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ODigraphic Topology On Directed Edges

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A R T I C L E I N F O

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A B S T R A C T. IN THIS PAPER, WE STUDY THE ODIGRAPHIC TOPOLOGY τ_{OD} FOR A DIRECTED EDGES OF A DIGRAPH. WE GIVE SOME PROPERTIES OF THIS TOPOLOGY, IN PARTICULAR WE PROVE THAT τ_{OD} IS AN ALEXANDROFF TOPOLOGY AND WHEN TWO DIGRAPHS ARE ISOMORPHIC, THEIR ODIGRAPHIC TOPOLOGIES WILL BE HOMEOMORPHIC. WE GIVE SOME PROPERTIES MATCHING DIGRAPHS AND HOMEOMORPHIC TOPOLOGY SPACES. FINALLY, WE INVESTIGATE THE CONNECTEDNESS OF THIS TOPOLOGY AND SOME RELATIONS BETWEEN THE CONNECTEDNESS OF THE DIGRAPH AND THE TOPOLOGY τ_{OD}.

MSC..

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1. Introduction.

Topological structures are mathematical models that can be used to analyze data without the concept of distance. Topological structures, in our opinion, are a crucial adjustment for the extraction and processing of knowledge [2]. This publication provides a few topological fundamentals that are pertinent to our study. One of the most crucial structures in discrete mathematics is the graph [1]. Two observations explain their pervasiveness. Graphs are mathematically elegant, to start, from a theoretical standpoint. Although a graph merely has a set of vertices and a relationship between pairs of vertices, it is a simple structure, yet graph theory is a vast and diverse field of study. This is partially because graphs can be thought of as topological spaces, combinatorial objects, and many other mathematical structures in addition to being relational structures [1]. This brings us to our second argument about the significance of graphs: many ideas may be abstractly represented by graphs[3], which makes them very helpful in real-world applications. Several earlier studies on the subject of topological graphs we can see in [4-11]. In this paper we discuss a new method to generate topology τ_{OD} on graph by using new method of taking neighborhood is determining a vertex on the digraph and calculate each vertex and its edges outdgree of it.

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2.Preliminaries

In this work, some basic notions of graph theory [1-2], and topology [2] are presented. A graph (resp., directed graph or digraph) Ɗ = (V̧, Ȩ) consists of a vertex set V̧ and an edge set Ȩ of unordered (resp., ordered) pairs of elements of V̧. To avoid ambiguities, we assume that the vertex and edge sets are disjoint. We say that two vertices v̧ and w of a graph (resp., digraph Ɗ) are adjacent if there is an edge of the form v̧w (rsep., $\overrightarrow{wv̧}$ or $\overrightarrow{v̧w}$) joining them, and the vertices v̧ and w are then incident with such an edge. A subdigraph H of a digraph Ɗ is a digraph, each of whose vertices belong to V̧ and each of whose edges belong to Ȩ. The degree of a vertex v̧ of Ɗ is the number of edges incident with v̧, and written deg(v̧). A vertex of degree zero is an isolated vertex. In digraph, the outdegree, of a vertex v̧ of Ɗ is the number of edges of the form $\overrightarrow{v̧w}$ and denoted by $d^+(v̧)$, similarly, the outdegree of a vertex v̧ of Ɗ is the number of edges of the form $\overrightarrow{wv̧}$, and denoted by $d^-(v̧)$. A vertex of out-degree and in-degree are zero is an isolated vertex". A topology τ on a set X is a combination of subset of X, called open, such that the union of the member of any subset of τ is a member of τ, the intersection of the members of any finite subset of τ is a member of τ, and both empty set and X are in τ and the ordered pair (X,τ) is called topological space. The topology $\tau = P(X)$ on X is called discrete topology while the topology $\tau = \{X, \emptyset\}$ on X is called indiscrete topology. A topology in which arbitrary intersection of open set is open called Alexandroff space.

3. Out Digraphic Topology.

New, we introduce our new subbasis family to generated a topology on the set of edges Ȩ $S_{OƊ}$ of a digraph Ɗ = (V̧,Ȩ).

Definition 3.1 : Let Ɗ = (V̧,Ȩ) be a digraph we defined $S_{OƊ} = \{\overleftarrow{Ȩ_{v̧}} \mid v̧ \in V̧\}$, where $\overleftarrow{Ȩ_{v̧}}$ is the set of all edges out indgree from v̧ we have $Ȩ = \bigcup_{v̧\in V̧} \overleftarrow{Ȩ_{v̧}}$. Hence $S_{OƊ}$ forms a subbasis for a topology $\tau_{OƊ}$ on Ȩ, called out digraphic topology (briefly odigtopology) of Ɗ.

Theorem 3.2: Let Ɗ = (V̧,Ȩ) be digraph then (Ȩ, $\tau_{OƊ}$) is topological space .

prove: we will prove that $\tau_{OƊ}$ is topological graph,

1) Since $Ȩ = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{OƊ}$ such that $\beta_{OƊ}$ is abasis for a topological graph $\tau_{OƊ}$,

then $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{OƊ}$ and $S_i = \overleftarrow{Ȩ_{v̧_i}}$, $v̧_i \in V̧$.then $Ȩ = \bigcup_{i\in I}(\overleftarrow{Ȩ_{v̧}})$ and so $Ȩ \in \tau_{OƊ}$. Also $\emptyset \in \tau_{OƊ}$ by complement of Ȩ.

2) Let $A_i \in \tau_{OƊ}$, $A_i = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{OƊ}$, $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{OƊ}$, $S_i = \overleftarrow{Ȩ_{v̧_i}}$, $v̧_i \in V̧$, then $A_i = \bigcup_{i\in I}(\bigcap_{j=1}^{n} \overleftarrow{Ȩ_{v̧_i}})$ where $\overleftarrow{Ȩ_{v̧_i}}$ is the set edges out directed from the vertices $A_i = \bigcup_{i\in I}(\bigcap_{j=1}^{n} \overleftarrow{Ȩ_{v̧_i}}) \in \tau_{OƊ}$, Therefore $\bigcup_{i\in I} A_i \in \tau_{OƊ}$.

