text{ƕ}))$ and defined by:

$$\zeta_{\mathrm{n}}(\mathcal{E}(\text{ƕ})) = 1 - \frac{|Bd_{\mathrm{n}}(\mathcal{E}(\text{ƕ}))|}{|\mathcal{E}(\Omega)|}$$

It is obvious that $0 \leq \zeta_{\mathrm{n}}(\mathcal{E}(\text{ƕ})) \leq 1$ Moreover, if $\zeta_{\mathrm{n}}(\mathcal{E}(\text{ƕ})) = 1$ then ƕ is called ƕ-definable (ƕ-exact) und. g. otherwise, it is called ƕ-rough.

Example 2.4. Let $\Omega = \{\Omega_i; i = 1,2,3\}$ such that $\mathcal{E}(\Omega) = \mathcal{E}(\Omega_1) = \mathcal{E}(\Omega_2) = \mathcal{E}(\Omega_3) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$, $\mho(\Omega_1) = \{\mathfrak{V}_1, \mathfrak{V}_2, \mathfrak{V}_3, \mathfrak{V}_4\}$

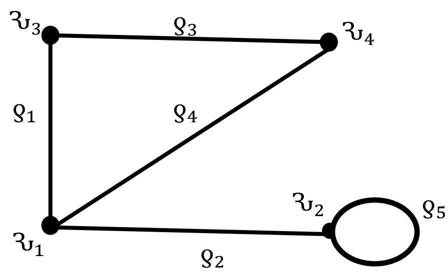

Figure 2.1: und. g. Ω_1 given in Example (2.4).

$Ʊ(\Omega_2) = \{Ʋ_1, Ʋ_2, Ʋ_3, Ʋ_4, Ʋ_5\}$

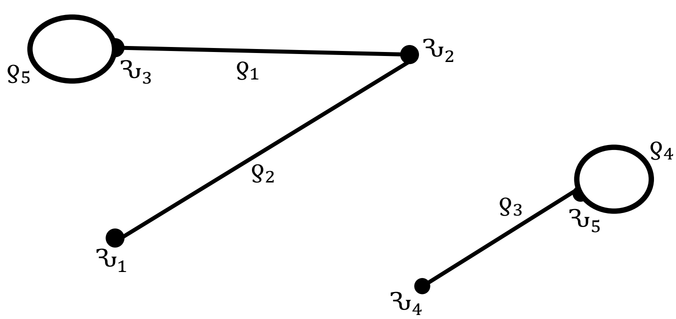

Figure 2.2: und. g. Ω_2 given in Example (2.4).

$Ʊ(\Omega_3) = \{Ʋ_1, Ʋ_2, Ʋ_3\}$

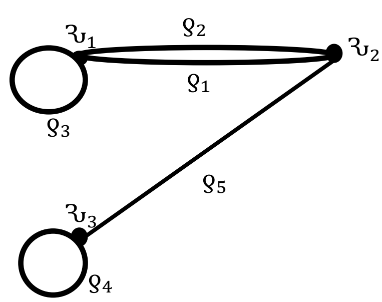

Figure 2.3: und. g. Ω_3 given in Example (2.4).

Are three arbitrary graphs on $\mathcal{E}(\Omega)$. So we have:

The combined edges systems based on Ω_1 are given by:

$Þ_{c_1}(\varrho_1) = \{\{\varrho_2, \varrho_3, \varrho_4\}, \{\varrho_5\}\}$, $Þ_{c_1}(\varrho_2) = \{\{\varrho_1, \varrho_4, \varrho_5\}, \{\varrho_3\}\}$, $Þ_{c_1}(\varrho_3) = \{\{\varrho_1, \varrho_4\}, \{\varrho_2, \varrho_5\}\}$, $Þ_{c_1}(\varrho_4) = \{\{\varrho_1, \varrho_2, \varrho_3\}, \{\varrho_5\}\}$, $Þ_{c_1}(\varrho_5) = \{\{\varrho_2, \varrho_5\}, \{\varrho_1, \varrho_3, \varrho_4\}\}$

The combined edges systems based on Ω_2 are given by:

$\text{Þ}_{c_2}(\varrho_1) = \{\{\varrho_2,\varrho_5\},\{\varrho_3,\varrho_4\}\}, \text{Þ}_{c_2}(\varrho_2) = \{\{\varrho_1\},\{\varrho_3,\varrho_4,\varrho_5\}\}, \text{Þ}_{c_2}(\varrho_3) = \{\{\varrho_4\},\{\varrho_1,\varrho_2,\varrho_5\}\}, \text{Þ}_{c_2}(\varrho_4) = \{\{\varrho_3,\varrho_4\},\{\varrho_1,\varrho_2,\varrho_5\}\}, \text{Þ}_{c_2}(\varrho_5) = \{\{\varrho_1,\varrho_5\},\{\varrho_2,\varrho_3,\varrho_4\}\}$

