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I-Pre- Cauchy Triple sequences of Fuzzy Number and Double Orlicz functions

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ABSTRACT

let Ж be a double Orlicz function and $x = (X_{\text{ӌ},\text{ʁ},\text{ɟ}})$ a triple seq of fuzzy numbers. We establish that X is. I-pre- Cauchy if and only if.

$$I\_\lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\text{ӌ},i>m} \sum_{\text{ʁ},p>n} \sum_{\text{ɟ},s>t} \left\{ \text{Ж}_1\, \tilde{d}\left(\frac{X_{\text{ӌ},\text{ʁ},\text{ɟ}}, X_{i,p,s}}{q}\right) V \text{Ж}_2 \tilde{d}\left(\frac{y_{\text{ӌ},\text{ʁ},\text{ɟ}}, y_{i,p,s}}{q}\right)\right\} = 0$$

This indicates a theorem returns to Connor, Vakeel A. Khan and Fridy and Klink[1] and Q.M.Danish Lohani [2]

$$I - \lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\text{ӌ},i\le m} \sum_{\text{ʁ},p\le n} \sum_{\text{ɟ},s\le t} \left\{ \left(\text{Ж}_1 \left(\frac{\tilde{d}(X_{\text{ӌ},\text{ʁ},\text{ɟ}}, X_{i,p,s})}{q} \right)\right)^{P_{\text{ӌ},\text{ʁ},\text{ɟ}}} V \left(\text{Ж}_2 \left(\frac{\tilde{d}(y_{\text{ӌ},\text{ʁ},\text{ɟ}}, y_{i,p,s})}{q} \right)\right)^{P_{\text{ӌ},\text{ʁ},\text{ɟ}}} \right\} = 0 .$$

MSC. 16D10, 16D70, 16D99

1.Introduction

For real and complex seq. the idea of st. con. was initially described by Steinhaus, [3] at a Symposium held in 1949 Poland, at wroclaw University and then independently by Fast [4],Schoenberg [6] and Buck[5].
Salat[7], Connor[9], Fridy [8] and others have all investigated this concept. Statistical con. is a Generalization of the conventional theory of con. that Parallels the conventional ,,theory of con..
A seq. $X = (X_{\text{ӌ}})$ is called st.con. to L if for a given $\epsilon > 0$,

$$\lim_{k} \frac{1}{k} |\{\text{ӌ} : \tilde{d}\ (X_{\text{ӌ}}, L)\ \ge \epsilon , \text{ӌ} \le k\}| = 0.$$

A seq. $X = (X_{\text{ӌ}})$ is called st. Pre -Cauchy if

$$\lim_{k} \frac{1}{k^2} |\{ (\text{ӌ}, \text{ʁ}) : \tilde{d}(X_{\text{ӌ}}, X_{\text{ʁ}}) \ge \epsilon , \text{ӌ}, \text{ʁ} \le k\}| = 0.$$

Connor, klin and Fridy [1] established that statistically convergent seq. are st.Pre – Cauchy so , any bounded st. Pre-Cauchy seq. with a no where dense collection of limit points is st.con..
They also gave an example demonstrating that st.Pre-Cauchy seq. are not always st.con. (see[10])
Throughout a triple seq of fuzzy number is denoted by $x = (X_{\text{ӌ},\text{ʁ},\text{ɟ}})$. Atriple seq of fuzzy number is a triple infinite array of elements $X_{\text{ӌ},\text{ʁ},\text{ɟ}} \in R$ for all $\text{ӌ}, \text{ʁ}, \text{ɟ} \in N$.
The first works on triple seq. of fuzzy number is found in Bromwich [11], Basarir and solancan[13] and many others.