3) Let $A_i , B_i \in \tau_{OƊ}$, $A_i = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{OƊ}$, $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{OƊ}$, $S_i = \overleftarrow{Ȩ_{v̧_i}}$, $v̧_i \in V̧$ then $A_i = \bigcup_{i\in I}(\bigcap_{j=1}^{n} \overleftarrow{Ȩ_{v̧_i}})$,and $B_i = \bigcup_{i\in I} W_i$ where $W_i \in \beta_{OƊ}$, $W_i = \bigcap_{i=1}^{n} S_i$ where $S_i \in S_{OƊ}$, $S_i = \overleftarrow{Ȩ_{v̧_i}}$, $v̧_i \in V̧$ then $B_i = \bigcup_{i\in I}(\bigcap_{j=1}^{n} I_{ȩ_i}$) then:

   i) If there are no element in intersection i.e., $A_i \cap B_i = \emptyset$ since $\emptyset \in \tau_{OƊ}$ then $A_i \cap B_i \in \tau_{OƊ}$.

   ii) If there exist element in intersection $U_i \cap W_i$ then we denote it $\overleftarrow{Ȩ_{v̧_i}}$, $v̧_i \in V̧$ since $A_i = \bigcup_{i\in I}(\bigcap_{j=1}^{n} \overleftarrow{Ȩ_{v̧_i}})$ and $B_i = \bigcup_{k\in I}(\bigcap_{j=1}^{n} \overleftarrow{Ȩ_{v̧_i}})$,Then $\overleftarrow{Ȩ_{v̧_i}}$ one of these Posts. Therefore $A_i \cap B_i \in \tau_{OƊ}$.

Example 3.3: Let Ɗ = (V̧,Ȩ) be digraph as in Figure (3.1), such that $V̧ = \{v̧_1, v̧_2, v̧_3, v̧_4, v̧_5, v̧_6\}$, $Ȩ = \{ȩ_1, ȩ_2, ȩ_3, ȩ_4, ȩ_5, ȩ_6\}$.

We have, $\overleftarrow{Ȩ_{v̧_1}} = \{ȩ_1\}$, $\overleftarrow{Ȩ_{v̧_2}} = \{ȩ_2, ȩ_6\}$, $\overleftarrow{Ȩ_{v̧_3}} = \{ȩ_3\}$, $\overleftarrow{Ȩ_{v̧_4}} = \{ȩ_4\}$, $\overleftarrow{Ȩ_{v̧_5}} = \{ȩ_5\}$, $\overleftarrow{Ȩ_{v̧_6}} = \emptyset$. and $S_{OƊ} = \{\emptyset, \{ȩ_1\}, \{ȩ_3\}, \{ȩ_4\}, \{ȩ_5\}, \{ȩ_2, ȩ_6\}\}$.

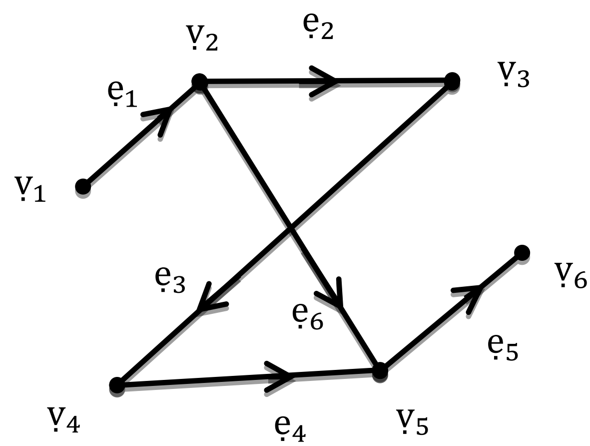
By taking finitely intersection the basis obtained is: $\{\emptyset, \{ȩ_1\}, \{ȩ_3\}, \{ȩ_4\}, \{ȩ_5\}, \{ȩ_2, ȩ_6\}\}$. Then by taking all unions the topology can be written as:

$\tau_{OƊ} = \{Ȩ(Ɗ), \emptyset, \{ȩ_1\}, \{ȩ_3\}, \{ȩ_4\}, \{ȩ_5\}, \{ȩ_2, ȩ_6\}, \{ȩ_1, ȩ_3\}, \{ȩ_1, ȩ_4\}, \{ȩ_1, ȩ_5\}, \{ȩ_3, ȩ_4\}, \{ȩ_3, ȩ_5\},$

$\{e_4, e_5\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_3, e_4, e_5\}, \{e_1, e_4, e_5\}, \{e_1, e_2, e_6\}, \{e_3, e_2, e_6\},$

$\{e_4, e_2, e_6\}, \{e_5, e_2, e_6\}, \{e_1, e_3, e_4, e_5\}, \{e_1, e_3, e_2, e_6\}, \{e_1, e_4, e_2, e_6\}, \{e_1, e_5, e_2, e_6\},$

$\{e_3, e_4, e_2, e_6\}, \{e_3, e_5, e_2, e_6\}, \{e_4, e_5, e_2, e_6\}, \{e_1, e_3, e_4, e_2, e_6\}, \{e_1, e_3, e_5, e_2, e_6\}, \{e_1, e_4, e_5, e_2, e_6\}, \{e_3, e_4, e_5, e_2, e_6\}\}$.
Then τ_{OD} is topology is called odigtopology τ_{OD} .



Figure(3.1), Digraph shows digtopology.

Remark 3.4: Let C_n be cyclic digraph if every edges are in the same directed then we get the odigtopology τ_{OD}on C_n is discrete, and if the edges are not all in the same direction we get the odigtopology τ_{OD}on C_n is not discrete.This remark illustrates in the next two examples .

Example 3.5: Let C_5 be cyclic digraph such that every edges are in the same direction, show in figure (2).

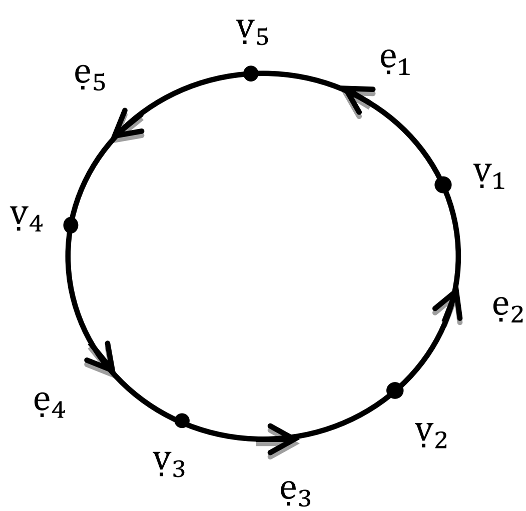


Figure (2), C_5 cyclic digraph it edges are same direction.

We have $E_{v_1} = \{e_1\}$, $E_{v_2}=\{e_2\}$, $E_{v_3}=\{e_3\}$, $E_{v_4}=\{e_4\}$, $E_{v_5}=\{e_5\}$.

And $S_{OD} = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\} , \{e_5\}\}$.