The combined edges systems based on Ω_3 are given by:

$\text{Þ}_{c_3}(\varrho_1) = \{\{\varrho_2,\varrho_3,\varrho_5\},\{\varrho_4\}\}, \text{Þ}_{c_3}(\varrho_2) = \{\{\varrho_1,\varrho_3,\varrho_5\},\{\varrho_4\}\}, \text{Þ}_{c_3}(\varrho_3) = \{\{\varrho_1,\varrho_2,\varrho_3\},\{\varrho_4,\varrho_5\}\}, \text{Þ}_{c_3}(\varrho_4) = \{\{\varrho_4,\varrho_5\},\{\varrho_1,\varrho_2,\varrho_3\}\}, \text{Þ}_{c_3}(\varrho_5) = \{\{\varrho_1,\varrho_2,\varrho_4\},\{\varrho_3\}\}$.

Accordingly ,can be obtain the following table

Table 2.1: $L_{c_1}(\mathcal{E}(\text{ƕ})), L_{c_2}(\mathcal{E}(\text{ƕ})), L_{c_3}(\mathcal{E}(\text{ƕ}))$ and $L_{c_n}(\mathcal{E}(\text{ƕ}))$ for all $\text{ƕ} \subseteq \Omega$.

$\mathcal{E}(\text{ƕ})$	$L_{c_1}(\mathcal{E}(\text{ƕ}))$	$L_{c_2}(\mathcal{E}(\text{ƕ}))$	$L_{c_3}(\mathcal{E}(\text{ƕ}))$	$L_{c_n}(\mathcal{E}(\text{ƕ}))$
$\{\varrho_1\}$	ϕ	$\{\varrho_2\}$	ϕ	$\{\varrho_2\}$
$\{\varrho_2\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_3\}$	$\{\varrho_2\}$	ϕ	$\{\varrho_5\}$	$\{\varrho_2,\varrho_5\}$
$\{\varrho_4\}$	ϕ	$\{\varrho_3\}$	$\{\varrho_1,\varrho_2\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	ϕ	ϕ	$\{\varrho_1,\varrho_4\}$
$\{\varrho_1,\varrho_2\}$	ϕ	$\{\varrho_2\}$	ϕ	$\{\varrho_2\}$
$\{\varrho_1,\varrho_3\}$	$\{\varrho_2\}$	$\{\varrho_2\}$	$\{\varrho_5\}$	$\{\varrho_2,\varrho_5\}$
$\{\varrho_1,\varrho_4\}$	$\{\varrho_3\}$	$\{\varrho_2,\varrho_3\}$	$\{\varrho_1,\varrho_2\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_1,\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	$\{\varrho_2,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_2,\varrho_3\}$	$\{\varrho_2\}$	ϕ	$\{\varrho_5\}$	$\{\varrho_2,\varrho_5\}$
$\{\varrho_2,\varrho_4\}$	ϕ	$\{\varrho_3\}$	$\{\varrho_1,\varrho_2\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1\}$	ϕ	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_3,\varrho_4\}$	$\{\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4\}$	ϕ	$\{\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	ϕ	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\{\varrho_2,\varrho_4\}$	$\{\varrho_2\}$	$\{\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\{\varrho_3\}$	$\{\varrho_2,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	ϕ	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_1,\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$	$\{\varrho_1,\varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_3,\varrho_4,\varrho_1\}$	$\{\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$
$\{\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_4,\varrho_5,\varrho_1\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$
$\{\varrho_4,\varrho_5,\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4\}$	$\{\varrho_2,\varrho_5\}$	$\{\varrho_2,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$

$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
ϕ	ϕ	ϕ	ϕ	ϕ

Table 2.2: $U_{c_1}(\mathcal{E}(\hbar))$, $U_{c_2}(\mathcal{E}(\hbar))$, $U_{c_3}(\mathcal{E}(\hbar))$ and $U_{c_n}(\mathcal{E}(\hbar))$ for all $\hbar \subseteq \Omega$.