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2. Definitions and Preliminaries

This section introduces several fundamental concepts that will be used throughout the article

Definition 2.1 A triple seq. $X = (X_{ʯ,ʁ,ɟ})$ of fuzzy numbers is called st. con. to a fuzzy number $X_\circ$ if for every $\epsilon > 0$,
$\lim_{n,m,t} \frac{1}{n\,m\,t} \left|\{ (ʯ,ʁ,ɟ), ʯ \leq n , ʁ \leq m \, and \, \leq t: \left(X_{ʯ,ʁ,ɟ}, X_\circ\right) \geq \epsilon \}\right| = 0.$
Where the number of elements in the set is indicated by vertical bars.

Definition 2.2. A triple seq. $X = (X_{ʯ,ʁ,ɟ})$ of fuzzy numbers is called st.pre-Cauchy if for every $\epsilon > 0$ there exist $i = i(\epsilon), p = p(\epsilon) \, and \, S = S(\epsilon)$:
$$\lim_{m,n,t\to\infty} \frac{1}{m^2n^2t^2} \left|(ʯ,ʁ,ɟ): \tilde{d}\left(X_{ʯ,ʁ,ɟ} - X_{i,p,s}\right) \geq \epsilon, ʯ \leq m, ʁ \leq n, ɟ \leq t\right| = 0.$$

Definition 2.3. An double Orlicz function is a function,.
$Ѫ: [0,\infty)X[0,\infty) \to [0,\infty)X[0,\infty)$:
$Ѫ(x,y) = \left(Ѫ_1(x), Ѫ_2(y)\right)$,
$Ѫ_1: [0,\infty) \to [0,\infty) \, , Ѫ_2: [0,\infty) \to [0,\infty)$,
which is continuous , non-decreasing, even ,convex and satisfy the following conditions.

i. $Ѫ_1(0) = 0 \, , \; Ѫ_2(0) = 0 \Rightarrow Ѫ(x,y) = \left(Ѫ_1(0), \; Ѫ_2(0) = (0,0)\right)$,

ii. $Ѫ_1(x) > 0, Ѫ_2(y) > 0 \Rightarrow \; Ѫ(x,y) = \left(Ѫ_1(x), \; Ѫ_2(y)\right) > (0,0)$,

for all $x, y \; > 0, so \; (x,y) > (0,0), mean \; that \; Ѫ_1(x) > 0 \, , Ѫ_2(y) > 0$

iii. $Ѫ_1(x) \to \infty \, , Ѫ_2(y) \to \infty \;\; as \; x, y \to \infty \, , then$