$\tau_{OD}= \{ E(D), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_1, e_5\}, \{e_2, e_3\}, \{e_2, e_4\},$

$\{e_2, e_5\}, \{e_3, e_4\}, \{e_3, e_5\}, \{e_4, e_5\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_5\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\},$

$\{e_1, e_4, e_5\}, \{e_2, e_3, e_4\}, \{e_2, e_3, e_5\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_5\},$

$\{e_1, e_2, e_4, e_5\}, \{e_1, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_5\}\}$ then we get the odigtopology τ_{OD} of C_5 is discrete topology.

Example 3.6: Let C_6 be cyclic digraph such that edges are not all in the same direction, show in figure (3).

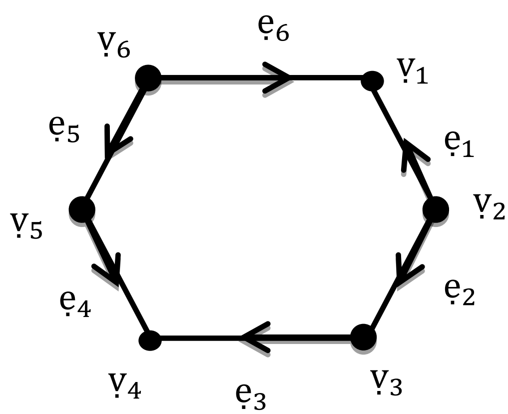


Figure (3), C_6 cyclic digraph it edges are not same direction.

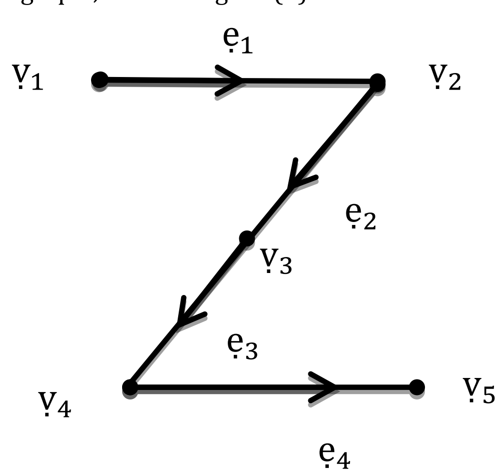
We have $E_{v_1} = \emptyset$, $E_{v_2} = \{e_1, e_2\}$, $E_{v_3} = \{e_3\}$, $E_{v_4} = \emptyset$, $E_{v_5} = \{e_4\}$, $E_{v_6} = \{e_5, e_6\}$. And $S_{OD} = \{\emptyset, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_5, e_6\}\}$.

$\tau_{OD} = \{ E(D), \emptyset, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_5, e_6\}, \{e_3, e_4\}, \{e_3, e_1, e_2\}, \{e_3, e_5, e_6\}, \{e_4, e_1, e_2\},$

$\{e_4, e_5, e_6\}, \{e_3, e_4, e_1, e_2\}, \{e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_5, e_6\}, \{e_3, e_1, e_2, e_5, e_6\}, \{e_4, e_1, e_2, e_5, e_6\} \}$ then we get the odigtopology τ_{OD} of C_6 is not discrete .

Remark 3.7: Let P_n be a path digraph if every edges are in the same directed then we get the odigtopology τ_{OD} on P_n is discrete, and if the edges are not all in the same direction we get the odigtopology τ_{OD} on P_n is not discrete .This remark illustrates in the next examples .

Example 3.8: Let P_5 be a path digraph , show in figure (4).



Figure(4), P_5 path digraph it edges are same direction.

We have $E_{v_1} = \{e_1\}$, $E_{v_2} = \{e_2\}$, $E_{v_3} = \{e_3\}$, $E_{v_4} = \{e_4\}$, $E_{v_5} = \emptyset$. And $S_{O\mathcal{D}} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$. $T_{O\mathcal{D}} = \{E(\mathcal{D}), \emptyset, \{e_1\}\{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\},$

$\{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_2, e_3, e_4\}\}$.We note the odigtopology $\tau_{O\mathcal{D}}$ is discret topology.

Example 3.9: Let P_5 be a path digraph, show in figure (5).

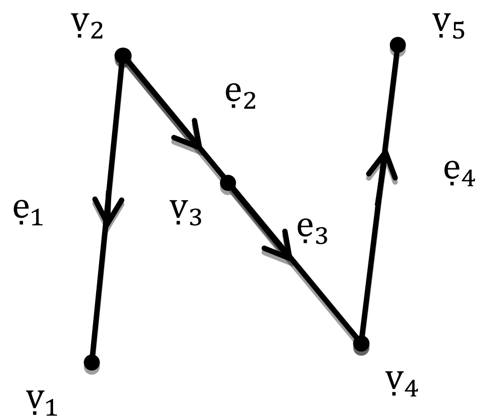


Figure (5), P_5 path digraph it one edge is not same direction.

We have $E_{v_1} = \emptyset, E_{v_2} = \{e_1, e_2\}$, $E_{v_3} = \{e_3\}$, $E_{v_4}\{e_4\} =$, $E_{v_5} = \emptyset$ And $S_{O\mathcal{D}} = \{\emptyset, \{e_3\}, \{e_4\}, \{e_1, e_2\}\}$. $\tau_{O\mathcal{D}} = \{E(\mathcal{D}), \emptyset, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_3, e_4\}, \{e_3, e_1, e_2\}, \{e_4, e_1, e_2\}\}$. We note the odigtopology $\tau_{O\mathcal{D}}$ is not discret topology.

Remark 3.10: Let S_n be a star digraph then :

(i) if every edges are direction to center vertex then the odigtopology $\tau_{O\mathcal{D}}$ is discreet topology.

(ii) If every edges are not direction to center vertex then the odigtopology $\tau_{O\mathcal{D}}$ is indiscreet topology.

(iii) If the edges are not same direction , then the odigtopology $\tau_{O\mathcal{D}}$ is not discrete topology.

This remark illustrates in the next examples.

Example 3.11: Let S_5 be a star digraph ,show in figure(6), (A)and(B).

