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Page 23 - 34 Noori. F / Kholod. M

# Some Results on Soft Seminorms

Noori F. AL-Mayahi Kholod M. Hassan

Department of Mathematics
College of Computer Science and Formation Technology
Al-Qadisiyah University

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**Abstract:** Our goal in this work , is to show that a family of soft seminorms on a soft linear space $SL(\tilde{X})$ induces a soft topology on that space and to find the relationship between another type of a soft functional called soft Minkowski's functional on a soft linear space over the soft scalar field.

**Keywords:** soft set , soft point , soft topology, soft topological linear space, soft Hausdorff space , soft linear functional , soft semi norm and soft Minkowski's functional.

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## 1. Introduction:

The concept of soft set has been initiated by Molodtsov D. [1] in 1999. After that in 2011, Shabir and Naz [3] came up with an idea of soft topological spaces. Later Janaki C. [8] , Nazmul S. [4] , Bedre O. and Demir I. [6] , Peyghan E. and others [7] , and Reddy B. and Sayyed J. [9] , studied on soft topological spaces . Also In 2012, sujoy D. [2] , introduced soft real sets and soft real numbers and theier properties.

In 2013, Sujoy D. and Smanta S. [13] , were introduced the concept of soft linear spaces , soft linear functional . The concept of soft balanced and soft absorbing sets was first introduced by Sanjay R. and Samanta T. [10], in 2015. Also, Irfan D. [11] in 2013 , were introduced the concept soft convex set and give some properties of this set. Later, Chiney S. and Smanta S. [12] in 2015, introduced the notation of vector soft topology and studied some of its basic properties and some facets of the system of soft nbhds of the soft zero element of linear soft topology are established. Also, Chiney S. and Smanta S. [15] in 2016 , were introduced a new notations namely soft semi norms and soft Minkowski's functionals on a soft linear space over a soft scaler fileds.Finally , Suioy D. and Samanta S. were introduced the concept of soft pseudo metric spaces.

## 2. Terminologies:

We introduced some elementary concept which we need in our work. In this section, we give basic definitions of soft sets and necessary their operations. Also we will introduced necessary notations introduced in soft topological space such as soft $T_1$-space , soft Hausdorff space and soft continuous mapping also, we introduced some results about soft convergence of soft net and investigated the relations between these concepts. In this section we will call that every soft element in a soft topological space of the term the soft point and the set of all soft points over a universe set $X$ is denoted by $S_P(X)$.

### Definition (2.1) [1]:

Let $X$ be a universe set and $E$ be a set of parameter , $A \subseteq E$. A pair $(F, A)$ is called a soft set over $X$ with respct to A , where $F$ is a mapping given by $F: A \to P(X)$ , $(F, A) = \{F(e) \in P(X), \forall e \in A\}$ .

## Remark (2.2) [2]:

**i.** $F(e)$ , $\forall e \in A$ may be arbitrary set , may be empty set.
**ii.** The soft set can be represented by two ways:
• $(F, A) = \{ F(e_i) \in P(X),\ \forall e_i \in A, i \in I \}$ .
• By ordered pairs; $(F, A) = \{ (e_i, F(e_i)) : \forall e_i \in A$, $F(e_i) \in P(X), i \in I \}$ .
**iii.** $S(X)$ denote to set of all soft sets over a universe set $X$.

## Definition (2.3) [3]:

A soft $(F, A)$ in $S(X)$ is called a soft element and denoted by $P_e^x = \{(e, F(e))\}$ if $e \in A$, $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for all $e' \in A \backslash \{e\}$ .
**i.** $P_e^x$ is a soft element of $(F, A)$ will be denoted by $P_e^x \tilde{\in} (F, A)$ , if $x \in F(e)\ \forall\ e \in A$.
**ii**. $SE(X)$ denote to set of all soft elements of a soft set $(F, A)$ over a universe set $X$.
**iii.** A soft set $(F, A)$ is called soft point if there is exactly one $e \in A$ such that $F(e) = \{x\}$ for some $x \in X$ and $F(e') = \emptyset$ for all $e' \in A \backslash \{e\}$ , denoted by $(F, A) = \{(e, \{x\})\}$.
**iv.** A soft set $(F, A)$ is called a singleton soft set , if there is $e \in A$ , then $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in A \backslash \{e\}$ , denoted by $(F, A) = \{(e, \{x\})\}$.

## Definition (2.4) [3]:

**i.** A soft set $(F, A)$ over $X$ is said to be a null soft set, denoted by $\tilde{\emptyset}$ , if $\forall e \in A$ , then $F(e) = \emptyset$.
**ii.** A soft set $(F, A)$ over $X$ is said to be the absolute soft set, denoted by $\tilde{X}$, if $\forall e \in A$, then $F(e) = X$.

## Definition (2.5)[3]:

The soft complement of a soft set $(F, A)$ over , denoted by $(F, A)^c$ defined by $(F, A)^c = (F^c, A)$ , where $F^c$ is a mapping given by $F^c : A \to P(X)$ , $F^c(e) = X \backslash F(e)$ ,$\forall e \in A$.
i.e $(F, A)^c = \{(e, X \backslash F(e)) : \forall\ e \in A\}$. It is clear that:
**i.** $\tilde{\emptyset}^{\,c} = \tilde{X}$ .
**ii.** $\tilde{X}^{\,c} = \tilde{\emptyset}$ .
**iii.** If $P_e^x \tilde{\in} (F, A)$ , then $P_e^x \notin (F, A)^c$ i.e. $x \notin F(e)$ for some $e \in A$ .

## Definition (2.6)[3]:

Let $(F, A)$ and $(G, B)$ be two soft sets over $X$, then:
**i.** $(F, A)$ is a soft subset of $(G, B)$ and denoted by $(F, A) \tilde{\subseteq} (G, B)$ , if:
• $A \subseteq B$ .
• $F(e) \subseteq G(e)$ , $\forall e \in A$ .
It is clear that:
**1.** $\tilde{\emptyset}$ is a soft subset of any soft set $(F, A)$.
**2.** Any soft set $(F, A)$ is a soft subset of $\tilde{X}$.
**ii.** $(F, A)$ , $(G, B)$ are soft equal denoted by $(F, A) = (G, B)$ , if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$. **iii.** The soft difference of $(F, A)$ and $(G, A)$ , denoted by $(H, A) = (F, A) \tilde{\backslash} (G, A)$ is defined as $H(e) = F(e) \backslash G(e)$ , $\forall\ e \in A$ .
**iv.** The union $(F, A)$ and $(G, B)$ is the soft set $(G, B)$ , where $C = A \cup B$ , $\forall\ e \in A$ , we have
$(H, C) = (F, A) \tilde{\cup} (G, B)$ such that:

$$H(e) = \begin{cases} F(e) & , if\ e \in A \backslash B \\ G(e) & , if\ e \in B \backslash A \\ F(e) \cup G(e) & , if\ e \in A \cap B \end{cases} .$$

**v.** The intersection of $(F, A)$ and $(G, B)$ is the soft set $(H, C) = (F, A) \tilde{\cap} (G, B)$ , where $C = A \cap B$ and $\forall\ e \in C$ , we write $(H, C) = (F, A) \tilde{\cap} (G, B)$ with $H(e) = F(e) \cap G(e)$.

## Definition (2.7)[2]:

Let $\mathbb{R}$ be the set of all real numbers and $P(\mathbb{R})$ be the collection of all non-empty bounded subset of $\mathbb{R}$ and $A$ be a set of parameters. A mapping $F: A \to P(\mathbb{R})$ , then $(F, A)$ is called a soft real set. If a soft real set is a singleton soft set it will be called a soft real number. We use notations $\tilde{r}$ , $\tilde{s}$ , $\tilde{t}$ etc. to denot soft real numbers. For example $\tilde{0}$ and $\tilde{1}$ are soft real numbers , where $\tilde{0}(e) = 0$ and $\tilde{1}(e) = 1$ , $\forall\ e \in A$ denote zero soft , unity soft respectively .The set of all soft real numbers is denoted by $\mathbb{R}(A)$ and the set of all non-negative soft real numbers by $\mathbb{R}(A)^*$ .
For two soft real numbers $\tilde{r}$ , $\tilde{s}$ , then:
**i.** $\tilde{r} \tilde{\leqq} \tilde{s}$ if $\tilde{r}(e) \leq \tilde{s}(e)$ ; $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(e) < \tilde{s}(e)$ , $\forall\ e \in A$.
**ii.** $\tilde{r} \tilde{\geqq} \tilde{s}$ if $\tilde{r}(e) \geq \tilde{s}(e)$ ; $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(e) > \tilde{s}(e)$ , $\forall\ e \in A$.

## Remark (2.8)[2]:

For two soft real numbers $\tilde{r}$ , $\tilde{s}$ , then:

**i.** $\tilde{r} \tilde{\mp} \tilde{s} = (e_1, F(e_1)) \tilde{\mp} (e_2, F(e_2)) = (e_1 \mp e_1 , F(e_1) \mp F(e_1))$ .

**ii.** According to these operations above know the soft real intervals.

**iii.** The inverse of any soft real $\tilde{r}$ denoted by $\tilde{r}^{-1}$ and defined as $\tilde{r}^{-1}(e) = [\tilde{r}(e)]^{-1}$, $\forall\, e \in A$.

**iv.** A soft set $(F, A)$ in $\mathbb{R}(A)$, is called soft bounded , if there are two soft real numbers $\tilde{m}$ , $\tilde{n}$ such that $\tilde{m} \lesssim \tilde{r} \lesssim \tilde{n}$ for any $\tilde{r} \,\tilde{\in}\, (F, A)$. A soft set which is not soft bounded is called soft unbounded set.

## Definition (2.9)[4]:

Let $\tilde{\lesssim}$ be a soft ordering of a soft set $(F, A)$ over a universe , let $(G, A) \,\tilde{\subseteq}\, (F, A)$:

**i.** For $\in A$ , $P_e^x$ is a soft lower bound of $(G, A)$ in the soft ordered $(F, A, \tilde{\lesssim})$ , if $P_e^x \,\tilde{\lesssim}\, P_e^y$ for all $P_e^y \,\tilde{\in}\, (G, A)$.