$Ѫ(x,y) = (Ѫ_1(x), Ѫ_2(y) \to (\infty,\infty) as (x,y) \to (\infty,\infty) \, , so \,)$
$Ѫ(x,y) \to (\infty,\infty), mean \; that \; Ѫ_1(x) \to \; \infty, Ѫ_2(y) \to \infty$
If convexity of double Orlicz function is replaced by
$Ѫ(x+y) < Ѫ(x) + Ѫ(y)$, then it is called a modulus function (see Maddox[14]).
An Bounded or unbounded double Orlicz functions are possible. For instance,
$Ѫ_1(x) = \; X^p, Ѫ_2(y) = y^p \Rightarrow \; Ѫ(x,y) = \left(Ѫ_1(x), \; Ѫ_2(y)\right) = (X^p, y^p)$
$(0 < p \leq 1)$ is unbounded and
$Ѫ_1(x) = \frac{x}{x+1}, Ѫ_2(y) = \frac{y}{y+1} \Rightarrow \; Ѫ(x,y) = (Ѫ_1(x), Ѫ_2(y)) = (\frac{x}{x+1}, \frac{y}{y+1})$
is bounded (see Maddox [14]).
A.H Battor, Neaman used the concept of a double Orlicz function to construct the triple seq. space
$L_{Ѫ^3}$,
$L_{Ѫ^3} = \left(2L_{Ѫ_1}, 2L_{Ѫ_2}\right) = \{(x,y) \in W^3 : \sum_{ʯ=1}^{\infty} \sum_{ʁ=1}^{\infty} \sum_{ɟ=1}^{\infty} \left\{Ѫ_1\left(\frac{|X_{ʯ,ʁ,ɟ}|}{q}\right) V Ѫ_1\left(\frac{|y_{ʯ,ʁ,ɟ}|}{q}\right)\right\} < \infty, for \; some \; q > 0\}$
The spacee $LѪ^3$is a Banach space with the norm $||x,y||_Ѫ = \inf\{q > 0 : \sum_{ʯ=1}^{\infty} \sum_{ʁ=1}^{\infty} \sum_{ɟ=1}^{\infty} \left\{Ѫ_1\left(\frac{|X_{ʯ,ʁ,ɟ}|}{q}\right) V Ѫ_1\left(\frac{|y_{ʯ,ʁ,ɟ}|}{q}\right) \; \leq 1\right\}$
The space $LѪ^3$is closely related to the space L_p^3 that is a double Orlicz triple seq. space with
$Ѫ(x,y) = \left(Ѫ_1(x), Ѫ_2(y)\right) = (x^p, y^p) : Ѫ_1(x) = X^p \; and \; Ѫ_2(y) = y^p$,
A double Orlicz function $Ѫ \, (\chi \, , \mathcal{Y} \,) \; = \left(Ѫ_1(\chi), Ѫ_2(\mathcal{Y})\right)$ is said to satisfy Δ_2-condition for all values of $\chi, \mathcal{Y}$ if there exists a constant r > 0 : $Ѫ_1(2\chi) \leq$ r $Ѫ_1(\chi)$ and $Ѫ_2(2\mathcal{Y}) \leq$ r $Ѫ_2(\mathcal{Y})$ for all $\chi \geq 0 \, , \mathcal{Y} \geq 0$, then

$Ѫ \; (2\chi, 2\mathcal{Y}) = \left(Ѫ_1(2\chi), Ѫ_2(2\mathcal{Y})\right) \leq \left(\text{r} \; Ѫ_1(\chi), \text{r} \; Ѫ_2(\mathcal{Y})\right) = \text{r}\left(Ѫ_1(\chi), Ѫ_2(\mathcal{Y})\right) = \text{R}Ѫ \; (\chi, \mathcal{Y})$, for all $\chi \geq 0 \, , \mathcal{Y} \geq 0$.

Various writers have lately investigated double Orliz triple seq. spaces [1,2, 16-20].

Connor, Fridy and Klin established in [1] that a bounded triple seq. $X = (X_{ʯ,ʁ,ɟ})$ is st.pre-Cauchy if and only if

$I - \lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{ʯ,i\leq m} \sum_{ʁ,p\leq n} \sum_{ɟ,s\leq t} \left(\tilde{d}\left(X_{ʯ,ʁ,ɟ}, X_{i,p,s}\right)\right) = 0.$
The concept of I-convergence is a generalization of st.con. It was initially investigated by Kostyrko, Salat and Wilezynski[21]. Salat, Ziman [22] Tripathy and Hazavika [24-26] ,Tripathy and Demirci [23] researched it later. We begin with some preliminaries on the concept of I-convergence.

Definition 2.4. [20,27]. If $X_{\text{ʮ},\text{ʁ},\text{ⅎ}}$ is a non-empty set, then a family of sets $I \leq 2^{x}$(2^{x} denoting the power set of x) is said to be an ideal in x if,

i. $\emptyset \in I$

ii. for any A, B $\in I$, we have $AUB \in I$.

iii. we have $B \in I$,for every $A \in I$ and each $B \subseteq A$.