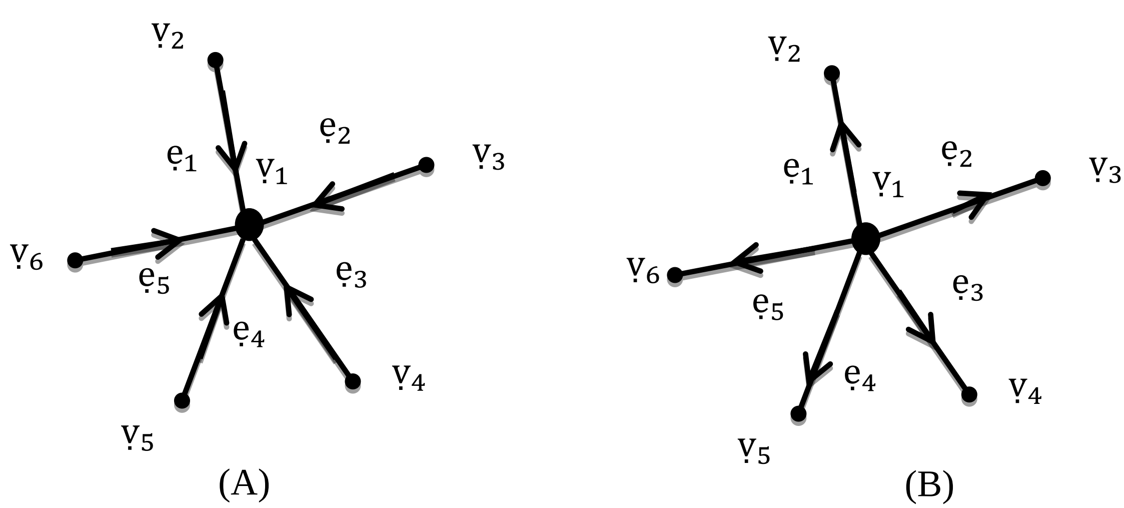


Figure (6), S_5 star digraph,it edges are not same directed.

In Figure(7)(A) every edges are directed to center vertex .

We have :$E_{v_1} = \emptyset$, $E_{v_2}=\{e_1\}$, $E_{v_3}=\{e_2\}$, $E_{v_4}=\{e_3\}$, $E_{v_5}=\{e_4\}$, $E_{v_6}=\{e_5\}$.

And $S_{O\mathcal{D}} = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}\}$.

$\tau_{O\mathcal{D}}=\{E(\mathcal{D}), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1,e_2\}, e_1, e_3\}, \{e_1,e_4\}, \{e_1,e_5\}, \{e_2,e_3\}, \{e_2,e_4\},$

$\{e_2,e_5\}, \{e_3,e_4\}, \{e_3,e_5\}, \{e_4,e_5\}, \{e_1,e_2,e_3\}, \{e_1,e_2,e_4\}, \{e_1,e_2,e_5\}, \{e_1,e_3,e_4\}, \{e_1,e_3,e_5\},$

$\{e_1,e_4,e_5\}, \{e_2,e_3,e_4\}, \{e_2,e_3,e_5\}, \{e_2,e_4,e_5\}, \{e_3,e_4,e_5\}, \{e_1,e_2,e_3,e_4\}, \{e_1,e_2,e_3,e_5\},$

$\{e_1,e_2,e_4,e_5\}, \{e_1,e_3,e_4,e_5\}, \{e_2,e_3,e_4,e_5\}\}$. we get the odigtopology $\tau_{O\mathcal{D}}$ of S_5 is discrete topology. But in figure (7)(B) every the edges not direction to center vertixe We note : $E_{v_1} = \{e_1,e_2,e_3,e_4,e_5\}$, $E_{v_2}=\emptyset$, $E_{v_3}=\emptyset$, $E_{v_4}=\emptyset$, $E_{v_5}=\emptyset$, $E_{v_6}=\emptyset$.

And $S_{O\mathcal{D}} = \{\emptyset, E(\mathcal{D})\}$.

$\tau_{O\mathcal{D}} = \{\emptyset, E(\mathcal{D})\}$then we get the odigtopology $\tau_{O\mathcal{D}}$ is indiscrete topology.

Example 3.12: Let S_4 be a star digraph ,show in figure (7).

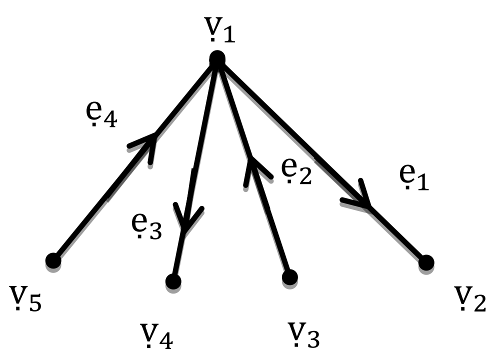


Figure (7), S_4 star digraph it edge be not same directed.

We note in figure (9), the edges is not same directed,$E_{v_1} = \{e_1,e_3\}$, $E_{v_2}=\emptyset$, $E_{v_3}=\{e_2\}$, $E_{v_4}=\emptyset$, $E_{v_5}=\{e_4\}$. And $S_{O\mathcal{D}} = \{\emptyset, \{e_2\}, \{e_4\}, \{e_1,e_3\}\}$.

$\tau_{O\mathcal{D}} = \{E(\mathcal{D}), \emptyset, \{e_2\}, \{e_4\}, \{e_1,e_3\}, \{e_2,e_4\}, \{e_2,e_1,e_3\}\}, \{e_4,e_1,e_3\}\}$. We get the odigtopology $\tau_{O\mathcal{D}}$ is not discret topology .

Proposition 3.13: Suppose that $\tau_{O\mathcal{D}}$ is the odigtopology of the a digraph $\mathcal{D} = (V,E)$, then $\{e\} \in \tau_{O\mathcal{D}}$ if $\overleftarrow{I_e}^{v} \neq \overleftarrow{I_{é}}^{v}$ for all $é \in E$.

Prove: Let $e \in E$ then $\overleftarrow{I_e}^{v} = \{v\}$ for some $v \in V$ and by hypothesis $\overleftarrow{I_e}^{v} \neq \overleftarrow{I_{é}}^{v}$ for all $é \in E$ then we get e is only edge is directed to v and hence then $E_v = \{e\}$ and by definition odigtopology $\tau_{O\mathcal{D}}$ we get $\{e\} \in \tau_{O\mathcal{D}}$.

Remark 3.14 : Let $\mathcal{D} = (V, E)$ be a digraph , then the odigtopology $\tau_{O\mathcal{D}}$ is not necessary to be discrete topology in general. The example 3.3, illustrate remark 3.14.

Corollary 3.15 : Let Ɗ = (V̧,Ȩ) be a digraph if $\overleftarrow{\mathrm{I}_{ȩ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distance pair of edge ȩ ,é ∈ Ȩ,then odigtopology $\tau_{OƊ}$ is discrete topology.