**ii.** For $e \in A$, $P_e^x$ is called soft infimum (for short $\widetilde{inf}$) of $(G, A)$ in $(F, A, \tilde{\lesssim})$ (or the greatest soft lower bound) if it is the greatest soft point of the set of all soft lower bounds of $(G, A)$ in $(F, A, \tilde{\lesssim})$.

**iii.** For $e \in A$, $P_e^x$ is a soft upper bound of $(G, A)$ in the soft ordered $(F, A, \tilde{\lesssim})$ , if $P_e^y \,\tilde{\lesssim}\, P_e^x$ for all $P_e^y \,\tilde{\in}\, (G, A)$.

**iv.** For $e \in A$ , $P_e^x$ is called soft supremum ( for short $\widetilde{sup}$ ) of $(G, A)$ in $(F, A, \tilde{\lesssim})$ (or the least soft upper bound) if it is the least soft point of the set of all soft upper bounds of $(G, A)$ in $(F, A, \tilde{\lesssim})$.

## Remark (2.10)[4]:

**i.** When a soft infimum of $(G, A)$ in $(F, A, \tilde{\lesssim})$ is a soft point of $(G, A)$ , then it is called soft minimum (for short $\widetilde{min}$) .

**ii.** When the soft supremum of $(G, A)$ in $(F, A, \tilde{\lesssim})$ is a soft point of $(G, A)$ , then it is called soft maximum ( for short $\widetilde{max}$ ) .

**iii.** For $e \in A$, $P_e^x$ is called a soft maximal point of $(G, A)$ in $(F, A, \tilde{\lesssim})$ , if there is no $P_e^y \,\tilde{\in}\, (G, A)$ such that $P_e^x \,\tilde{\lesssim}\, P_e^y$ and $P_e^x \neq P_e^y$ .

## Definition (2.11) [5]:

Let $\tilde{\tau}$ be the collection of soft sets over $X$, then $\tilde{\tau}$ is said to be a soft topology on $X$, if :

**i.** $\tilde{\emptyset}, \tilde{X} \,\tilde{\in}\, \tilde{\tau}$ .

**ii.** If $(F, A)_\alpha \,\tilde{\in}\, \tilde{\tau}$ for all $\alpha \in \Lambda$ , then $\tilde{\bigcup}_{\alpha \in \Lambda} (F, A)_\alpha \,\tilde{\in}\, \tilde{\tau}$.

**iii.** If $(F, A)_1 , (F, A)_2 \,\tilde{\in}\, \tilde{\tau}$ , then $(F, A)_1 \,\tilde{\cap}\, (F, A)_2 \,\tilde{\in}\, \tilde{\tau}$ .

The triple $(X, \tilde{\tau}, A)$ is called a soft topological space, the members of $\tilde{\tau}$ is called soft open sets. The complements of the members of $\tilde{\tau}$ is called soft closed sets. If $(X, \tilde{\tau}, A)$ be a soft topological space over $X$ and $(F, A)$ be a soft set over $X$ , then:

- The soft closure of $(F, A)$ ( for short $\overline{(F, A)}$ ) is the intersection of all soft closed sets containing $(F, A)$.
- A soft point $P_e^x \,\tilde{\in}\, (F, A)$ is said to be a soft interior point of $(F, A)$ , if there is a soft open set $(G, A)$ such that $P_e^x \,\tilde{\in}\, (G, A) \,\tilde{\subseteq}\, (F, A)$ . The soft set $(F, A)$ which contains all soft interior points is called soft interior set and it is denoted by $(F, A)^o$.
- A soft set $(F, A)$ is said to be a soft neighborhood ( soft nbhd ) of a soft point $P_e^x$ in set of all soft points over $X$ [ $S_P(X)$ ] if there is a soft open set $(G, A)$ such that $P_e^x \,\tilde{\in}\, (G, A) \,\tilde{\subseteq}\, (F, A)$ set. The soft nbhd system of $P_e^x$, denoted by $\mathcal{N}_{P_e^x}$ is the collection of all soft nbhds of $P_e^x$.

## Definition (2.12)[5]:

Let $(F, A)$ be a soft set of $(X, \tilde{\tau}, A)$ , $P_e^x \,\tilde{\in}\, S_P(X)$ , then $P_e^x \,\tilde{\in}\, \overline{(F, A)}$ if and only if $(F, A) \tilde{\cap} (G, A) \neq \tilde{\emptyset}$ , for all soft open set $(G, A)$ of $(X, \tilde{\tau}, A)$ soft contain $P_e^x$.

## Definition (2.13)[5],[6]:

**i.** Let $(F, A)$ be a soft set over $X$ and $Y$ be a non-empty set of $X$, then a soft set $(F, A)$ over $Y$ denoted by $(F, A)' = \tilde{Y} \tilde{\cap} (F, A)$ is defined as follows $F'(e) = Y \cap F(e)$ , $\forall e \in A$. In other words $F'(e) = \tilde{Y} \cap F(e)$

**ii.** $(F, A)$ , $(G, A)$ are said to be soft disjoint , if $(F, A) \tilde{\cap} (G, A) = \tilde{\emptyset}$ .

**iii.** A non-null soft sets $(F, A)$ , $(G, A)$ are said to be soft separated of $\tilde{X}$ iff $(F, A) \tilde{\cap} \overline{(G, A)} = \tilde{\emptyset}$ , $\overline{(F, A)} \tilde{\cap} (G, A) = \tilde{\emptyset}$ and $(F, A) \tilde{\cap} (G, A) = \tilde{\emptyset}$ .

**iv.** Two soft closed (soft open) sets $(F, A)$ , $(G, A)$ are soft separated if and only if they are soft disjoint.

**v.** $(X, \tilde{\tau}, A)$ is soft Hausdorff space if for all $P_e^x, P_e^y \in S_P(X)$ with $P_e^x \neq P_e^y$ , there are two soft disjoint soft open sets $(F, A),(G, A)$ in $(X, \tilde{\tau}, A)$ with $P_e^x \,\tilde{\in}\, (F, A)$ and $P_e^y \,\tilde{\in}\, (G, A)$.

**i.** $(X,\tilde{\tau},A)$ is soft Hausdorff if and only if $\tilde{\Delta}(X)=\{(P_e^x,P_e^x): P_e^x \tilde{\in} S_P(X)\}$ is soft closed.
**ii.** $(X,\tilde{\tau},A)$ is soft $T_1$-space if and only if $\{P_e^x\}$ is soft closed for every $P_e^x \tilde{\in} S_P(X)$.
**iii.** Every soft Hausdorff space is soft $T_1$-space ,but the converse is not true.
.

## Definition (2.15)[7]:

Let $(F,A)$ , $(G,A)$ be two soft sets over a universes $X$, $Y$ respectively. A soft mapping $\tilde{f}:(X,\tilde{\tau}_X,A) \tilde{\rightarrow} (Y,\tilde{\tau}_Y,A)$ is called soft continuous at $P_e^x \tilde{\in} S_P(X)$ , if for each soft nbhd $(F,A)$ of $P_e^x$ , there is soft nbhd $(G,A)$ of $\tilde{f}(P_e^x)$ such that $\tilde{f}((F,A)) \tilde{\subseteq} (G,A)$. If $\tilde{f}$ is soft continuous mapping at $P_e^x$, then it is called soft continuous.

## Theorem (2.16)[7]:

The following statements are equivalent for any soft mappin $\tilde{f}:(X,\tilde{\tau}_X,A) \tilde{\rightarrow} (Y,\tilde{\tau}_Y,A)$ :
**i.** $\tilde{f}$ is soft continuous .
**ii.** $\tilde{f}^{-1}((G,A))$ is soft open set over $X$ for each soft open set $(G,A)$ over $Y$.
**iii.** $\tilde{f}^{-1}((G,A))$ is soft closed set over $X$ for each soft closed set $(G,A)$ over $Y$.

## Definition (2.17)[8]:

A soft mapping $\tilde{f}:(X,\tilde{\tau}_X,A) \tilde{\rightarrow} (Y,\tilde{\tau}_Y,A)$ is called soft homeomorphism, if $\tilde{f}$ is soft bijective, and both $\tilde{f}$ and $\tilde{f}^{-1}$ are soft continuous mapping.
.

## Definition (2.18)[9]:

A soft net in a set $S_P(X)$ is a soft mapping $P_{e_d}^{x_d} : D \tilde{\rightarrow} S_P(X)$ , $D$ is directed set and $F(e_d)=x_d$ , $(x_d)_{d\in D}$ is a net in a universe set $X$ and $(e_d)_{d\in D}$ be a net in $A$. The soft net $P_{e_d}^{x_d}(d)$ is denoted by $(P_{e_d}^{x_d})_{d\in D}$.

## Definition (2.19) [9]:

**i.** $(P_{e_d}^{x_d})_{d\in D}$ is said to be soft convergence to $P_e^x$ , if $(P_{e_d}^{x_d})_{d\in D}$ is eventually in each soft nbhd of $P_e^x$ and denoted by $(P_{e_d}^{x_d} \tilde{\rightarrow} P_e^x)$ . The soft point $P_e^x$ is called a limit soft point of $(P_{e_d}^{x_d})_{d\in D}$ .

**ii.** $\tilde{f}:(X,\tilde{\tau}_X,A) \tilde{\rightarrow} (Y,\tilde{\tau}_Y,A)$ be a soft mapping and , $(P_{e_d}^{x_d})_{d\in D}$ be a soft net in $(X,\tilde{\tau}_X,A)$ , then $\{f(P_{e_d}^{x_d})\}_{d\in D}$ is a soft net in $(Y,\tilde{\tau}_Y,A)$ .