A non-trivial ideal I is maximal if there can't exist eany non-trivia ideal $J \neq I$ containing I as a subset.
For each ideal I , there is a filter F(I) corresponding to I.i.e.

$$F(I) = \{K \subseteq N : K^{c} \in I\},$$

where $K^{c} = N - K$.
Definition 2.5. [10, 21, 28] A triple seq. $(x, y) = (X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, Y_{\text{ʮ},\text{ʁ},\text{ⅎ}})$ of fuzzy numbers is called I-convergent to fuzzy number$(x_{\circ}, y_{\circ})$if for any $\epsilon > 0$,

$$\left\{\text{ʮ}, \text{ʁ}, \text{ⅎ} \in N : d\left(\left(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, Y_{\text{ʮ},\text{ʁ},\text{ⅎ}}\right), (x_{\circ}, y_{\circ})\right) \geq \epsilon\right\} \in I.$$

In this case we write $I - \lim\left(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, Y_{\text{ʮ},\text{ʁ},\text{ⅎ}}\right) = (x_{\circ}, y_{\circ})$.
Definition 2.6. [21] A non-empty family of sets $F(I) \subseteq 2^{x}$ is called filter on X if and only if

i. $\emptyset \notin F(I)$

ii. we've got $A \cap B \in F$, for each $A , B \in F$

iii. we have $B \in F$, for each $A \in F$ and each $A \subseteq B$.

3. Main Results

We establish the I-pre-Cauchy condition for every arbitrary triple seq. of fuzzy number in this article.

Theorem 3.1. Let $(x , y) = (X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, Y_{\text{ʮ},\text{ʁ},\text{ⅎ}})$ be the a triple set of fuzzy number and let $\text{Ѫ} = (\text{Ѫ}_1, \text{Ѫ}_2)$ be a bounded double Orlict function then X is I-pre Cauchy if and only if

$$I - \lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\text{ʮ},i \leq m} \sum_{\text{ʁ},p \leq n} \sum_{\text{ⅎ},s \leq t} \left\{ \left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} \right\} = 0, \text{ for some } q > 0.$$

Proof: assume that

$$I - \lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\text{ʮ},i \leq m} \sum_{\text{ʁ},p \leq n} \sum_{\text{ⅎ},s \leq t} \left\{ \left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} \right\} = 0, \text{ for some } q > 0.$$

For any $\epsilon > 0, q > 0$ and $m, n, t \in IN$ we have that

$$\text{A}_1 = \left\{m, n, t \in N : \left\{\left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}}\right\} \geq \frac{\epsilon}{2mnt}, \text{ʮ}, i \leq m, \text{ʁ}, p \leq n, \text{ⅎ}, s \leq t\right\} \in I, \qquad (1)$$

$$A_1^C = \left\{m, n, t \in N : \left\{\left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}}\right\} < \frac{\epsilon}{2mnt}, \text{ʮ}, i \leq m, \text{ʁ}, p \leq n, \text{ⅎ}, s \leq t\right\} \in I, \qquad (2)$$

$$\lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\text{ʮ},i \leq m} \sum_{\text{ʁ},p \leq n} \sum_{\text{ⅎ},s \leq t} \left\{ \left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} \right\}$$

$$= \lim_{mnt} \frac{1}{m^2n^2t^2} \sum_{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s}) < \frac{\epsilon}{2mnt}} \left\{ \left(\text{Ѫ}_1\left(\frac{\tilde{d}(X_{\text{ʮ},\text{ʁ},\text{ⅎ}}, X_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} V \left(\text{Ѫ}_2\left(\frac{\tilde{d}(y_{\text{ʮ},\text{ʁ},\text{ⅎ}}, y_{i,p,s})}{q}\right)\right)^{P_{\text{ʮ},\text{ʁ},\text{ⅎ}}} \right\} +$$

$$\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})\geq\frac{\epsilon}{2mnt}}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}\geq$$

$$\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})\geq\frac{\epsilon}{2mnt}}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}.$$

Now by (1) and(2) we have

$$\left\{m\,,n\,,t\ \in N:\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ӌ,i\leq m}\sum_{ʁ,p\leq n}\sum_{Ⅎ,s\leq t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}\geq\epsilon, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right\}\subset A_1\cup A_1^C\in I.$$

Thus X is I-pre-Cauchy.