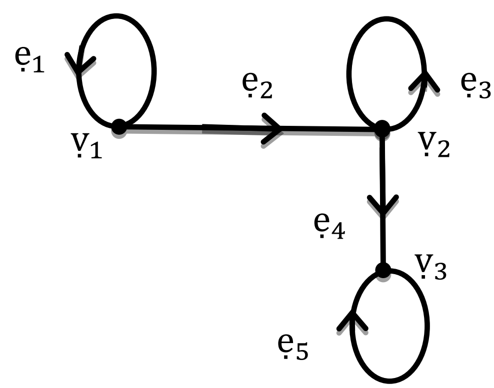
Prove: since $\overleftarrow{\mathrm{I}_{ȩ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distance pair of edge in digraph Ɗ then by proposition 3.13, {ȩ} ∈ $\tau_{OƊ}$ for all ȩ ∈ Ȩ, hence we get the odigtopology $\tau_{OƊ}$ is discrete topology.

Example 3.16: according to example 3.4 we note that $\overleftarrow{\mathrm{I}_{ȩ_1}}^{v} = \{v̧_1\}$, $\overleftarrow{\mathrm{I}_{ȩ_2}}^{v} = \{v̧_2\}$, $\overleftarrow{\mathrm{I}_{ȩ_3}}^{v} = \{v̧_3\}$, $\overleftarrow{\mathrm{I}_{ȩ_4}}^{v} = \{v̧_4\}$, $\overleftarrow{\mathrm{I}_{ȩ_5}}^{v} = \{v̧_5\}$ i.e $\overleftarrow{\mathrm{I}_{ȩ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ ∀ȩ , é ∈ Ȩ, hence odigtopology $\tau_{OƊ}$ is discrete topology.

Remark 3.17: If Ɗ = (V̧, Ȩ) be reflexive digraph then odigtopology $\tau_{OƊ}$ is not discrete topology .

The example illustrate remark 3.17 .

Example 3.18 : Let Ɗ = (V̧, Ȩ) be reflexive digraph ,show in figure(10).



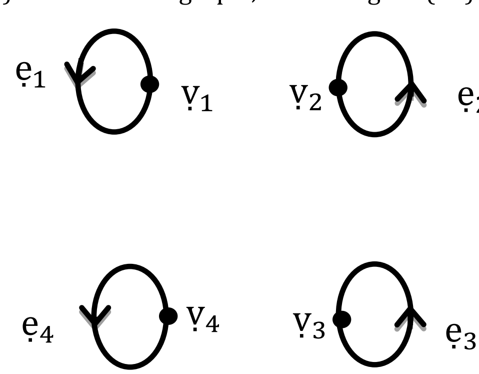
Figure(10), reflexive digraph

We note $Ȩ_{v̧_1} = \{ȩ_1, ȩ_2\}$, $Ȩ_{v̧_2} = \{ȩ_3, ȩ_4\}$, $Ȩ_{v̧_3} = \{ȩ_5\}$. And $S_{OƊ} = \{\{ȩ_1, ȩ_2\}, \{ȩ_3, ȩ_4\}, \{ȩ_5\}\}$. $\tau_{OƊ} = \{Ȩ(Ɗ), \emptyset, \{ȩ_1, ȩ_2\}, \{ȩ_3, ȩ_4\}, \{ȩ_5\}, \{ȩ_1, ȩ_2, ȩ_5\}, \{ȩ_3, ȩ_4, ȩ_5\}\}$. We get the odigtopology $\tau_{OƊ}$ is not discret toplogy.

Proposition 3.19: Let Ɗ = (V̧, Ȩ) be reflexive digraph and $d(v̧) \leq 2$ then the odigtopology $\tau_{OƊ}$ is discrete topology .

Prove : since the Ɗ = (V̧, Ȩ) reflexive digraph and $d(v̧) \leq 2$ for all v̧ ∈V̧ there exist only loop on every vertex and hence ∀ v̧ ∈ V̧ we get $Ȩ_{v̧} = \{ȩ\}$, where $ȩ = (v̧, v̧)$ and by definition odigtopology $\tau_{OƊ}$ implies {ȩ} ∈ $\tau_{OƊ}$ for all ȩ∈ Ȩ, thus odigtopology $\tau_{OƊ}$ is discrete topology. The example illustrate proposition 3.18.

Example 3.20 : Let Ɗ = (V̧, Ȩ) be reflexive digraph ,show in figure (11).



Figure(11),reflexive digraph such that $d(v̧) \leq 2$

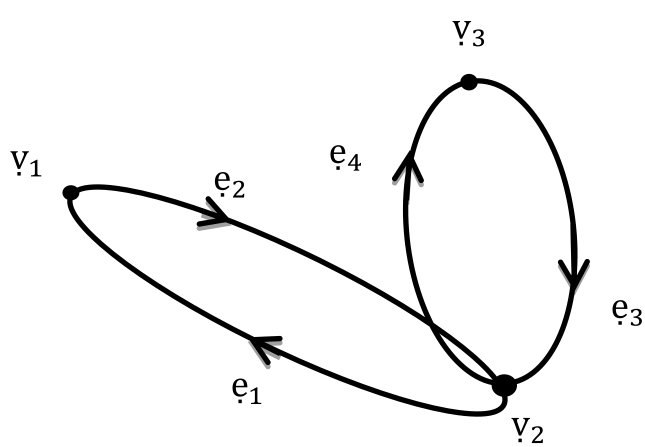
We note $E_{v_1} = \{e_1\}$, $E_{v_2} = \{e_2\}$, $E_{v_3} = \{e_3\}$, $E_{v_4} = \{e_4\}$. And $S_{O\mathcal{D}} = \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}\}$.

$\tau_{O\mathcal{D}} = \{E(\mathcal{D}), \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\},$

$\{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_3, e_4\}, \{e_2, e_3, e_4\}\}$. We get the odigtopology $\tau_{O\mathcal{D}}$ is discrete topology.

Remark 3.21: Let $\mathcal{D} = (V, E)$ be a symmetric digraph, then digtopology $\tau_{O\mathcal{D}}$ not necessary to be discrete intopology in general. Ṯhe following example show remark

Example 3.22: Let $\mathcal{D} = (V, E)$ be symmetric digraph in Figure (12) such that $V = \{v_1, v_2, v_3\}$, $E = \{e_1, e_2, e_3, e_4\}$



Figure(12), symmetric digraph

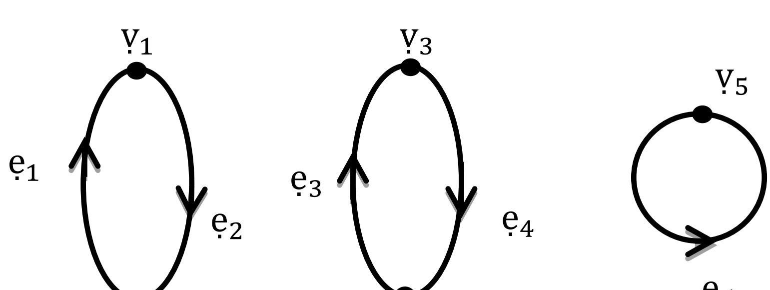
We have : $E_{v_1} = \{e_2\}$, $E_{v_2} = \{e_1, e_4\}$, $E_{v_3} = \{e_3\}$. Implies $S_{O\mathcal{D}} = \{\{e_2\}, \{e_3\}, \{e_1, e_4\}\}$.