$\mathcal{E}(\hbar)$	$U_{c_1}(\mathcal{E}(\hbar))$	$U_{c_2}(\mathcal{E}(\hbar))$	$U_{c_3}(\mathcal{E}(\hbar))$	$U_{c_n}(\mathcal{E}(\hbar))$
$\{\varrho_1\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_2\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_3\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_4\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_1, \varrho_2\}$	$\{\varrho_3, \varrho_5\}$	$\{\varrho_5\}$	ϕ	ϕ
$\{\varrho_1, \varrho_3\}$	$\{\varrho_2\}$	$\{\varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_5\}$	ϕ
$\{\varrho_1, \varrho_4\}$	ϕ	$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_2, \varrho_3, \varrho_4\}$	ϕ
$\{\varrho_1, \varrho_5\}$	$\{\varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_2\}$	$\{\varrho_3, \varrho_4\}$	ϕ
$\{\varrho_2, \varrho_3\}$	$\{\varrho_5\}$	$\{\varrho_1, \varrho_4\}$	$\{\varrho_5\}$	ϕ
$\{\varrho_2, \varrho_4\}$	$\{\varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$	$\{\varrho_3\}$
$\{\varrho_2, \varrho_5\}$	$\{\varrho_1, \varrho_4\}$	$\{\varrho_5\}$	$\{\varrho_3, \varrho_4\}$	ϕ
$\{\varrho_3, \varrho_4\}$	$\{\varrho_2\}$	ϕ	$\mathcal{E}(\Omega)$	ϕ
$\{\varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_4, \varrho_5\}$	$\{\varrho_3, \varrho_4\}$	$\{\varrho_4\}$
$\{\varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_2\}$	$\{\varrho_1\}$
$\{\varrho_1, \varrho_2, \varrho_3\}$	$\{\varrho_2, \varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_5\}$	$\{\varrho_5\}$
$\{\varrho_1, \varrho_2, \varrho_4\}$	$\{\varrho_3, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_3\}$
$\{\varrho_1, \varrho_2, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_2, \varrho_5\}$	$\{\varrho_3, \varrho_4\}$	ϕ
$\{\varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_2, \varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_3\}$
$\{\varrho_2, \varrho_3, \varrho_5\}$	$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_4, \varrho_5\}$	$\{\varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_4, \varrho_5\}$
$\{\varrho_3, \varrho_4, \varrho_1\}$	$\{\varrho_2\}$	$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_2\}$
$\{\varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_4, \varrho_5, \varrho_1\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$
$\{\varrho_4, \varrho_5, \varrho_2\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$
$\{\varrho_1, \varrho_3, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$	$\{\varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_3, \varrho_5\}$
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_2, \varrho_3, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\{\varrho_2, \varrho_3, \varrho_5\}$
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_3, \varrho_4, \varrho_5\}$	$\{\varrho_4, \varrho_5\}$
$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$
$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	$\{\varrho_1, \varrho_3, \varrho_4\}$

$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
ϕ	ϕ	ϕ	ϕ	ϕ

Table 2.3: $Bd_{c_1}(\mathcal{E}(\text{ƕ}))$, $Bd_{c_2}(\mathcal{E}(\text{ƕ}))$, $Bd_{c_3}(\mathcal{E}(\text{ƕ}))$ and $Bd_{c_n}(\mathcal{E}(\text{ƕ}))$ for all $\text{ƕ} \subseteq \Omega$.

$\mathcal{E}(\text{ƕ})$	$Bd_{c_1}(\mathcal{E}(\text{ƕ}))$	$Bd_{c_2}(\mathcal{E}(\text{ƕ}))$	$Bd_{c_3}(\mathcal{E}(\text{ƕ}))$	$Bd_{c_n}(\mathcal{E}(\text{ƕ}))$
$\{\varrho_1\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_2\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_3\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_4\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_1,\varrho_2\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_3\}$	ϕ	$\{\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_4\}$	ϕ	$\{\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_1,\varrho_5\}$	$\{\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_2,\varrho_3\}$	$\{\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	ϕ	ϕ
$\{\varrho_2,\varrho_4\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_2,\varrho_5\}$	ϕ	$\{\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_3,\varrho_4\}$	ϕ	ϕ	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_3,\varrho_5\}$	$\{\varrho_5\}$	$\{\varrho_1,\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\{\varrho_5\}$	$\{\varrho_1,\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_1,\varrho_2,\varrho_5\}$	ϕ	ϕ	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\{\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_2,\varrho_3,\varrho_5\}$	ϕ	$\{\varrho_1,\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_3,\varrho_4,\varrho_1\}$	ϕ	$\{\varrho_5\}$	$\{\varrho_3,\varrho_4\}$	ϕ
$\{\varrho_3,\varrho_4,\varrho_5\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_5\}$	ϕ	ϕ
$\{\varrho_4,\varrho_5,\varrho_1\}$	$\{\varrho_5\}$	$\{\varrho_1,\varrho_4\}$	ϕ	ϕ
$\{\varrho_4,\varrho_5,\varrho_2\}$	ϕ	$\{\varrho_4,\varrho_5\}$	ϕ	ϕ
$\{\varrho_1,\varrho_3,\varrho_5\}$	$\{\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_3\}$	$\{\varrho_3,\varrho_4\}$	$\{\varrho_3\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	ϕ	ϕ	ϕ	ϕ
$\mathcal{E}(\Omega)$	ϕ	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ	ϕ