Consider the following scenario: X is I-pre-Cauchy, and that ϵ has been granted. Then there is

$$\left\{m,n,t\ \in N:\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ӌ,i\leq m}\sum_{ʁ,p\leq n}\sum_{Ⅎ,s\leq t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}\geq\epsilon, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right\}\subset A_1\cup A_1^C\in I.$$

Where,

$$A_1=\left\{\left\{m,n,t\ \in N:\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}\geq\frac{\epsilon}{2mnt}, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right\}\in I$$

$$A_1^C=\left\{\left\{m,n,t\ \in N:\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}<\frac{\epsilon}{2mnt}, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right\}\in I$$

Let $(\delta,\delta)>(0,0)$ be such that $Ӿ(\delta)=\left(Ӿ_1(\delta),Ӿ_2(\delta)<(\frac{\epsilon}{2},\frac{\epsilon}{2})\right)$.

Since $Ӿ$ is a There is an Orlicz function that is bounded.

integer B such that $Ӿ(x,y)\ =(Ӿ_1(x),Ӿ_2(y)<(\frac{B}{2},\frac{B}{2})$ for all $(x,y)>(0,0)$, therefore, for each $m,n,t\ \in IN$,

$$\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ӌ,i\leq m}\sum_{ʁ,p\leq n}\sum_{Ⅎ,s\leq t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}$$

$$=\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})<\frac{\epsilon}{2mnt}}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}$$

$+$

$$\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})\geq\frac{\epsilon}{2mnt}}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}\leq$$

$$\left(Ӿ_1(\delta),Ӿ_2(\delta)\right)+\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ӌ,i\leq m}\sum_{ʁ,p\leq n}\sum_{Ⅎ,s\leq t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}$$

$$\leq\left(\frac{\epsilon}{2},\frac{\epsilon}{2}\right)+\left(\frac{B}{2},\frac{B}{2}\right)\left(\frac{1}{m^2n^2t^2}\left|(ӌ,ʁ,Ⅎ):\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})\geq\epsilon, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right|\right)$$

$$\leq(\epsilon,\epsilon)+(B,B)\left(\frac{1}{m^2n^2t^2}\left|(ӌ,ʁ,Ⅎ):\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})\geq\epsilon, ӌ,i\leq m, ʁ,p\leq n, Ⅎ,s\leq t\right|\right).(3)$$

Since X is I-pre – Cauchy there is an IN on the right hand side of Cauchy of (3) is less than (ϵ,ϵ) for all $m,n,t\in IN$. Hence

$$I-\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ӌ,i\leq m}\sum_{ʁ,p\leq n}\sum_{Ⅎ,s\leq t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_{i,p,s})}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}.$$

Theorem 2.2. Let $(x,y)=(X_{ӌ,ʁ,Ⅎ},y_{ӌ,ʁ,Ⅎ})$ be a triple seq. of fuzzy number and let $Ӿ=(Ӿ_1,Ӿ_2)$ be bounded double Orlicz function then (x, y) is I – convergent to a fuzzy number
$(x_\circ,y_\circ)$ if and only if .

$$I-\lim_{mnt}\frac{1}{mnt}\sum_{ӌ=1}^{m}\sum_{ʁ=1}^{n}\sum_{Ⅎ=1}^{t}\left\{\left(Ӿ_1\left(\frac{\tilde{d}(X_{ӌ,ʁ,Ⅎ},X_\circ)}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}} V \left(Ӿ_2\left(\frac{\tilde{d}(y_{ӌ,ʁ,Ⅎ},y_\circ)}{q}\right)\right)^{P_{ӌ,ʁ,Ⅎ}}\right\}=0 \text{ for some } q>0.$$

Proof: Suppose that

$$I-\lim_{mnt}\frac{1}{mnt}\sum_{ʯ=1}^{m}\sum_{ʁ=1}^{n}\sum_{ɖ=1}^{t}\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}=0 \text{ for some } q>0.$$