$\tau_{O\mathcal{D}} = \{E(\mathcal{D}), \emptyset, \{e_2\}, \{e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_1, e_4\}, \{e_3, e_1, e_4\}\}$. We note the odigtopology $\tau_{O\mathcal{D}}$ is not discrete topology.

Proposition 3.23 : If $\mathcal{D} = (V, E)$ be a symmetric digraph and $d(v) \le 2$ for all $v \in V$ then the digtopology $\tau_{I\mathcal{D}}$ is discrete topology.

Prove: since $\mathcal{D} = (V, E)$ be a symmetric digraph and $d(v) \le 2$ implies we get for all $v \in V$ there is more one edge emerging from the v for all $v \in V$ [because if there exsit two edge emerging from the v and $d(v) \le 2$ implies $\mathcal{D} = (V, E)$ is not symmetric] and hence $\overleftarrow{I}^{v}_{e} \neq \overleftarrow{I}^{v}_{é}$ for all $é \in E$, by proposition 3.11 then the odigtopology $\tau_{O\mathcal{D}}$ is discrete topology. the following example show the proposition .

Example 3.24 : Let $\mathcal{D} = (V, E)$ be symmetric digraph in figure (12) such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$



We note $E_{v_1} = \{e_2\}$, $E_{v_2}=\{e_1\}$, $E_{v_3}=\{e_4\}$, $E_{v_4}=\{e_3\}$, $E_{v_5}=\{e_5\}$.

And $S_{O\mathcal{D}} = \{\{e_1\},\{e_2\},\{e_3\},\{e_4\},\{e_5\}\}$

$\tau_{O\mathcal{D}}=\{ E(\mathcal{D}),\emptyset, \{e_1\},\{e_2\},\{e_3\},\{e_4\},\{e_5\},\{e_1,e_2\},\{e_1,e_3\},\{e_1,e_4\},\{e_1,e_5\},\{e_2,e_3\},$

$\{e_2,e_4\},\{e_2,e_5\},\{e_3,e_4\},\{e_3,e_5\},\{e_4,e_5\},\{e_1,e_2,e_3\},\{e_1,e_2,e_4\},\{e_1,e_2,e_5\},\{e_1,e_3,e_4\},$

$\{e_1,e_3,e_5\},\{e_1,e_4,e_5\},\{e_2,e_3,e_4\},\{e_2,e_3,e_5\},\{e_2,e_4,e_5\},\{e_3,e_4,e_5\},\{e_1,e_2,e_3,e_4\},\{e_1,e_2,e_3,e_5\},$

$\{e_1,e_2,e_4,e_5\},\{e_1,e_3,e_4,e_5\},\{e_2,e_3,e_4,e_5\}\}$. we get the odigtopology $\tau_{O\mathcal{D}}$ is discrete topology.

Proposition 3.25 : The odigtopology space $(E, \tau_{O\mathcal{D}})$ of digraph $\mathcal{D} = (V, E)$ is Alexandroff space .

Prove : It is adequate to show that arbitrary intersection of elements of $S_{O\mathcal{D}}$ is open, Let $A \subseteq V$: $\bigcap_{v\in A} E_v = \begin{cases} E_v & \text{if } A \text{ contin one vertex } v \\ \emptyset & \text{owther wise} \end{cases}$ And by definition odigtopology $\tau_{O\mathcal{D}}$ we get $\emptyset, E_v \in \tau_{O\mathcal{D}}$, then $\bigcap_{v\in A} E_v$ is open Hence the odigtopology $\tau_{O\mathcal{D}}$ is satisfies property of Alexandroff .

Definition 3.26 : In any digraph $\mathcal{D} = (V,E)$ since $(E, \tau_{O\mathcal{D}})$ is Alexandroff space, for each $e \in E$, the intersection of all open set containing e is the smallest open set containing e and denoted by U_e , also the family $M_{\mathcal{D}} = \{U_e | e \in E\}$ is the minimal basis for the odigtopology $\tau_{O\mathcal{D}}$.

Proposition 3.27 : In any digraph $\mathcal{D} = (V, E)$, $U_e = E_v$ where $\overleftarrow{I_e}^{v} = \{v\}$ for every $e \in E$.

Prove : Here since every $e \in E$ then $\overleftarrow{I_e}^{v} = \{v\}$ for some $v \in V$ and by definition digtopology $\tau_{O\mathcal{D}}$,E_v is open contain e and by definition U_e , then we get $U_e \subseteq E_v$

On the other hand since U_e is open set and contain e then by definition odigtopology $\tau_{O\mathcal{D}}$ there exist E_v for some $v \in V$ such that $e \in E_v \subseteq U_e$, and since $\acute{e} \notin U_e$ implies $\acute{e} \notin E_v$ when $e \in E_v$, then $\acute{e} \notin E_v$ when $\overleftarrow{I_e}^{v} = \{v\}$ thus $E_v \subseteq U_e$ when $\overleftarrow{I_e}^{v} = \{v\}$, then $U_e = E_v$ where $\overleftarrow{I_e}^{v} = \{v\}$ for every $e \in E$.

Remark 3.28 : Let $\mathcal{D} = (V, E)$ be a digraph, if $\overleftarrow{I_e}^{v} \neq \overleftarrow{I_{\acute{e}}}^{v}$ for all $\acute{e} \in E$, then $U_e = \{e\}$.

Prove : Clear

Theorem 3.29 : For any $e,\acute{e} \in E$ in a digraph $\mathcal{D} = (V, E)$ we have $\overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v}$ iff $e \in U_{\acute{e}}$ i.e $U_e = \{\acute{e} \in E | \overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v}$

Prove : will prove it, let $\overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v}$ to prove $e \in U_{\acute{e}}$ Since $\overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v} = \{v\}$, implies $e,\acute{e} \in E_v$ and by properties 3.26 $U_e = E_v$ and $U_{\acute{e}} = E_v$, and hence $U_e = U_{\acute{e}} = E_v$ then $e \in U_{\acute{e}}$.