Table 2.4: $\zeta_{c_1}(\mathcal{E}(\text{ƕ}))$, $\zeta_{c_2}(\mathcal{E}(\text{ƕ}))$, $\zeta_{c_3}(\mathcal{E}(\text{ƕ}))$ and $\zeta_{c_n}(\mathcal{E}(\text{ƕ}))$ for all ƕ ⊆ Ω.

$\mathcal{E}(\text{ƕ})$	$\zeta_{c_1}(\mathcal{E}(\text{ƕ}))$	$\zeta_{c_2}(\mathcal{E}(\text{ƕ}))$	$\zeta_{c_3}(\mathcal{E}(\text{ƕ}))$	$\zeta_{c_n}(\mathcal{E}(\text{ƕ}))$
$\{\varrho_1\}$	1	1	1	1
$\{\varrho_2\}$	1	1	1	1
$\{\varrho_3\}$	1	1	1	1
$\{\varrho_4\}$	1	1	1	1
$\{\varrho_5\}$	1	1	1	1
$\{\varrho_1, \varrho_2\}$	3/5	4/5	1	1
$\{\varrho_1, \varrho_3\}$	1	3/5	1	1
$\{\varrho_1, \varrho_4\}$	1	3/5	3/5	1
$\{\varrho_1, \varrho_5\}$	3/5	1	3/5	1
$\{\varrho_2, \varrho_3\}$	4/5	3/5	1	1
$\{\varrho_2, \varrho_4\}$	3/5	3/5	3/5	1
$\{\varrho_2, \varrho_5\}$	1	4/5	3/5	1
$\{\varrho_3, \varrho_4\}$	1	1	3/5	1
$\{\varrho_3, \varrho_5\}$	4/5	2/5	3/5	1
$\{\varrho_4, \varrho_5\}$	3/5	1/5	1	1
$\{\varrho_1, \varrho_2, \varrho_3\}$	3/5	2/5	1	1
$\{\varrho_1, \varrho_2, \varrho_4\}$	4/5	2/5	3/5	1
$\{\varrho_1, \varrho_2, \varrho_5\}$	1	1	3/5	1
$\{\varrho_2, \varrho_3, \varrho_4\}$	3/5	1	3/5	1
$\{\varrho_2, \varrho_3, \varrho_5\}$	1	2/5	3/5	1
$\{\varrho_3, \varrho_4, \varrho_1\}$	1	4/5	3/5	1
$\{\varrho_3, \varrho_4, \varrho_5\}$	3/5	4/5	1	1
$\{\varrho_4, \varrho_5, \varrho_1\}$	4/5	3/5	1	1
$\{\varrho_4, \varrho_5, \varrho_2\}$	1	3/5	1	1
$\{\varrho_1, \varrho_3, \varrho_5\}$	3/5	3/5	3/5	4/5
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$	1	1	1	1
$\{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$	1	1	1	1
$\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}$	1	1	1	1
$\{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}$	1	1	1	1
$\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}$	1	1	1	1
$\mathcal{E}(\Omega)$	1	1	1	1
ϕ	1	1	1	1

Theorem 2.5. Let $\Omega = \{\Omega_i; i = 1,2,3, \dots, n\}$ be a finite family of arbitrary non-empty finite und. g. then the following hold for every ƕ, ƙ $\subseteq \Omega_i$

(1) $L_{c_n}(\mathcal{E}(\Omega)) = \mathcal{E}(\Omega)$.

(2) if $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$,then $L_{c_n}(\mathcal{E}(ƕ)) \subseteq L_{c_n}(\mathcal{E}(ƙ))$.

(3) $L_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq L_{c_n}(\mathcal{E}(ƕ)) \cap L_{c_n}(\mathcal{E}(ƙ))$.