With an double Orlicz functions Ѫ, then(x ,y) is I-convergent to $(x_{\circ}, y_{\circ})$ (see [1]).
Consider the following scenario: (x , y) is 1- convergent to $(x_{\circ}, y_{\circ})$. This can be proved in the same way as theorem 2.1, providing that

$$I-\lim_{mnt}\frac{1}{mnt}\sum_{ʯ=1}^{m}\sum_{ʁ=1}^{n}\sum_{ɖ=1}^{t}\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}=0 \text{ for some } q>0.$$

and Ѫ being an double Orlicz function that is bounded,

Corollary 2.3. A triple seq. $(x\,,y)=(X_{ʯ,ʁ,ɖ},y_{ʯ,ʁ,ɖ})$ of fuzzy numbers is I-convergent if and only if

$$I-\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ʯ,i\le m}\sum_{ʁ,p\le n}\sum_{ɖ,s\le t}\left(\tilde{d}\left(X_{ʯ,ʁ,ɖ},X_{i,p,s}\right)\right)=0.$$

Proof : Let $Ѫ(x,y)=\left(Ѫ_1(x\,),Ѫ_2(y)\right)=(x,y\,)$such that $Ѫ_1(x)=x\;\; Ѫ_2(y)=y$ then

$$\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}\le\left\{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})\;V\;\tilde{d}(y_{ʯ,ʁ,ɖ}-y_{i,p,s})\right\} \text{ for all } ʯ,i\le m,ʁ,p\le n,ɖ,s\le t$$

and for $m,n,t\in IN$.
Let

$$B_1=\left\{m,n,t\in N:\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}<\epsilon,ʯ,i\le m,ʁ,p\le n,ɖ,s\le t\right\}\in I \qquad (4)$$

And

$$B_1^C=\left\{m,n,t\in N\left\{:\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}\ge\epsilon,ʯ,i\le m,ʁ,p\le n,ɖ,s\le t\right\}\in I \qquad (5)$$

Therefor from (4) and (5) we have,

$$\left\{m,n,t\in N:\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}\ge\epsilon,ʯ,i\le m,ʁ,p\le n,ɖ,s\le t\right\}\subset B_1\cup B_1^C\in I.$$

Hence

$$I-\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ʯ,i\le m}\sum_{ʁ,p\le n}\sum_{ɖ,s\le t}\left\{\tilde{d}\left(X_{ʯ,ʁ,ɖ},X_{i,p,s}\right)V\;\tilde{d}\left(y_{ʯ,ʁ,ɖ},y_{i,p,s}\right)\right\}=0.$$

If and only if

$$I-\lim_{mnt}\frac{1}{m^2n^2t^2}\sum_{ʯ,i\le m}\sum_{ʁ,p\le n}\sum_{ɖ,s\le t}\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ}-y_{i,p,s})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}=0$$

We get the desired outcome by using theorem 2.1 right away.

Corollary 2.4. A triple seq. $(x\,,y)=(X_{ʯ,ʁ,ɖ},y_{ʯ,ʁ,ɖ})$ is I-convergent to fuzzy number$(x_{\circ},y_{\circ})$ if and only if

$$I-\lim_{mnt}\frac{1}{mnt}\sum_{ʯ=1}^{m}\sum_{ʁ=1}^{n}\sum_{ɖ=1}^{t}\left\{\left(Ѫ_1\left(\frac{\tilde{d}(X_{ʯ,ʁ,ɖ},X_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}} V\left(Ѫ_2\left(\frac{\tilde{d}(y_{ʯ,ʁ,ɖ},y_{\circ})}{q}\right)\right)^{P_{ʯ,ʁ,ɖ}}\right\}=0.$$

Proof: Let $Ѫ(x,y)=\left(Ѫ_1(x),Ѫ_2(y)\right)=(x,y)$such that
$Ѫ_1(x)=x\,,\;Ѫ_2(y))=y$.
We can prove this in the same way that we proved corollary 2.3.

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