To prove the inverse, let $e \in U_{\acute{e}}$ and by proposition 3.26 we get $U_{\acute{e}} = E_v$ where $\overleftarrow{I_{\acute{e}}}^{v} = \{v\}$, since $e \in U_{\acute{e}}$ we get $e \in E_v$ where $\overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v} = \{v\}$ and hence $\overleftarrow{I_e}^{v} = \overleftarrow{I_{\acute{e}}}^{v}$.

Remark 3.30: The odigtopology $\tau_{OƊ}$ in any digraph Ɗ = (Ṿ, Ẹ) is not necessary T_0 in general.

Example 3.31: according to example 3.11 in figure (B) we get the odigtopology $\tau_{OƊ} = \{ \emptyset , Ẹ(Ɗ)\}$ is not T_0 because $ẹ_2 , ẹ_3 \in Ẹ(Ɗ)$ But ∄ open set A such that $ẹ_2 \in A$ *and* $ẹ_3 \notin A$ or $ẹ_3 \in A$ *and* $ẹ_2 \notin A$.

Remark 3.32 : Let C_n be a cyclic digraph such that every edges are in the same direction then we get the odigtopology $\tau_{IƊ}$ is T_0 and if the edges are not all in the same direction we get the odigtopology $\tau_{IƊ}$ is not T_0.

The next example are illustrates the remark 3.32 .

Example 3.33 :

(i). by according example 3.5, we note that the cyclic digraph C_5 all edges in the same direction and hence we note the odigtopology $\tau_{OƊ}$ on C_5 is T_0 .

(ii). By according example 3.6, we note that the cyclic digraph C_6 all edges are not in the same direction and hence we note the odigtopology $\tau_{OƊ}$ is not T_0 , because $ẹ_5, ẹ_6 \in Ẹ$ but $\nexists\, u \in \tau_{OƊ}$ such that $ẹ_5 \in u$ *and* $ẹ_6 \notin u$ *or* $ẹ_5 \notin u$ *and* $ẹ_6 \in u$

Remark 3.34 : Let P_n be a path digraph such that every edges are in the same direction then we get the odigtopology $\tau_{OƊ}$ is T_0 , and if the edges are not all in the same direction we get the odigtopology $\tau_{OƊ}$ is not T_0.

The next two example are illustrates the remark 3.34 .

Example 3.35:

(i). By according example 3.7 we note that the path digraph P_5 all edges in the same direction and hence we note the digtopology $\tau_{OƊ}$ on P_5 is T_0.

(ii). By according example 3.8 we note that the path digraph P_5 all edges are not in the same direction and hence we note the odigtopology $\tau_{OƊ}$ is not T_0, because $ẹ_1, ẹ_2 \in Ẹ$ but $\nexists\, u \in \tau_{OƊ}$ such that $ẹ_1 \in u$ *and* $ẹ_2 \notin u$ *or* $ẹ_1 \notin u$ *and* $ẹ_2 \in u$

Remark 3.36 : Let S_n be a star digraph such that every edges are indgree to the center vertex then we get the odigtopology $\tau_{OƊ}$ is T_0, and if the every edges are not indgree to the center vertex we get the odigtopology $\tau_{OƊ}$ is not T_0. also,if the edges are not same direction then we have the odigtopology $\tau_{OƊ}$ is not T_0.

The next example are illustrates the remark 3.36

Example 3.37 :

(i). By according example 3.12 in figure (9)(A), we note that the star digraph S_5 every edges are indgree to the center vertex and hence we note the odigtopology $\tau_{OƊ}$ is T_0 ,

(ii). By according example 3.11 in figure (7)(B), we note that the star digraph S_5 every edges are not indgree to the center vertex we get the digtopology $\tau_{OƊ}$ is not T_0, because $ẹ_2, ẹ_4 \in Ẹ$ but $\nexists\, u \in \tau_{OƊ}$ such that $ẹ_2 \in u$ *and* $ẹ_4 \notin u$ *or* $ẹ_2 \notin u$ *and* $ẹ_4 \in u$.

Proposition 3.38 : The odigtopology $\tau_{OƊ}$ in any digraph Ɗ = (Ṿ,Ẹ) is T_0 if and only if $\overleftarrow{\mathrm{I}_{ẹ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distinct pair of edges ẹ, é ∈ Ẹ.

Prove: will prove it, Suppos the odigtopology $\tau_{OƊ}$ is T_0 to prove $\overleftarrow{\mathrm{I}_{ẹ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distinct pair of edges ẹ, é ∈ Ẹ. If $\overleftarrow{\mathrm{I}_{ẹ}}^{v} = \overleftarrow{\mathrm{I}_{é}}^{v}$ then by theorem 3.28 $ẹ \in U_{é}$ and we get there exist u is open set such that $é \in u$ and $ẹ \in u$ implies the odigtopology $\tau_{OƊ}$ is not T_0 this contradiction. then $\overleftarrow{\mathrm{I}_{ẹ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distinct pair of edges ẹ, é ∈ Ẹ .

*A*nd to prove that $\tau_{OƊ}$ is T_0, since $\overleftarrow{\mathrm{I}_{ẹ}}^{v} \neq \overleftarrow{\mathrm{I}_{é}}^{v}$ for every distinct pair of edges then by corollary 3.14 odigtopology $\tau_{OƊ}$ is discrete implies the digtopology $\tau_{OƊ}$ is T_0 . The next example is illusteted this proposition 3.36.

Example 3.39: Acorroding example 3.5, we note that $\overleftarrow{I_{e}}^{v} \neq \overleftarrow{I_{é}}^{v}$ for every distinct pair of edges

[since $\overleftarrow{I_{e_1}}^{v} = \{v_1\}$, $\overleftarrow{I_{e_2}}^{v}=\{v_2\}$, $\overleftarrow{I_{e_3}}^{v}=\{v_3\}$, $\overleftarrow{I_{e_4}}^{v}=\{v_4\}$, $\overleftarrow{I_{e_5}}^{v}=\{v_5\}$] then we get the odigtopology τ_{OD} is T_0, and we note that in the example 3.5 [$\overleftarrow{I_{e_1}}^{v} = \{v_2\}$, $\overleftarrow{I_{e_2}}^{v}=\{v_2\}$, $\overleftarrow{I_{e_3}}^{v}=\{v_3\}$, $\overleftarrow{I_{e_4}}^{v}=\{v_5\}$, $\overleftarrow{I_{e_5}}^{v}=\{v_6\}$, $\overleftarrow{I_{e_6}}^{v}=\{v_6\}$]there exists $\overleftarrow{I_{e_1}}^{v} = \overleftarrow{I_{e_2}}^{v}$ and $\overleftarrow{I_{e_5}}^{v} = \overleftarrow{I_{e_6}}^{v}$,hence the odigtopology τ_{OD} is not T_0 .