(4) $L_{c_n}(\mathcal{E}(ƕ)) \cup L_{c_n}(\mathcal{E}(ƙ)) \subseteq L_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

Proof:

(1) It is clear $L_{c_n}(\mathcal{E}(\Omega)) \subseteq \mathcal{E}(\Omega) --- (1)$. And by Definition $L_{c_i}(\mathcal{E}(ƕ))$ and by Definition (2.1). We get $\mathcal{E}(\Omega) \subseteq L_{c_n}(\mathcal{E}(\Omega)) ---(2)$. From (1) and (2) we get $L_{c_n}(\mathcal{E}(\Omega)) = \mathcal{E}(\Omega)$.

(2) Let $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$ and $\varrho \in L_{c_n}(\mathcal{E}(ƕ)) \Rightarrow \exists i = 1,2,3,\dots,n \ni \varrho \in L_{c_i}(\mathcal{E}(ƕ)) \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \exists c_i \mathcal{E}(\varrho) \subseteq \mathcal{E}(ƕ), i = 1,2,3,\dots,n\} \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \exists c_i \mathcal{E}(\varrho) \subseteq \mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ), i = 1,2,3,\dots,n\} \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \exists c_i \mathcal{E}(\varrho) \subseteq \mathcal{E}(ƙ), i = 1,2,3,\dots,n\} \Rightarrow \exists i \ni \varrho \in L_{c_i}(\mathcal{E}(ƙ)) \Rightarrow \varrho \in \cup_{i=1}^{n} L_{c_i}(\mathcal{E}(ƙ)) \Rightarrow \varrho \in L_{c_n}(\mathcal{E}(ƙ)) \Rightarrow L_{c_n}(\mathcal{E}(ƕ)) \subseteq L_{c_n}(\mathcal{E}(ƙ))$.

(3) Since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƕ)$ by (2) above we get $L_{c_n}((\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ))) \subseteq L_{c_n}(\mathcal{E}(ƕ)) ---(1)$. And since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƙ)$ by (2) above we get $L_{c_n}((\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ))) \subseteq L_{c_n}(\mathcal{E}(ƙ)) --- (2)$. From (1) and (2) we get $L_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq L_{c_n}(\mathcal{E}(ƕ)) \cap L_{c_n}(\mathcal{E}(ƙ))$.

(4) Since $\mathcal{E}(ƕ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$by (2) above we get $L_{c_n}(\mathcal{E}(ƕ)) \subseteq L_{c_n}((\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))) --- (1)$. And since $\mathcal{E}(ƙ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by (2) above we get $L_{c_n}(\mathcal{E}(ƙ)) \subseteq L_{c_n}((\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))) --- (2)$. From (1) and (2) we get $L_{c_n}(\mathcal{E}(ƕ)) \cup L_{c_n}(\mathcal{E}(ƙ)) \subseteq L_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

Proposition 2.6: Let $\Omega = \{\Omega_i; i = 1,2,3,\dots,n\}$ be a finite family of arbitrary non-empty finite und. g.then the following hold for every $ƕ, ƙ \subseteq \Omega_i$

(1) $U_{c_n}(\emptyset) = \emptyset$.

(2) if $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$,then $U_{c_n}(\mathcal{E}(ƕ)) \subseteq U_{c_n}(\mathcal{E}(ƙ))$.

(3) $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ)) \cap U_{c_n}(\mathcal{E}(ƙ))$.

(4) $U_{c_n}(\mathcal{E}(ƕ)) \cup U_{c_n}(\mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

Proof:

(1) By Definition $U_{c_i}(\mathcal{E}(ƕ))$ and by Definition (2.1). We get $U_{c_n}(\emptyset) = \emptyset$.

(2) Let $\mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)$ and $\varrho \in U_{c_n}(\mathcal{E}(ƕ)) \Rightarrow \forall i = 1,2,3,\dots,n \ni \varrho \in U_{c_i}(\mathcal{E}(ƕ)) \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \forall c_i \mathcal{E}(\varrho) \cap \mathcal{E}(ƕ) \neq \emptyset, i = 1,2,3,\dots,n\} \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \forall c_i \mathcal{E}(\varrho) \cap \mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ) \neq \emptyset, i = 1,2,3,\dots,n\} \Rightarrow \varrho \in \{\varrho \in \mathcal{E}(\Omega); \forall c_i \mathcal{E}(\varrho) \cap \mathcal{E}(ƙ) \neq \emptyset, i = 1,2,3,\dots,n\} \Rightarrow \varrho \in U_{c_i}(\mathcal{E}(ƕ)) \; \forall i = 1,2,3,\dots,n \Rightarrow \varrho \in \cap_{i}^{n} U_{c_i}(\mathcal{E}(ƙ)) \Rightarrow \varrho \in U_{c_n}(\mathcal{E}(ƙ)) \Rightarrow L_{c_n}(\mathcal{E}(ƕ)) \subseteq L_{c_n}(\mathcal{E}(ƙ))$.