## Theorem (2.14)[6]:

**iii.** $P_e^x \tilde{\in} \overline{(F,A)}$ if and only if there is a soft net $(P_{e_d}^{x_d})_{d\in D}$ in $(F,A)$ such that $P_{e_d}^{x_d} \tilde{\rightarrow} P_e^x$ .
**iv.** A soft mapping $\tilde{f}:(X,\tilde{\tau}_X,A) \tilde{\rightarrow} (Y,\tilde{\tau}_Y,A)$ is soft continuous at $P_e^x$ if and only if , whenever a soft net $(P_{e_d}^{x_d})_{d\in D}$ in $(X,\tilde{\tau}_X,A)$ and $P_{e_d}^{x_d} \tilde{\rightarrow} P_e^x$ , then $\tilde{f}(P_{e_d}^{x_d}) \tilde{\rightarrow} f(P_e^x)$ in $(Y,\tilde{\tau}_Y,A)$ .

## Definition (2.20) [10]:

A soft set $(F,A)$ over a linear space $X$ is said to be:
**i.** Soft convex, if $\lambda(F,A)\tilde{+}(1-\lambda)(F,A) \tilde{\subseteq} (F,A)$ , for all $\lambda \in [0,1]$ .
**ii.** Soft balanced if $\alpha(F,A) \tilde{\subseteq} (F,A)$ $\forall\, \alpha \in \mathbb{K}$ with $|\alpha|\le 1$. If $|\alpha|=1$ , then $\alpha(F,A)=(F,A)$.
**iii.** Soft absorbing , if $\bigcup_{\alpha>0}\alpha(\mathrm{F},\mathrm{A})=\tilde{\mathrm{X}}$ .
**iv.** Soft symmetric if $(F,A)=-(F,A)$, $-F(e)=\{-x: x\in F(e)\}$. If $P_e^x \tilde{\in} (F,A)$ , then $-P_e^x \in (F,A)$ .

## Theorem (2.21) [11]:

**i** If $(F,A)$ and $(G,A)$ are two soft convex (soft balanced) , then $\alpha_1(F,A)\tilde{+}\alpha_2(G,A)$ and $(F,A)\tilde{\cap}(G,A)$ is soft convex (soft balanced) for all$\alpha_1,\alpha_2 \in \mathbb{K}$ .
**ii** If $(F,A)$ and $(G,A)$ are two soft balanced , then $(F,A)\tilde{\cup}(G,A)$ is soft balanced.

## Definition (2.22) [11]:

Let $X$ be a linear space over $\mathbb{K}$ , $A$ be a parameter set , then $X$ with the soft topology $\tilde{\tau}$ is said to be linear soft topology on $X$ . A triple $(X,\tilde{\tau},A)$ is called soft topological linear space ( for short $STLS$ ) , if the mappings:
**i.** $\tilde{+}: SL(\tilde{X}) \tilde{\times} SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{X})$ which is defined as $\tilde{+}(P_e^x,P_e^y)=P_e^x\tilde{+}P_e^y$.
**ii.** $\tilde{\cdot}: \tilde{\mathbb{K}} \times SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{X})$ defined as $\tilde{\cdot}(\tilde{\lambda},P_e^x)=\tilde{\lambda}\tilde{\cdot}P_e^x$ are soft continuous , for all $P_e^x,P_e^y \tilde{\in} SL(\tilde{X})$ and for all $\lambda \tilde{\in} \tilde{\mathbb{K}}$.

## Theorem (2.23) [12]:

Let $(X,\tilde{\tau},A)$ be $STLS$ over $\mathbb{K}$. For given $x_0 \in X$ and $\tilde{0} \ne \tilde{k},\tilde{\alpha} \tilde{\in} \tilde{\mathbb{K}}$, then:
**i.** A soft translation ; $\tilde{T}_{x_0}: SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{X})$ , defined by $\tilde{T}_{x_0}(P_e^x)=x_0\tilde{+}P_e^x$;
**ii.** A soft multiplication; $\tilde{M}_{\tilde{k}}: SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{X})$ , defined by $\tilde{M}_{\tilde{k}}(P_e^x)=\tilde{k}\tilde{\cdot}P_e^x$ , are soft homeomorphism.

For given $x_o \in X$ and $0 \neq t \in \mathbb{K}$, if $(F,A)$ be a soft open set of $STL(\tilde{X})$, then:

- $(x_o + F, A)$ is a soft open (soft nbhd) of $(X,\tilde{\tau},A)$.
- $(tF,A)$ is a soft open (soft nbhd) of $(X,\tilde{\tau},A)$.
- If $(F,A)$ be a soft nbhd of $P_e^x$, then $(-x+F,A)$ is a soft nbhd of $P_e^0$.

## Definition (2.24)[12]:

A collection $\mathcal{B}_{P_e^x}$ of soft nbhds of $P_e^x$ in $STLS$ $(X,\tilde{\tau},A)$ is called soft local base of $P_e^x$ if for any soft nbhd $(G,A)$ of $P_e^x$, there is $(F,A) \tilde{\in} \mathcal{B}_{P_e^x}$ such that $(F,A) \tilde{\subseteq} (G,A)$.

It is clear that in a $STLS$ $(X,\tilde{\tau},A)$:

**i.** $\mathcal{B}_{P_e^0}$ be a soft local base at $P_e^0$, if $(F,A) \tilde{\in} \mathcal{B}_{P_e^o}$, then $(-F,A) \tilde{\in} \mathcal{B}_{P_e^o}$.

**ii.** If $\mathcal{B}_{P_e^0}$ be a soft local base at $P_e^0$, then $\mathcal{B}_{P_e^x} = \{(x+F,A):(F,A) \tilde{\in} \mathcal{B}_{P_e^o}; x \in X\}$ is a soft local base at $P_e^x$.

**iii.** If $\mathcal{B}_{P_e^x}$ be a soft local base at $P_e^x$, then $\mathcal{B}_{tP_e^x} = \{(tF,A):(F,A) \tilde{\in} \mathcal{B}_{P_e^x}\}$ is a soft local base at $P_e^{tx}$.

**iv.** For any soft nbhd $(F,A)$ of $P_e^o$, there is a soft nbhd $(G,A)$ of $P_e^o$ with $\overline{(G,A)} \tilde{\subseteq} (F,A)$.

**v.** For any soft nbhd $(F,A)$ of $P_e^o$, $\bar{F}(e) = \overline{F(e)}^e = \bigcap_{(G,A)\tilde{\in}\mathcal{B}_{P_e^o}}[F(e)+G(e)]$, $\forall\, e \in A$.

## Theorem (2.25)[12]:

Let $(X,\tilde{\tau},A)$ be a $STLS$ over $\mathbb{K}$, then:

**i.** Every soft nbhd of $P_e^o$ contains a soft balanced nbhd of $P_e^o$.

**ii.** Every soft convex nbhd of $P_e^o$ contains a soft convex nbhd of $P_e^o$.

**iii.** Every soft nbhd of $P_e^o$ is soft symmetric.

**iv.** For every soft nbhd $(G,A)$ of $P_e^o$, there is a soft symmetric ( which is soft nbhd of $P_e^o$) $(F,A)$ such that $(F,A)\tilde{+}(F,A) \tilde{\subseteq} (G,A)$.

**v.** For every soft nbhds $(F,A)$ and $(G,A)$ of $P_e^o$, the following is true $\overline{(F,A)}\tilde{+}\overline{(G,A)} \tilde{\subseteq} \overline{(F,A)\tilde{+}(F,A)}$.

## Theorem (2.26):

For any soft set $(F,A)$ in a $STLS$ $SL(\tilde{X})$ and for any $P_e^x \tilde{\in} \overline{(F,A)}$, there is a soft net $(P_{e_d}^{x_d})_{d\in D}$ in $(F,A)$ with $P_{e_d}^{x_d} \tilde{\to} P_e^x$. (Soft partially ordered by reverse soft inclusion ) .

**Proof:** For any soft nbhd $(G,A)$ of $P_e^o$ and by theorem (2.23.i), then $P_e^x\tilde{+}(G,A)$ is a soft nbhd of $P_e^x$.

Since $P_e^x \tilde{\in} \overline{(F,A)}$, so $P_e^x\tilde{+}(G,A)\tilde{\cap}(F,A) \neq \tilde{\emptyset}$. Let $P_{e_d}^{x_d}$ be any soft points of $P_e^x\tilde{+}(G,A)\tilde{\cap}(F,A)$. Hence $(P_{e_d}^{x_d})_{d\in D}$ is a soft net in $(F,A)$ such that $P_{e_d}^{x_d} \tilde{\to} P_e^x$. Indeed, for each soft nbhd $(G,A)$ of $P_e^o$, we have $P_{e_d}^{x_d} \tilde{-} P_e^x \tilde{\in} (G,A)$ and so $P_{e_d}^{x_d} \tilde{-} P_e^x \tilde{\to} P_e^0$ along $\mathcal{B}_{P_e^o}$.

## Theorem (2.27):

A soft closed , soft balanced and soft nbhds of $P_e^o$ forms a soft local base of $P_e^o$ in a $STLS$ $SL(\tilde{X})$ .

**Proof:** Let $(F,A)$ be a soft nbhd of $P_e^o$, by theorem (2.25.iv) , there is a soft nbhd $(G,A)$ of $P_e^o$ such that $(G,A)\tilde{+}(G,A) \tilde{\subseteq} (F,A)$. Also , by theorem (2.25.i) , there is a soft balanced set $(F,A)$ which is a soft nbhd of $P_e^o$, with $(H,A) \tilde{\subseteq} (F,A)$, thus $(H,A)\tilde{+}(H,A) \tilde{\subseteq} (F,A)$. Then by definition (2.24.iv), implies that $\overline{(H,A)} \tilde{\subseteq} (H,A)\tilde{+}(H,A) \tilde{\subseteq} (F,A)$ where $\overline{(H,A)}$ is a soft nbhd of $P_e^o$ . We claim that $\overline{(H,A)}$ is a soft balanced , let $P_e^x \tilde{\in} \overline{(H,A)}$ and let , $\alpha \in \mathbb{K}$, with $|\alpha| \leq 1$ . By theorem (2.26) , there is a soft net $(P_{e_d}^{x_d})_{d\in D}$ in $(H,A)$ such that $P_{e_d}^{x_d} \tilde{\to} P_e^x$ . Then $\alpha P_{e_d}^{x_d} \tilde{\to} \alpha P_e^x$, but $\alpha P_{e_d}^{x_d} \tilde{\in} (H,A)$ and it is a soft balanced nbhd , so $\alpha P_e^x \tilde{\in} (H,A)$. It follows that $\overline{(H,A)}$ is a soft balanced as claimed.