Corollary 3.40 : The odigtopology τ_{OD} in any digraph $D = (V, E)$ is T_0 if and only if it is discrete .

Prove : The proof is easy by properties 3.37 and corollary 3.14 .

Remark 3.41: The odigtopology τ_{OD} in any digraph $D = (V,E)$ is not necessary T_1 in general.

Example 3.42: according to example 3.3 we get the odigtopology τ_{OD} is not T_1 because $e_1 , e_3 \in E(D)$ But $\nexists$ open set u such that $e_1 \in u , e_3 \notin u$ and $e_3 \in u , e_1 \notin u$.

Remark 3.43 : Let C_n be a cyclic digraph such that every edges are in the same direction then we get the odigtopology τ_{OD} is T_1 , and if the edges are not all in the same direction we get the odigtopology τ_{OD} is not T_1. The next example are illustrates the remark 3.43 .

Example 3.44 :

(i). By according example 3.4 we note that the cyclic digraph C_5 all edges in the same direction and hence we note the odigtopology τ_{OD} on C_5 is T_1.

(ii). By according example 3.5 we note that the cyclic digraph C_6 all edges are not in the same direction and hence we note the odigtopology τ_{OD} is not T_1 , because $e_5, e_6 \in E$ but $\nexists\, u \in \tau_{OD}$ such that $e_5 \in u , e_6 \notin u$ and $e_6 \in u , e_5 \notin u$.

Remark 3.45 : Let P_n be a path digraph such that every edges are in the same direction then we get the odigtopology τ_{OD} is T_1, and if the edges are not all in the same direction we get the odigtopology τ_{OD} is not T_0. This remark illustrates in the next example.

Example 3.46 :

(i). By according example 3.7 we note that the path digraph P_5 all edges in the same direction and hence we note the odigtopology τ_{OD} on P_5 is T_1.

(ii). By according example 3.8 we note that the path digraph P_5

all edges are not in the same direction and hence we note the odigtopology τ_{OD} is not T_1. because $e_1, e_2 \in E$ but $\nexists\, u \in \tau_{OD}$ such that $e_1 \in u , e_2 \notin u$ *and* $e_2 \notin u , e_1 \in u$.

Proposition 3.47 : The odigtopology τ_{OD} in any digraph $D = (V, E)$ is T_1 if and only if $\overleftarrow{I_{e}}^{v} \neq \overleftarrow{I_{é}}^{v}$ for every distinct pairs of edges $e, é \in E$.

Prove: will prove that, suppos the odigtopology τ_{OD} is T_1 to prove $\overleftarrow{I_{e}}^{v} \neq \overleftarrow{I_{é}}^{v}$ for every distinct pair of edges $e, é \in E$. If $\overleftarrow{I_{e}}^{v} = \overleftarrow{I_{é}}^{v}$ then by theorem 3.29 $e \in U_{é}$ and we get u is open set such that $é \in u$ and $e \in u$ implies the odigtopology τ_{OD} is not T_1 this contradiction , thus $\overleftarrow{I_{e}}^{v} \neq \overleftarrow{I_{é}}^{v}$ for every distinct pair of edges $e, é \in E$.

And prove τ_{OD} is T_1, let $\overleftarrow{I_{e}}^{v} \neq \overleftarrow{I_{é}}^{v}$ for every distinct pair of edge by corallay 3.15 implies the odigtopology τ_{OD} is discrete and hence τ_{OD} is T_1.

Corollary 3.48 : The odigtopology τ_{OD} in any digraph $D = (V, E)$ is T_1 if and only if it is discrete .

Prove : The proof is easy by properties 3.44 and corollary 3.14 .

Proposition 3.49: The odigtopology τ_{OD} in any digraph $D = (V, E)$ is T_0 if and only if T_1.

Prove: by proposition 3.45 and proposition 3.40.

Corollary 3.50: Let $Ɗ = (Ṿ,Ẹ)$ be a digraph, For every $ẹ \in Ẹ$ we have $\overline{U_{ẹ}} = \overline{Ẹ_{ṿ}}$ where $\overleftarrow{I_{ẹ}}^{v} = \{ṿ\}$.

Prove: let $ẹ \in Ẹ$ by proposition 3.27 we get $U_{ẹ} = Ẹ_{ṿ}$ where $\overleftarrow{I_{ẹ}}^{v} = \{ṿ\}$ Therefor $\overline{U_{ẹ}} = \overline{Ẹ_{ṿ}}$ where $\overleftarrow{I_{ẹ}}^{v} = \{ṿ\}$.

Corollary 3.51 : given a digraph $Ɗ = (Ṿ,Ẹ)$. for every $ẹ \in Ẹ$, $\overline{\{ẹ\}} \subseteq \overline{U_{ẹ}} = \overline{Ẹ_{ṿ}}$ where $\overleftarrow{I_{ẹ}}^{v} = \{ṿ\}$.

Prove : let $u \in \overline{\{ẹ\}}$ this implies $U \cap \{ẹ\} \neq \emptyset$ for all open set U containing ẹ. since $\{ẹ\} \subseteq U_{ẹ}$ this implies $U \cap U_{ẹ} \neq \emptyset$ for all open set U containing ẹ . hence $ẹ \in \overline{U_{ẹ}}$ and so, $\overline{\{ẹ\}} \subseteq \overline{U_{ẹ}}$. then by corollary 3.42, $\overline{\{ẹ\}} \subseteq \overline{U_{ẹ}} = \overline{Ẹ_{ṿ}}$ where $\overleftarrow{I_{ẹ}}^{v} = \{ṿ\}$.

Corollary 3.52 : for any $ẹ, é \in Ẹ$ in a digraph $Ɗ = (Ṿ, Ẹ)$ we have, $é \in \overline{\{ẹ\}}$ if and only if $\overleftarrow{I_{ẹ}}^{v} = \overleftarrow{I_{é}}^{v}$.

Prove: $é \in \overline{\{ẹ\}} \Leftrightarrow U \cap \{ẹ\} \neq \emptyset$ for all open set U containing $é \Leftrightarrow ẹ \in U_{é} \Leftrightarrow \overleftarrow{I_{ẹ}}^{v} = \overleftarrow{I_{é}}^{v}$ by corollary 3.27 .

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