(3) Since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƕ)$ by (2) above we get $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ)) - - - (1)$. And since $(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq \mathcal{E}(ƙ)$ by (2) above we get $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƙ)) - - - (2)$. From (1) and (2) we get $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ)) \cap U_{c_n}(\mathcal{E}(ƙ))$.

(4) Since $\mathcal{E}(ƕ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by (2) above we get $U_{c_n}(\mathcal{E}(ƕ)) \subseteq U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (1)$. And since $\mathcal{E}(ƙ) \subseteq (\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$ by (2) above we get $U_{c_n}(\mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) - - - (2)$. From (1) and (2) we get $U_{c_n}(\mathcal{E}(ƕ)) \cup U_{c_n}(\mathcal{E}(ƙ)) \subseteq U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ))$.

Remark 2.7. Let $\Omega = \{\Omega_i; i = 1,2,3, \ldots, n\}$ be a finite family of arbitrary non-empty finite und. g. $ƕ, ƙ \subseteq \Omega_i$ then the following are not true in general:

(1) $L_{c_n}(\mathcal{E}(ƕ)) \subseteq \mathcal{E}(ƕ)$.

(2) $L_{c_n}(\emptyset) = \emptyset$.

(3) $L_{c_n}(\mathcal{E}(ƕ)) = L_{c_n}\left(L_{c_n}(\mathcal{E}(ƕ))\right)$.

(4) $L_{c_n}(\mathcal{E}(ƕ)) = U_{c_n}\left(L_{c_n}(\mathcal{E}(ƕ))\right)$.

(5) $L_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = L_{c_n}(\mathcal{E}(ƕ)) \cup L_{c_n}(\mathcal{E}(ƙ))$.

(6) $L_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) = L_{c_n}(\mathcal{E}(ƕ)) \cap L_{c_n}(\mathcal{E}(ƙ))$.

(7) $\mathcal{E}(ƕ) \subseteq U_{c_n}(\mathcal{E}(ƕ))$.

(8) $U_{c_n}(\mathcal{E}(\Omega)) = \mathcal{E}(\Omega)$.

(9) $U_{c_n}(\mathcal{E}(ƕ)) = L_{c_n}\left(U_{c_n}(\mathcal{E}(ƕ))\right)$.

(10) $U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = U_{c_n}(\mathcal{E}(ƕ)) \cup U_{c_n}(\mathcal{E}(ƙ))$.

(11) $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) = U_{c_n}(\mathcal{E}(ƕ)) \cap U_{c_n}(\mathcal{E}(ƙ))$. ,

The following example is applied to show this remark.

Example 2.9. Let $\Omega = \{\Omega_i; i = 1,2,3\}$ such that $\mathcal{E}(\Omega) = \mathcal{E}(\Omega_1) = \mathcal{E}(\Omega_2) = \mathcal{E}(\Omega_3) = \{ϱ_1, ϱ_2, ϱ_3, ϱ_4, ϱ_5\}$, $Ʊ(\Omega_1) = \{Ʋ_1, Ʋ_2, Ʋ_3, Ʋ_4\}$

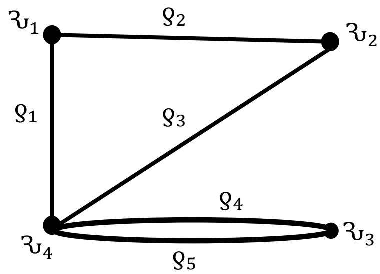

Figure 2.4: und. g. Ω_1 given in Example (2.9).

$\mho(\Omega_2) = \{\mathfrak{V}_1, \mathfrak{V}_2, \mathfrak{V}_3, \mathfrak{V}_4, \mathfrak{V}_5, \mathfrak{V}_6\}$

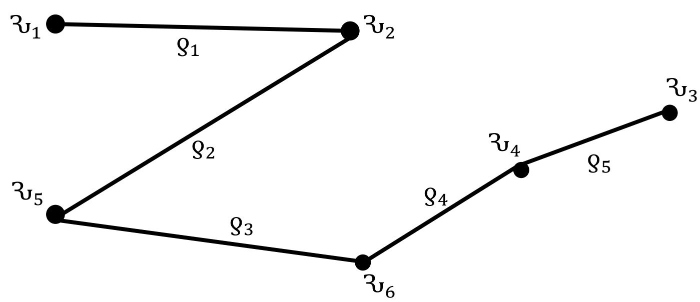

Figure 2.5: und. g. Ω_2 given in Example (2.9).