## Theorem (2.28):

A $STLS$ $SL(\tilde{X})$ is soft Hausdorff space if and only if $\{P_e^x\}$ is soft closed set for all $P_e^x \tilde{\in} SL(\tilde{X})$.

**Proof:** Of course , if $SL(\tilde{X})$ is a soft Hausdorff , by Theorem (2.14. iii) , we obtained the result. Conversely, since $\tilde{f}: SL(\tilde{X}) \tilde{\times} SL(\tilde{X}) \tilde{\to} SL(\tilde{X})$ defined as $\tilde{f}(P_e^x, P_e^y) = P_e^x \tilde{-} P_e^y$ be a soft continuous mapping for all $P_e^x, P_e^y \in SL(\tilde{X})$ and $\{P_e^0\}$ be a soft closed. By theorem (2.16.iii) , $\tilde{f}^{-1}(\{P_e^0\})$ is soft closed set in $SL(\tilde{X}) \tilde{\times} SL(\tilde{X})$. But $\tilde{f}^{-1}(\{P_e^0\}) = \{(P_e^x, P_e^x): P_e^x \tilde{\in} SL(X)\} = \tilde{\Delta}(X)$, then by theorem (2.14.i) completes the proof .

## Corollary (2.29):

A $STLS\ SL(\tilde{X})$ is soft Hausdorff if and only if it is a soft separated space.

**Proof:** Clearly, if $SL(\tilde{X})$ be a soft Hausorff , by definition (2.13.iv) we get the required.
For converse, let $P_e^x , P_e^y \tilde{\in} SL(\tilde{X})$ with $P_e^x \neq P_e^y$ , put $P_e^z = P_e^x \tilde{-} P_e^y$ , so that $P_e^z \neq P_e^0$ . Now, if $\{P_e^z\}$ be a soft closed , then $SL(\tilde{X})\tilde{\setminus}\{P_e^z\}$ is a soft open set which is soft nbhd of $P_e^0$. A soft continuity of an
addition at $P_e^0$ this means there two soft nbhds $(F,A),(G,A)$ of $P_e^0$ with $(F,A)\tilde{+}(G,A) \tilde{\subseteq} SL(\tilde{X})\tilde{\setminus}\{P_e^z\}$ .
Since $(G,A)$ be a soft nbhd of $P_e^0$, by theorem (2.25.iii) , so is $(-G,A)$ , then by theorem (2.23.i) $P_e^z\tilde{+}(-G,A)$ is a soft nbhd of $P_e^z$. For any $P_e^w \tilde{\in} P_e^z\tilde{+}(-G,A)$ has the form $P_e^w = P_e^z \tilde{-} P_e^u$ , where $P_e^u \tilde{\in} (G,A)$ , and so $P_e^z = P_e^w \tilde{+} P_e^u$ , since $\{P_e^z\} \tilde{\notin} (F,A)\tilde{+}(G,A)$ , we must have $\{P_e^u\} \tilde{\notin} (F,A)$. Thus
$(F,A)\tilde{\cap} P_e^z\tilde{+}(-G,A) = \tilde{\emptyset}$ by soft translation we have $[ P_e^y\tilde{+}(F,A)]\tilde{\cap}[P_e^y\tilde{+}P_e^z\tilde{+}(-G,A)] = \tilde{\emptyset}$ , implies that $[P_e^x\tilde{+}(F,A)]\tilde{\cap}[P_e^y\tilde{+}(-G,A)] = \tilde{\emptyset}$ .
i. e. $[P_e^x\tilde{+}(F,A)] , [P_e^y\tilde{+}(-G,A)]$are soft disjoint nbhds of $P_e^x$ and $P_e^y$ respectively, this complete proof.

## Corollary (2.30):

A $STLS\ SL(\tilde{X})$ is soft separated space if and only if $\{P_e^x\}$ is soft closed set for all $P_e^x \tilde{\in} SL(\tilde{X})$.

**Proof:** By using theorem (2.28) and corollary (2.29).

## 3. The Soft Topologies Induced By Seminorms:

In this section we introduced the important concept is called soft seminorm on a soft linear space over a soft scalar field and we give some results on about this concept. Also, we study a new type of a soft topology by using a soft seminors denoted by a soft topology which induced by the family of soft seminorms.

## Definition (3.1)[13],[14]:

Let $\tilde{X}$ , $\tilde{Y}$ be two $SLS$s over $\tilde{\mathbb{K}}$.

**i.** A soft mapping $\tilde{\Lambda}: SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{Y})$ is said to be a soft linear, if $\tilde{\Lambda}(\tilde{\alpha}P_e^x\tilde{+}\tilde{\delta}P_e^y) = \tilde{\alpha}\tilde{\Lambda}(P_e^x) + \tilde{\delta}\tilde{\Lambda}(P_e^x)$ ; $\forall P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ , $\forall \tilde{\alpha} , \tilde{\delta} \in \tilde{\mathbb{K}}$ .

**ii.** A soft linear functional is a soft linear $\tilde{\Lambda}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{K}(A)$ , it follows that the difference between a soft linear mapping and a soft linear functional is that in the case of soft linear functional , the range $\mathbb{K}(A)$.

## Definition (3.2) :

Let $SL(\tilde{X})$ be a $SLS$ over $\tilde{\mathbb{K}}$. A soft mapping $\tilde{\Lambda}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{K}(A)$ is called a soft convex functional on $SL(\tilde{X})$, if:

**i.** $\tilde{\Lambda}(P_e^x) \geq \tilde{0}$ ; $\forall P_e^x \tilde{\in} SL(\tilde{X})$ , (soft positive).

**ii.** $\tilde{\Lambda}\left(P_e^x\tilde{+}P_e^y\right) \leq \tilde{\Lambda}(P_e^x)\tilde{+}\tilde{\Lambda}(P_e^y)$ ; $\forall P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ , (soft sub-additive).

**iii.** $\tilde{\Lambda}(\tilde{\alpha}P_e^x) = \tilde{\alpha}\tilde{\Lambda}(P_e^x)$ ; $\forall P_e^x \tilde{\in} SL\left(\tilde{X}\right)$ , $\forall \tilde{\alpha} \tilde{\in} \tilde{\mathbb{R}}$ ( soft homogeneous).

## Example (3.3)

Let $SL(\tilde{\mathbb{R}}^n)$ be a $SLS$ over $\tilde{\mathbb{R}}$ with the parameter set $\mathbb{N}$. Define $\tilde{\Lambda}: SL(\tilde{\mathbb{R}}^n) \tilde{\rightarrow} \mathbb{R}(A)$ by $\tilde{\Lambda}(P_e^x) = \sum_{i=0}^{n}\left|P_{e_i}^{x_i}\right|$ , $\forall P_e^x \tilde{\in} SL(\tilde{\mathbb{R}}^n)$, where $P_e^x = (P_{e_1}^{x_1}, P_{e_2}^{x_2}, \dots, P_{e_n}^{x_n})$ , $n \in \mathbb{N}$ , then $\tilde{\Lambda}$ is soft convex functional.
.

## Definition (3.4)

Let $SL(\tilde{X})$ be a $SLS$ over $\tilde{\mathbb{K}}$. A soft mapping $\tilde{\varphi}: SL(\tilde{X}) \tilde{\times} SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ is called a soft Hermitian functional , if satisfying $\forall P_e^x, P_e^y, P_e^z \tilde{\in} SL(\tilde{X})$ , $\forall \tilde{\alpha} \tilde{\in} \tilde{\mathbb{R}}$ , then:

**i.** $\tilde{\varphi}(P_e^x\tilde{+}P_e^y, P_e^z) = \tilde{\varphi}(P_e^x, P_e^z)\tilde{+}\tilde{\varphi}(P_e^y, P_e^z)$ ; $\tilde{\varphi}(P_e^x, P_e^y\tilde{+}P_e^z) = \tilde{\varphi}(P_e^x, P_e^y)\tilde{+}\tilde{\varphi}(P_e^x, P_e^z)$ .

**ii.** $\tilde{\varphi}\left(\tilde{\alpha}P_e^x, P_e^y\right) = \tilde{\alpha}\tilde{\varphi}(P_e^x, P_e^y)$ ; $\tilde{\varphi}\left(P_e^x, \tilde{\alpha}P_e^y\right) = \tilde{\alpha}\tilde{\varphi}\left(P_e^x, P_e^y\right)$ .

**iii.** $\tilde{\varphi}(P_e^x, P_e^y) = \tilde{\varphi}(P_e^y, P_e^x)$ ; $\forall P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ .

## Remark (3.5)

If $\tilde{\varphi}: SL(\tilde{X}) \tilde{\times} SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ be a soft Hermitian functional , then:

**i.** $\tilde{\varphi}\left(P_e^x\tilde{+}P_e^y, P_e^x\tilde{+}P_e^y\right) =$

$$\tilde{\varphi}(P_e^x, P_e^x)\tilde{+}\tilde{\varphi}\left(P_e^x, P_e^y\right)\tilde{+}\tilde{\varphi}\left(P_e^y, P_e^x\right)\tilde{+}\tilde{\varphi}\left(P_e^y, P_e^y\right)$$
$$= \tilde{\varphi}(P_e^x, P_e^x) + \tilde{2}\tilde{\varphi}\left(P_e^x, P_e^y\right)\tilde{+}\tilde{\varphi}\left(P_e^y, P_e^y\right).$$

**ii.** For any a soft linear functional $\tilde{\Phi}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ defined by $\tilde{\Phi}(P_e^x) = \tilde{\varphi}(P_e^x, P_e^x)$ , $\forall P_e^x \tilde{\in} SL(\tilde{X})$.
Then $\tilde{\Phi}$ is called a soft linear functional associated with $\tilde{\varphi}$.