$\mho(\Omega_3) = \{\mathfrak{V}_1, \mathfrak{V}_2, \mathfrak{V}_3, \mathfrak{V}_4, \mathfrak{V}_5, \mathfrak{V}_6\}$

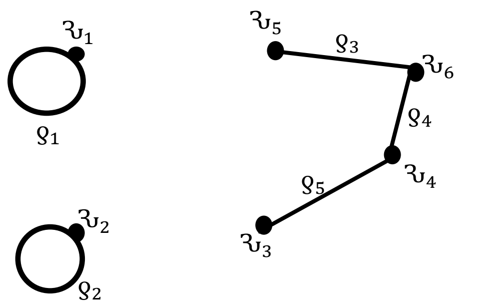

Figure 2.6: und. g. Ω_3 given in Example (2.9).

Are three arbitrary graphs on $\mathcal{E}(\Omega)$. So we have:

The combined edges systems based on Ω_1 are given by:

$Þ_{c_1}(\varrho_1) = \{\{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}, \phi\}, Þ_{c_1}(\varrho_2) = \{\{\varrho_1, \varrho_3\}, \{\varrho_4, \varrho_5\}\}, Þ_{c_1}(\varrho_3) = \{\{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}, \phi\}, Þ_{c_1}(\varrho_4) = \{\{\varrho_1, \varrho_3, \varrho_5\}, \{\varrho_2\}\}, Þ_{c_1}(\varrho_5) = \{\{\varrho_1, \varrho_3, \varrho_4\}, \{\varrho_2\}\}$

The combined edges systems based on Ω_2 are given by:

$Þ_{c_2}(\varrho_1) = \{\{\varrho_2\}, \{\varrho_3, \varrho_4, \varrho_5\}\}, Þ_{c_2}(\varrho_2) = \{\{\varrho_1, \varrho_3\}, \{\varrho_4, \varrho_5\}\}, Þ_{c_2}(\varrho_3) = \{\{\varrho_2, \varrho_4\}, \{\varrho_1, \varrho_5\}\}, Þ_{c_2}(\varrho_4) = \{\{\varrho_3, \varrho_5\}, \{\varrho_2, \varrho_1\}\}, Þ_{c_2}(\varrho_5) = \{\{\varrho_4\}, \{\varrho_1, \varrho_2, \varrho_3\}\}$

The combined edges systems based on Ω_3 are given by:

$Þ_{c_3}(\varrho_1) = \{\{\varrho_1\}, \{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}\}, Þ_{c_3}(\varrho_2) = \{\{\varrho_2\}, \{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}\}, Þ_{c_3}(\varrho_3) = \{\{\varrho_4\}, \{\varrho_1, \varrho_2, \varrho_5\}\}, Þ_{c_3}(\varrho_4) = \{\{\varrho_3, \varrho_5\}, \{\varrho_1, \varrho_2\}\}, Þ_{c_3}(\varrho_5) = \{\{\varrho_4\}, \{\varrho_1, \varrho_2, \varrho_3\}\}$

We get

1) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $L_{c_n}(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_3\}$ .Therefore, $L_{c_n}(\mathcal{E}(ƕ)) \nsubseteq \mathcal{E}(ƕ)$.

2) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \emptyset$ and $\mathcal{E}(ƕ) = \emptyset$. Then $L_{c_n}(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_3\}$.Therefore, $L_{c_n}(\mathcal{E}(ƕ)) \neq \emptyset$.

3) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $L_{c_n}(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_3\}, L_{c_n}\big(L_{cn}(\mathcal{E}(ƕ))\big) = \{\varrho_1, \varrho_2, \varrho_3\}$. Therefore $L_{c_n}(\mathcal{E}(ƕ)) \neq L_{c_n}\left(L_{c_n}(\mathcal{E}(ƕ))\right)$.

4) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $L_{c_n}(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_3\}, U_{c_n}\big(L_{cn}(\mathcal{E}(ƕ))\big) = \emptyset$. Therefore $L_{c_n}(\mathcal{E}(ƕ)) \neq U_{c_n}\left(L_{c_n}(\mathcal{E}(ƕ))\right)$.

5) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_1\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_2, Ʋ_4, Ʋ_5, Ʋ_6\}$ and $\mathcal{E}(ƙ) = \{\varrho_3\}$ Then $L_{c_n}(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_3\}$, $L_{c_n}(\mathcal{E}(ƙ)) = \{\varrho_1, \varrho_3\}$, $L_{c_n}(\mathcal{E}(ƕ)) \cup L_{c_n}\big(\mathcal{E}(ƙ)\big) = \{\varrho_1, \varrho_3\}$, $L_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = \{\varrho_1, \varrho_2, \varrho_3\}$Therefore, $L_{c_n}\big(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)\big) \neq L_{c_n}\big(\mathcal{E}(ƕ)\big) \cup L_{c_n}\big(\mathcal{E}(ƙ)\big)$.

6) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_5\}$ and $\mathcal{E}(ƕ) = \{\varrho_2\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_3, Ʋ_4, Ʋ_6\}$ and $\mathcal{E}(ƙ) = \{\varrho_4\}$ Then $L_{c_n}(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega)$, $L_{c_n}(\mathcal{E}(ƙ)) = \{\varrho_1, \varrho_3, \varrho_5\}$, $L_{c_n}(\mathcal{E}(ƕ)) \cap L_{c_n}\big(\mathcal{E}(ƙ)\big) = \{\varrho_1, \varrho_3, \varrho_5\}$, $L_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) = \{\varrho_1, \varrho_3\}$Therefore, $L_{c_n}\big(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)\big) \neq L_{c_n}\big(\mathcal{E}(ƕ)\big) \cap L_{c_n}\big(\mathcal{E}(ƙ)\big)$.

7) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $U_{c_n}(\mathcal{E}(ƕ)) = \emptyset$ .Therefore, $\mathcal{E}(ƕ) \nsubseteq U_{c_n}(\mathcal{E}(ƕ))$.

8) Let $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ such that $Ʊ(\Omega) = \{Ʋ_1, Ʋ_2, Ʋ_3, Ʋ_4, Ʋ_5, Ʋ_6\}$ and $\mathcal{E}(ƕ) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$. Then $U_{c_n}(\mathcal{E}(\Omega)) = \{\varrho_2, \varrho_4, \varrho_5\}$.Therefore, $\mathcal{E}(\Omega) \neq U_{c_n}(\mathcal{E}(\Omega))$.

9) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $U_{c_n}(\mathcal{E}(ƕ)) = \emptyset$, $L_{c_n}\big(U_{cn}(\mathcal{E}(ƕ))\big) = \{\varrho_1, \varrho_3\}$.Therefore, $U_{c_n}(\mathcal{E}(ƕ)) \neq L_{c_n}\left(U_{c_n}(\mathcal{E}(ƕ))\right)$.

10) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_5\}$ and $\mathcal{E}(ƕ) = \{ϱ_2\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_3, Ʋ_4\}$ and $\mathcal{E}(ƙ) = \{ϱ_5\}$ Then $U_{c_n}(\mathcal{E}(ƕ)) = \emptyset$, $U_{c_n}(\mathcal{E}(ƙ)) = \emptyset$, $U_{c_n}(\mathcal{E}(ƕ)) \cup U_{c_n}(\mathcal{E}(ƙ)) = \emptyset$, $U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) = \{ϱ_4\}$ Therefore, $U_{c_n}(\mathcal{E}(ƕ) \cup \mathcal{E}(ƙ)) \neq U_{c_n}(\mathcal{E}(ƕ)) \cup U_{c_n}(\mathcal{E}(ƙ))$.

11) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{Ʋ_1, Ʋ_2, Ʋ_4, Ʋ_5, Ʋ_6\}$ and $\mathcal{E}(ƕ) = \{ϱ_2, ϱ_3\}$.And $ƙ = (Ʊ(ƙ), \mathcal{E}(ƙ))$ such that $Ʊ(ƙ) = \{Ʋ_1, Ʋ_2, Ʋ_3, Ʋ_4, Ʋ_5\}$ and $\mathcal{E}(ƙ) = \{ϱ_1, ϱ_2, ϱ_5\}$ Then $U_{c_n}(\mathcal{E}(ƕ)) = \{ϱ_4\}$, $U_{c_n}(\mathcal{E}(ƙ)) = \{ϱ_2, ϱ_4\}$, $U_{c_n}(\mathcal{E}(ƕ)) \cap U_{c_n}(\mathcal{E}(ƙ)) = \{ϱ_4\}$, $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) = \emptyset$ Therefore, $U_{c_n}(\mathcal{E}(ƕ) \cap \mathcal{E}(ƙ)) \neq U_{c_n}(\mathcal{E}(ƕ)) \cap U_{c_n}(\mathcal{E}(ƙ))$.

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