**iii**. For any $P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ a Caushy-Schwarz soft inequality

$$|\tilde{\varphi}(P_e^x, P_{e'}^y)| \tilde{\leq} \sqrt[\tilde{2}]{\tilde{\Phi}(P_e^x)} \tilde{\cdot} \sqrt[\tilde{2}]{\tilde{\Phi}(P_{e'}^y)} .$$

**iv.** For any $P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ a Minkowski's soft inequality

$$\sqrt[\tilde{2}]{\tilde{\Phi}(P_e^x) \tilde{+} \tilde{\Phi}(P_e^y)} \tilde{\leq} \sqrt[\tilde{2}]{\tilde{\Phi}(P_e^x)} \tilde{+} \sqrt[\tilde{2}]{\tilde{\Phi}(P_e^y)} .$$

## Definition (3.6) [15]:

A soft seminorm on $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$ is a soft mapping $\tilde{p}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ having the following:
**i.** $\tilde{p}(P_e^x) \geq \tilde{0}$ ; $\forall P_e^x \tilde{\in} SL(\tilde{X})$, (soft positive).
**ii.** $\tilde{p}(P_e^x \tilde{+} P_e^y) \leq \tilde{p}(P_e^x) \tilde{+} \tilde{p}(P_e^y)$ ; $\forall P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ , (soft sub-additive).
**iii.** $\tilde{p}(\tilde{\alpha} P_e^x) = |\tilde{\alpha}| \tilde{p}(P_e^x)$ , $\forall P_e^x \tilde{\in} SL(\tilde{X}), \tilde{\alpha} \tilde{\in} \tilde{\mathbb{K}}$ ( soft homogeneous).

## Remark (3.7) [15]:

Let $\tilde{p}$ be a soft seminorm on a soft linear space $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$, then:
**i.** If $P_e^x = P_e^0$, then $\tilde{p}(P_e^x) = \tilde{0}$.
**ii.** $|\tilde{p}(P_e^x) \tilde{-} \tilde{p}(P_e^y)| \tilde{\leq} \tilde{p}(P_e^x - P_e^y)$ , $\forall P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$.

## Example (3.8):

Let $SL(\tilde{X})$ be a $SLS$ over $\tilde{\mathbb{K}}$.
**i.** A soft non-negative soft Hermitian functional, which defined in definition (3.4) , then a soft mapping $\tilde{p}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ , which is defined by $\tilde{p}(P_e^x) = \sqrt[\tilde{2}]{\tilde{\varphi}(P_e^x, P_e^x)}$ is soft seminorm.
**ii.** A soft mapping $SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ with $P_e^x \tilde{\rightarrow} \tilde{0}$ , $\forall P_e^x \tilde{\in} SL(\tilde{X})$, is soft seminorm.
**iii.** Every soft norm $\|\tilde{\cdot}\|: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ is soft seminorm. The converse is not true in general , as the example in paragraph (ii) above.

## Remark (3.9) :

A soft seminorm on a $SLS$s is related to a special kind of a soft convex functional. But the converse is not true as the following example shows:
A soft mapping $\tilde{p}: SL(\tilde{X}) \tilde{\rightarrow} \mathbb{R}(A)$ with $P_e^x \tilde{\rightarrow} |P_e^x|$ , $\forall P_e^x \tilde{\in} SL(\tilde{X})$, $\tilde{p}$ is soft seminorm , but it is not soft convex functional.

## Theorem (3.10)[15]: (Decomposition theorem)

Every soft seminorm $\tilde{p}$ on a $SLS$ $SL(\tilde{X})$ with a parameter set $A$ satisfies the following condition: For $x \in X$ and $e \in A$, $\{[\tilde{p}(P_e^x)](e): F(e) = x\}$ is singleton set. Then each soft seminorm $\tilde{p}$ can be decomposed into a family $\{p_e: e \in A\}$ of seminorms on a linear space $X$, where $p_e: X \rightarrow \mathbb{R}$ is defined by: For $x \in X$ , $p_e(x) = [\tilde{p}(P_e^x)](e)$ with $P_e^x \tilde{\in} SL(\tilde{X})$ such that $F(e) = x$.

## Remark (3.11) [15]:

From the condition above and the soft seminorms axioms gives the seminorms condition of $p_e$.
Thus , every soft seminorms gives a parameterized family of seminorms.

## Theorem (3.12) [15]:

Every parameterized family of seminorms $\{p_e: e \in A\}$ on a linear space $X$ can be considered as soft seminorms on a $SLS$ $SL(\tilde{X})$.

## Theorem (3.13) [15]:

If $\tilde{p}: SL(\tilde{X}) \tilde{\rightarrow} \tilde{\mathbb{R}}$ be a soft seminorm on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$ , then:
**i.** $ker\tilde{p} = \{ P_e^x : \tilde{p}( P_e^x) = \tilde{0}\}$ is soft linear subspace of $SL(\tilde{X})$.
**ii.** A soft set $(F, A) = \{ P_e^x : \tilde{p}(P_e^x) \tilde{<} \tilde{1}\}$ is soft convex , soft balanced and soft absorbing.

## Remark (3.14):

Let $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ be a family of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$ , consider the soft set $(F, A)(P_e^{x_o}, \tilde{p}_1 , \tilde{p}_2 , \dots, \tilde{p}_n ; \tilde{r}) = \{ P_e^x \tilde{\in} SL(\tilde{X}) : \tilde{p}_i (P_e^x \tilde{-} P_e^{x_o}) \tilde{<} \tilde{r} \}$ , for all $i = 1,2, \dots, n$ , where
$P_e^{x_o} \tilde{\in} SL(\tilde{X})$ , $\tilde{r} \tilde{>} \tilde{0}$ and is a finite collection of soft semi norms in $\mathcal{P}$. Notice that:
**i.** $(F, A)(P_e^{x_o}, \tilde{p}_1 , \tilde{p}_2, \dots, \tilde{p}_n ; \tilde{r}) = P_e^{x_o} \tilde{+} (F, A)(P_e^0, \tilde{p}_1 , \tilde{p}_2 , \dots, \tilde{p}_n ; \tilde{r})$.
**ii.** $(F, A)(P_e^{x_o}, \tilde{p}_1 , \tilde{p}_2 , \dots, \tilde{p}_n ; \tilde{r}) = \{ P_e^x \tilde{\in} SL(\tilde{X}) : \tilde{p}_i(P_e^x \tilde{-} P_e^{x_o}) \tilde{<} \tilde{r} \}$ is soft convex , soft balanced and soft absorbing set.

## Theorem (3.15):

Let $\mathcal{P}=\{\tilde{p}_\alpha : \alpha \in I\}$ be a family of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$ . For $P_e^x \widetilde{\in} SL(\tilde{X})$ the collection of all soft sets $(F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r})$ , $\tilde{r} \widetilde{>} \tilde{0}$ . Let $\tilde{\tau}_{\mathcal{P}}$ be the collection of all soft sets over $X$ consisting of $\widetilde{\emptyset}$ together with all those soft sets $(H,A) \widetilde{\in} SL(\tilde{X})$ such that for any $P_e^x \widetilde{\in} (H,A)$ , there is $(F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r})$ such that $(F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r}) \widetilde{\subseteq} (H,A)$. Then $\tilde{\tau}_{\mathcal{P}}$ is a soft topology on $SL(\tilde{X})$ compatible with the soft linear space $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$.

**Proof:**

Evidently $\tilde{X} \widetilde{\in} \tilde{\tau}_{\mathcal{P}}$ , and it is clear that the soft union of any family soft sets of $\tilde{\tau}_{\mathcal{P}}$ is also a soft member of $\tilde{\tau}_{\mathcal{P}}$. We shall that if $(G,A),(H,A) \widetilde{\in} \tilde{\tau}_{\mathcal{P}}$ ,then $(G,A)\widetilde{\cap}(H,A) \widetilde{\in} \tilde{\tau}_{\mathcal{P}}$. If $(G,A)\widetilde{\cap}(H,A)=\widetilde{\emptyset}$ , there is no more to be done , so suppose that $P_e^x \widetilde{\in} (G,A)\widetilde{\cap}(H,A)$ . Then $P_e^x \widetilde{\in} (G,A)$ , $P_e^x \widetilde{\in} (H,A)$, and so there is $(F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r})$ , $(W,A)\left(P_e^x,\widetilde{p'}_1,\dots,\widetilde{p'}_m;\tilde{s}\right)$ such that $(F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r}) \widetilde{\subseteq} (G,A)$ and $(W,A)\left(P_e^x,\widetilde{p'}_1,\dots,\widetilde{p'}_m;\tilde{s}\right) \widetilde{\subseteq} (H,A)$.

Put $(M,A)\left(P_e^x,\tilde{p}_1 \dots,\tilde{p}_n,\widetilde{p'}_1,\dots,\widetilde{p'}_m;\tilde{t}\right)$ with $\tilde{t}=\widetilde{min}\{\tilde{r},\tilde{s}\}$

$\Rightarrow$ $(M,A)\left(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n,\widetilde{p'}_1,\dots,\widetilde{p'}_m;\tilde{t}\right) \widetilde{\subseteq} (F,A)(P_e^x,\tilde{p}_1,\dots,\tilde{p}_n;\tilde{r})\widetilde{\cap}(W,A)\left(P_e^x,\widetilde{p'}_1,\dots,\widetilde{p'}_m;\tilde{s}\right)$ $\widetilde{\subseteq} (G,A)\widetilde{\cap}(H,A)$ . It follows that $\tilde{\tau}_{\mathcal{P}}$ is soft topology on $SL(\tilde{X})$.

## Remark (3.16):

A soft topology $\tilde{\tau}_{\mathcal{P}}$ on a $SLS$ $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$ is said to be a soft topology induced by the family of soft seminorms $\mathcal{P}=\{\tilde{p}_\alpha : \alpha \in I\}$.

## Theorem (3.17):

For a $SLS$ $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$ . A soft seminorm induces a soft pseudo metric on $SL(\tilde{X})$ , as following $\tilde{d}_p(P_e^x,P_e^y)=\tilde{p}(P_e^x \tilde{-} P_e^y)$, $\forall$ $P_e^x,P_e^y \widetilde{\in} SL(\tilde{X})$.

**Proof:** Clear.

## Theorem (3.18):

The soft topology $\tilde{\tau}_{\mathcal{P}}$ on a $SLS$ $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$ is $SLT$ determined by the family $\mathcal{P}=\{\tilde{p}_\alpha : \alpha \in I\}$ of a soft seminorms.

**Proof:** Sufficient to show that a soft vector soft addition and soft scalar multiplication are soft continuous with respect to this soft topology. Suppose $(P_{e_d}^{x_d},P_{e'_d}^{y_d}) \widetilde{\rightrightarrows} (P_e^x,P_{e'}^y)$ in $SL(\tilde{X}) \widetilde{\times} SL(\tilde{X})$. We want to show that $P_{e_d}^{x_d} \widetilde{\mp} P_{e_d}^{y_d} \widetilde{\rightrightarrows} P_e^x \widetilde{\mp} P_e^y$ .

Let $(F,A)\left(P_e^x \widetilde{\mp} P_{e'}^y,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\tilde{r}\right)$ be a soft basic soft nbhd of $P_e^x \widetilde{\mp} P_e^y$ . Since $P_{e_d}^{x_d} \widetilde{\mp} P_{e_d}^{y_d} \widetilde{\rightrightarrows} P_e^x \widetilde{\mp} P_e^y$ , then there is $d \geq d_0$ such that $(P_{e_d}^{x_d},P_{e_d}^{y_d}) \widetilde{\in} (F,A)\left(P_e^x,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\frac{\tilde{r}}{\tilde{2}}\right) \widetilde{\times} (F,A)\left(P_e^y,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\frac{\tilde{r}}{\tilde{2}}\right)$ whenever $d \geq d_0$ . For any $i=1,2,\dots,n$ and $d \geq d_0$ , implies

$\tilde{p}_i(P_e^x \widetilde{\mp} P_e^y \tilde{-} (P_{e_d}^{x_d} \widetilde{\mp} P_{e_d}^{y_d})) \widetilde{\leq} \tilde{p}_i(P_e^x \tilde{-} P_{e_d}^{x_d}) \widetilde{\mp} \tilde{p}_i(P_e^y \tilde{-} P_{e_d}^{y_d}) \widetilde{<} \frac{\tilde{r}}{\tilde{2}} \widetilde{\mp} \frac{\tilde{r}}{\tilde{2}}$ ;

so $P_{e_d}^{x_d} \widetilde{\mp} P_{e_d}^{y_d} \widetilde{\in} (F,A)\left(P_e^x \widetilde{\mp} P_{e'}^y,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\tilde{r}\right)$.

Thus we get the required.

Now, suppose that $(\tilde{t}_d,P_{e_d}^{x_d}) \widetilde{\rightrightarrows} (\tilde{t},P_e^x)$ in $\widetilde{\mathbb{K}} \widetilde{\times} SL(\tilde{X})$ .

Let $(F,A)(\tilde{t} \,\tilde{\cdot}\, P_e^x,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\tilde{r})$ be a soft basic soft nbhd of $\tilde{t} \,\tilde{\cdot}\, P_e^x$ . For any $\tilde{\epsilon} \widetilde{>} \tilde{0}$ and $\tilde{\delta} \widetilde{>} \tilde{0}$ , there is $d \geq d_0$ such that $\left(\tilde{t}_d,P_{e_d}^{x_d}\right) \widetilde{\in} \{\tilde{\gamma} \widetilde{\in} \widetilde{\mathbb{K}} : |\tilde{\gamma} \tilde{-} \tilde{t}| \widetilde{<} \tilde{\epsilon}\} \widetilde{\times} (F,A)\left(P_e^x,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\tilde{\delta}\right)$ . Hence for all $i=1,2,\dots,n$ and $d \geq d_0$ , we have

$\tilde{p}_i\left(\tilde{t} \,\tilde{\cdot}\, P_e^x \tilde{-} \tilde{t}_d \,\tilde{\cdot}\, P_{e_d}^{x_d}\right) \widetilde{\leq} \tilde{p}_i(\tilde{t} \,\tilde{\cdot}\, P_e^x \tilde{-} \tilde{t}_d . P_e^x) \widetilde{\mp} \tilde{p}_i\left(\tilde{t}_d \,\tilde{\cdot}\, P_e^x \tilde{-} \tilde{t}_d \,\tilde{\cdot}\, P_{e_d}^{x_d}\right)$ $\widetilde{\leq} |\tilde{t} - \tilde{t}_d| \,\tilde{\cdot}\, \tilde{p}_i(P_e^x) \widetilde{\mp} |\tilde{t}_d| \,\tilde{\cdot}\, \tilde{p}_i\left(P_e^x \tilde{-} P_{e_d}^{x_d}\right) \widetilde{<} \tilde{\epsilon} \,\tilde{\cdot}\, \tilde{p}_i(P_e^x) + (|\tilde{t}| \widetilde{\mp} \tilde{\varepsilon})\tilde{\delta} \widetilde{<} \tilde{r}$ ; if we choose $\tilde{\epsilon} \widetilde{>} \tilde{0}$ such that $\tilde{\epsilon} \,\tilde{\cdot}\, \tilde{p}_i(P_e^x) \widetilde{<} \frac{\tilde{r}}{\tilde{2}}$ and then $\tilde{\delta} \widetilde{>} \tilde{0}$ such that $(|\tilde{t}| \widetilde{\mp} \tilde{\varepsilon}) \,\tilde{\cdot}\, \tilde{\delta} < \frac{\tilde{r}}{\tilde{2}}$ .

Thus $\tilde{t}_d \,\tilde{\cdot}\, P_{e_d}^{x_d} \widetilde{\in} (F,A)(\tilde{t} \,\tilde{\cdot}\, P_e^x,\tilde{p}_1,\tilde{p}_2,\dots,\tilde{p}_n;\tilde{r})$, whenever $d \geq d_0$. We have $\tilde{t}_d \,\tilde{\cdot}\, P_{e_d}^{x_d} \widetilde{\rightrightarrows} \tilde{t} \,\tilde{\cdot}\, P_e^x$.

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## Theorem (3.19):

Let $SL(\tilde{X})$ be a $SLS$ over $\widetilde{\mathbb{K}}$ and $\mathcal{P}=\{\tilde{p}_\alpha : \alpha \in I\}$ be a family of soft seminorms on $SL(\tilde{X})$. Then $\tilde{\tau}_{\mathcal{P}}$ is the coarsest soft topology on $SL(\tilde{X})$ with each $\tilde{p}_\alpha$ is soft continuous.

**Proof:** To show that each $\tilde{p}_\alpha$ is a soft continuous , let $\tilde{\delta} \widetilde{>} \tilde{0}$ and suppose that $(P_{e_d}^{x_d})_{d \in D}$ be a soft net in $SL(\tilde{X})$ with $P_{e_d}^{x_d} \widetilde{\rightarrow} P_e^x$. Then there $d_0$ such that $P_{e_d}^{x_d} \in (F,A)(P_e^x,\tilde{p}_\alpha;\tilde{\delta})$ whenever $d \geq d_0$ . Hence by remark (3.7.ii) , we have $\left|\tilde{p}_\alpha(P_e^x) - \tilde{p}_\alpha(P_{e_d}^{x_d})\right| \widetilde{\leq} \tilde{p}_\alpha(P_e^x \tilde{-} P_{e_d}^{x_d}) \widetilde{\leq} \tilde{\delta}$ , it follows that $\tilde{p}_\alpha$ is soft continuous.

## Theorem (3.20):

Let $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ be a family of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$. For $P_e^x \tilde{\in} SL(\tilde{X})$ a soft sets $(F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r})$, constitute of a soft local base at $P_e^x$ for soft topology $\tilde{\tau}_\mathcal{P}$ compatible with the soft linear space $SL(\tilde{X})$.

**Proof:** Put $\mathcal{B} = \{(H,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r}) : \tilde{r} \tilde{>} \tilde{0}\}$ and $(F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r}) \tilde{\in} \mathcal{N}_{P_e^x}$ such that $P_e^z \tilde{\in} (F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r})$.
Let $\tilde{\delta} \tilde{>} \tilde{0}, \tilde{p}_i(P_e^x \tilde{-} P_e^z) \tilde{<} \tilde{r}, \forall\ i = 1, 2, \dots, n$, such that $\tilde{\delta} \tilde{<} \tilde{r} \tilde{-} \tilde{p}_i(P_e^z \tilde{-} P_e^x)$. Suppose that : $(H,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{\delta}) \in \mathcal{B}$. For any $P_e^y \tilde{\in} (H,A)(P_e^z, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{\delta})$ with $\tilde{p}_i(P_e^y \tilde{-} P_e^z) \tilde{<} \tilde{\delta}$, by definition (3.2.iii) , we have
$\tilde{p}_i(P_e^y \tilde{-} P_e^x) \tilde{\leq} \tilde{p}_i(P_e^y \tilde{-} P_e^z) \tilde{+} \tilde{p}_i(P_e^z \tilde{-} P_e^x) \tilde{<} \tilde{\delta} \tilde{+} \tilde{p}_i(P_e^z \tilde{-} P_e^x) \tilde{<} \tilde{r}$. $P_e^y \tilde{\in} (F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r})$. i.e. $(H,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{\delta}) \tilde{\subseteq} (F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_n; \tilde{r})$.

## Definition (3.21):

A family $\mathcal{P} = \{\tilde{p}_i : i \in I\}$ of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$ is called soft separated, if for each $P_e^0 \neq P_e^x \tilde{\in} SL(\tilde{X})$, there is $i \in I$ such that $\tilde{p}_i(P_e^x) \neq \tilde{0}$.

## Theorem (3.22):

Let $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ be a family of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$. A $STLS$ $(X, \tilde{\tau}_\mathcal{P}, A)$ is soft Hausdorff if and only if $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ is soft separating family.

**Proof:** Suppose that $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ be a soft separating family , let $P_e^x, P_e^y \tilde{\in} SL(\tilde{X})$ with $P_e^x \neq P_e^y$. Then there is some $\tilde{p}_\delta \in \mathcal{P}$ such that $\tilde{p}_\alpha(P_e^x \tilde{-} P_e^y) = \tilde{r} \tilde{>} \tilde{0}$.
The soft sets $(F,A)(P_e^x, \tilde{p}_\alpha; \frac{\tilde{r}}{\tilde{2}})$ and $(F,A)(P_e^y, \tilde{p}_\alpha; \frac{\tilde{r}}{\tilde{2}})$ are soft disjoint nbhds of $P_e^x, P_e^y$ respectively, so that $SL(\tilde{X})$ is soft Hausdorff.
Conversely , if $SL(\tilde{X})$ be a soft Hausdorff , then by definition (2.13.iv) and theorem (2.14.iii) , we have the soft set $\{P_e^x\}$ is soft closed for $P_e^x \tilde{\in} SL(\tilde{X})$ .
For $P_e^0 \neq P_e^x \tilde{\in} SL(\tilde{X})$ , then there is a soft nbhd $(H,A)(P_e^0, \tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n; \tilde{r})$ of $P_e^0$ is not containing $P_e^x$. In particular, there is $\tilde{p}_i \in \mathcal{P}, i = 1, 2, \dots, n$ and $\tilde{r} \tilde{>} \tilde{0}$ with $P_e^x \tilde{\notin} (H,A)(P_e^0, \tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n; \tilde{r})$.

It follows that $\tilde{p}_i(P_e^x \tilde{-} P_e^0) = \tilde{p}_i(P_e^x) \tilde{\geq} \tilde{r}$ for some $i \in I$ and so certainly $\tilde{p}_i(P_e^x) \tilde{>} \tilde{0}$ and we see that $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ is soft separating family of seminorms on $SL(\tilde{X})$.

## Theorem (3.23):

Let $\mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$ be a soft separating family of soft seminorms on a $SLS$ $SL(\tilde{X})$ over $\tilde{\mathbb{K}}$. Associated to each $\tilde{p}_\alpha \in \mathcal{P}$ and each $\tilde{n} \tilde{\in} \tilde{\mathbb{N}}$ the soft set $(F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_m; \tilde{n}) = \{P_e^x : \tilde{p}_i(P_e^x) \tilde{<} \frac{\tilde{1}}{\tilde{n}}\}$.
Let $\mathcal{B}$ be the collection of all finite soft intersection of a soft sets of form $(F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_m; \tilde{n})$.
Then $\mathcal{B}$ is soft local base at $P_e^0$ , each members are soft convex , soft balanced.

**Proof:** For every soft nbhd $(H,A)$ of $P_e^0$ can be expressed as the form
$(H,A) = \tilde{\bigcap}_{i=1}^{m} (F,A)(P_e^x, \tilde{p}_1, \dots, \tilde{p}_m; \tilde{n}_i)$.
Then by theorem (3.20) , $\mathcal{B}$ is soft local base at $P_e^0$. Also by remark (3.14.ii) , each members of $\mathcal{B}$ is soft convex , soft balanced.

## Theorem (3.24):

A soft net $(P_{e_d}^{x_d})_{d \in D}$ in a $SLS$ $SL(\tilde{X})$ with the soft topology $\tilde{\tau}_\mathcal{P}$ is soft converging to $P_e^0$ in $SL(\tilde{X})$
if and only if $\tilde{p}(P_{e_d}^{x_d}) \tilde{\rightarrow} P_e^0$ for each $\tilde{p}_\alpha \in \mathcal{P} = \{\tilde{p}_\alpha : \alpha \in I\}$.

**Proof:** Assume that $P_{e_d}^{x_d} \tilde{\rightarrow} P_e^0$ in $SL(\tilde{X})$ by soft continuity of $\tilde{p}_\alpha \in \mathcal{P}$, then $\tilde{p}_\alpha(P_{e_d}^{x_d}) \tilde{\rightarrow} \tilde{p}_\alpha(P_e^0) = P_e^0$. Conversely, suppose the condition is holds, let $\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n \in \mathcal{P}$ and $\tilde{r} \tilde{>} \tilde{0}$. Then there is $d_0$ such that $\tilde{p}_i(P_{e_d}^{x_d}) \tilde{<} \tilde{r}$, whenever $d \geq d_0$ , for $i = 1, 2, \dots, n$ . Hence $P_{e_d}^{x_d} \tilde{\in} (H,A)(P_e^0, \tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n; \tilde{r})$ , whenever $d \geq d_0$ , for every $(H,A)(P_e^0, \tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n; \tilde{r})$ be a soft nbhd of $P_e^0$ . It follows the result is obtain.

## Remark (3.25):

The soft converge of a soft net $(P_{e_d}^{x_d})_{d \in D}$ to $P_e^x$ is not necessarily implied by the soft convergent of $\tilde{p}((-1)^n P_e^x) \tilde{\rightarrow} \tilde{p}(P_e^x)$ for each $\tilde{p} \in \mathcal{P}$.
Indeed, for any $P_e^x \neq P_e^0$ and any $\tilde{p} \in \mathcal{P}$, as $n \to \infty$ , but is not true that $((-1)^n P_e^x) \tilde{\rightarrow} P_e^x$ , if $(X, \tilde{\tau}_\mathcal{P}, A)$ is soft separated.

## 4. The relationship between soft seminorms and soft Minkwski's functional

### Definition (4.1):

Let $(X,\tilde{\tau},A)$ be a $STLS$ over $\tilde{\mathbb{K}}$ and $(F,A)$ be a soft set over $X$, then $(F,A)$ is said to be a soft bounded set ,if for every soft nbhd $(G,A)$ of $P_e^0$ there is $\tilde{\gamma} \tilde{>} \tilde{0}$ such that $(F,A) \tilde{\subseteq} \tilde{\gamma}(G,A)$.

### Example (4.2):

**i.** For every soft set of $SL(\tilde{\mathbb{R}})$ with the soft usual topology $\tilde{\tau}_U$ is soft bounded.

**ii.** Every finite soft set of a $STLS$ $(X,\tilde{\tau},A)$ is soft bounded.

### Proposition (4.3):

Let $(X,\tilde{\tau},A)$ be a $STLS$ over $\tilde{\mathbb{K}}$ and let $(F,A)$, $(G,A)$ be two soft sets over $X$, then:

**i.** If $(F,A)$ be a soft bounded and $(G,A) \tilde{\subseteq} (F,A)$ , then $(G,A)$ is soft bounded.

**ii.** If $(F,A)$ be a soft bounded , then $\tilde{\alpha}(F,A)$ is soft bounded for all $\tilde{\alpha} \tilde{\in} \tilde{\mathbb{K}}$.

**iii.** If $(F,A)$ be a soft bounded, then $\overline{(F,A)}$ is soft bounded.

**iv.** If $(F,A)$ and $(G,A)$ be two soft bounded , then $(F,A)\tilde{\cap}(G,A)$ , $(F,A)\tilde{\cup}(G,A)$ and $(F,A)\tilde{+}(G,A)$are soft bounded.

**Proof:**

i. Since $(F,A)$ be a soft bounded , there is $\tilde{\gamma} \tilde{>} \tilde{0}$ such that $(F,A) \tilde{\subseteq} \tilde{\gamma}(H,A)$ for every soft nbhd $(H,A)$ of $P_e^0$. Since $(G,A) \tilde{\subseteq} (F,A)$, we have $(G,A) \tilde{\subseteq} \tilde{\gamma}(H,A)$. We easily get that $(G,A)$ is soft bounded.

ii. If $\tilde{\alpha} = \tilde{0}$, this follows immediately from the observation that every finite soft set is soft bounded.

If $\tilde{\alpha} \neq \tilde{0}$ , let $(H,A)$ be a soft nbhd of $P_e^0$. By theorem (2.25.i) , there is a soft balanced nbhd $(W,A)$ of $P_e^0$ such that $(W,A) \tilde{\subseteq} (H,A)$. Since $(F,A)$ be a soft bounded, by (i) above sthere is $\tilde{\gamma} \tilde{>} \tilde{0}$ such that $(F,A) \tilde{\subseteq} \tilde{\gamma}(W,A)$.Put $\tilde{r} = \tilde{\gamma}|\tilde{\alpha}| \Longrightarrow \tilde{r} \tilde{>} \tilde{0}$, since $(W,A)$ soft balanced and $\tilde{\alpha} \tilde{\leq} |\tilde{\alpha}| \Longrightarrow \tilde{\alpha}(W,A) \tilde{\subseteq} |\tilde{\alpha}|(W,A) \Longrightarrow \tilde{\gamma}\tilde{\alpha}(W,A) \tilde{\subseteq} \tilde{\gamma}|\tilde{\alpha}|(W,A)$. Since $(F,A) \tilde{\subseteq} \tilde{\gamma}(W,A) \Longrightarrow$ $\tilde{\alpha}(F,A) \tilde{\subseteq} \tilde{\gamma}\tilde{\alpha}(W,A) \tilde{\subseteq} \tilde{\gamma}|\tilde{\alpha}|(W,A) = \tilde{r}(W,A)$. i.e. $\tilde{r}(W,A) \tilde{\subseteq} \tilde{r}(H,A) \Longrightarrow \tilde{\alpha}(F,A) \tilde{\subseteq} \tilde{r}(H,A)$, proving $\tilde{\alpha}(F,A)$ is soft bounded.

iii. Let $(H,A)$ be a soft nbhd of $P_e^0$ in $SL(\tilde{X})$ , there is a soft nbhd $(W,A)$ of $P_e^0$ , by definition (2.24.iv), implies that $\overline{(W,A)} \tilde{\subseteq} (H,A)$. Since$(F,A)$ be a soft bounded, there is $\tilde{\gamma} \tilde{>} \tilde{0}$ with $(F,A) \tilde{\subseteq} \tilde{\gamma}(W,A)$.

By definition (2.24.v), we have $\overline{(F,A)} \tilde{\subseteq} \overline{\tilde{\gamma}(W,A)} = \tilde{\gamma}\overline{(W,A)} \tilde{\subseteq} \tilde{\gamma}(H,A)$. We end up we have reached that $\overline{(F,A)}$ is soft bounded.

iv. $(F,A)\tilde{\cap}(G,A)$ produced directly from (i). let $(H,A)$ be a soft nbhd of $P_e^0$, then by theorem (2.25.i) there is a soft balanced $(W,A)$ which is a soft nbhd of $P_e^0$ such that $(W,A) \tilde{\subseteq} (H,A)$.

Since $(F,A)$,$(G,A)$ are soft bounded , then there are $\tilde{\gamma}_1,\tilde{\gamma}_2 \tilde{>} \tilde{0}$ such that $(F,A) \tilde{\subseteq} \tilde{\gamma}_1(W,A)$ and $(G,A) \tilde{\subseteq} \tilde{\gamma}_2(W,A)$. Take $\tilde{\gamma} \tilde{>} \widetilde{max}\{\tilde{\gamma}_1,\tilde{\gamma}_2\}$. Since $(W,A)$ be a soft balanced , by theorem (2.21.ii) , then $(F,A)\tilde{\cup}(G,A) \tilde{\subseteq} \tilde{\gamma}(W,A) \tilde{\subseteq} \tilde{\gamma}(H,A)$ , therefore $(F,A)\tilde{\cup}(G,A)$ is soft bounded.

Now , let $(H,A)$ be a soft nbhd of $P_e^0$, then by theorem (2.25.iv) there is a soft symmetric which is soft nbhd $(W,A)$ of $P_e^0$ with $(W,A)\tilde{+}(W,A) \tilde{\subseteq} (H,A)$. By theorem (2.25.i) , there a soft balanced nbhd $(U,A)$ of $P_e^0$ such that $(U,A) \tilde{\subseteq} (W,A)$. Since $(F,A)$,$(G,A)$ are soft bounded , there are $\tilde{\gamma}_1,\tilde{\gamma}_2 \tilde{>} \tilde{0}$ such that $(F,A) \tilde{\subseteq} \tilde{\gamma}_1(U,A)$ and $(G,A) \tilde{\subseteq} \tilde{\gamma}_2(U,A)$. Take $\tilde{\gamma} \tilde{>} \widetilde{max}\{\tilde{\gamma}_1,\tilde{\gamma}_2\}$. Since $(U,A)$ be a soft balanced , then $\tilde{\gamma}_1(U,A)\tilde{+}\tilde{\gamma}_2(U,A) \tilde{\subseteq} \tilde{\gamma}[(U,A)\tilde{+}(U,A)] \tilde{\subseteq} \tilde{\gamma}[(W,A)\tilde{+}(W,A)] \tilde{\subseteq} \tilde{\gamma}(H,A)$ , thus $(F,A)\tilde{+}(G,A)$ is soft bounded.

### Theorem (4.4):

Let $SL(\tilde{X})$ be a $STLS$ over $\tilde{\mathbb{K}}$ . For any $P_e^0 \neq P_e^x \tilde{\in} SL(\tilde{X})$, then a soft set $(F,A) = \{\tilde{n}P_e^x : \tilde{n} \tilde{\in} \tilde{\mathbb{N}}\}$ is not soft bounded.

**Proof:** Since $P_e^x \neq P_e^0$, there is a soft nbhd $(G,A)$ of $P_e^0$ such that $P_e^x \tilde{\notin} (G,A)$. Hence $\tilde{n}P_e^x \tilde{\notin} \tilde{n}(G,A)$, for all $\tilde{n} \tilde{\in} \tilde{\mathbb{N}}$ i.e. $(F,A) \tilde{\nsubseteq} \tilde{n}(G,A)$.

### Definition (4.5):

For a $SLS$ $SL(\tilde{X})$ and $SL(\tilde{Y})$ over $\tilde{\mathbb{K}}$ . A soft linear mapping $\tilde{\Lambda}: SL(\tilde{X}) \tilde{\rightarrow} SL(\tilde{Y})$ is said to be a soft bounded, if $\tilde{\Lambda}((F,A))$ is soft bounded set in $SL(\tilde{Y})$ for every soft bounded set $(F,A)$ in $SL(\tilde{X})$ .

**Example (4.6):**

**i.** For soft identity linear mapping $\widetilde{\Lambda}: SL(\tilde{X}) \rightrightarrows SL(\tilde{X})$ is soft bounded.

**ii.** A soft linear mapping $\widetilde{\Lambda}: SL(\widetilde{\mathbb{R}}) \rightrightarrows SL(\widetilde{\mathbb{R}})$, defined by $\widetilde{\Lambda}(\tilde{r}) = \tilde{2}\tilde{r}$, for every $\tilde{r} \tilde{\in} SL(\widetilde{\mathbb{R}})$ is soft bounded.

**iii.** A soft linear mapping $\tilde{\Lambda}: SL(\tilde{X}) \rightrightarrows SL(\tilde{Y})$, defined by $\widetilde{\Lambda}(P_e^x) = P_e^0$, for every $P_e^x \tilde{\in} SL(\tilde{X})$ is soft bounded.

**Definition (4.7)[15]:**

For any soft convex , soft absorbing set $(H, A)$ in a $SLS$ $SL(\tilde{X})$ over $\widetilde{\mathbb{K}}$. A soft convex functional mapping $\tilde{\Lambda}_{(H,A)}: SL(\tilde{X}) \rightrightarrows \mathbb{R}(A)$ defined by $\tilde{\Lambda}_{(H,A)}(P_e^x) = \widetilde{\inf}\{\tilde{\eta} \tilde{>} \tilde{0}: P_e^x \tilde{\in} \tilde{\eta}(H, A)\}$ called a soft Minkowski's functional associated with $(H, A)$.

Now, we wil put condition to requirement to the relationship between soft seminorms and soft Minkowski's functionals.

**Theorem (4.7)[15]:**

Let $SL(\tilde{X})$ be a $SLS$ over $\widetilde{\mathbb{K}}$ and $\tilde{p}: SL(\tilde{X}) \rightrightarrows \mathbb{R}(A)$ be a soft seminorm, then:

**i.** If $(H, A)$ be a soft convex , soft absorbing and any $\tilde{r} \tilde{\geq} \tilde{0}$, $\tilde{\Lambda}_{(H,A)}(\tilde{r}^{-1} P_e^x) = \tilde{r}^{-1}\tilde{\Lambda}_{(H,A)}(P_e^x)$.

**ii.** If $(H, A) = \{P_e^x \tilde{\in} SL(\tilde{X}): \tilde{p}(P_e^x) \tilde{<} \tilde{1}\}$ , then $\tilde{\Lambda}_{(H,A)}(P_e^x) = \tilde{p}(P_e^x)$.

**Theorem (4.8)[15]:**

Let $SL(\tilde{X})$ be a $SLS$ over $\widetilde{\mathbb{K}}$ and $\tilde{\Lambda}_{(F,A)}$ be a soft Minkowski's functional associated with a soft convex , soft absorbing set $(F, A)$. $\forall\, \tilde{r} \tilde{\geq} \tilde{0}$ , $(W, A) = \tilde{r}(G, A)$
, $(W, A) = \{P_e^x: \tilde{\Lambda}_{(F,A)}(P_e^x) \tilde{<} \tilde{r}\}$ and $(G, A) = \{P_e^x: \tilde{\Lambda}_{(F,A)}(P_e^x) \tilde{<} \tilde{1}\}$.

**Remark (4.9):**

**i.** A soft seminors is a special case of a soft Minkowski's functional.

**ii.** The soft Minkowski's functional associated to a soft balanced , soft convex and soft absorbing set, in a $SLS$ is a soft seminorm.

**Example (4.10):**

Let $SL(\tilde{X})$ be a $SLS$ over $\widetilde{\mathbb{K}}$ and $\tilde{\rho}: SL(X) \rightrightarrows \widetilde{\mathbb{R}}$ be a soft linear functional. Fix $\tilde{s} \tilde{>} \tilde{0}$ , let $(H, A)$ the soft set given by $(H, A) = \{P_e^x \tilde{\in} SL(\tilde{X}): |\tilde{\rho}(P_e^x)| \tilde{\leq} \tilde{s}\}$.
Define $\tilde{\Lambda}_{(H,A)}: SL(\tilde{X}) \rightrightarrows \mathbb{R}(A)$
by $\tilde{\Lambda}_{(H,A)}(P_e^x) = \widetilde{\inf}\{\tilde{r} \tilde{>} \tilde{0}: P_e^x \tilde{\in} (H, A)\}$.
Then $\tilde{\Lambda}_{(H,A)}$ is another instance of a soft Minkowski's functional. It is clear that the proof is produced directly apply of theorem (4.7.ii).

**References:**

**[1]** Molodtsov D. , " Soft set theory – First results" , Comp. and Math. Appl. , 1999.
**[2]** Sujoy D.,"Soft real sets, soft real numbers and their properties", J.fuzzy Math.Vol. 20, No.3, 2012.
**[3]** Shabir M. and Naz M. " On soft topological spaces" , Comp. Math. Appl. , 2011.
**[4]** Bekir T. and Yaylayi G. , " New structures on partially ordered soft sets and soft Scott topology " , Ann. of fuzzy Math. and Inf. , 2013.
**[5]** Nazmul S." Neighborhood properties of soft topological spaces ", Ann fuzzy Math. Inf. , Vol. 6, N0.1, 2013.
**[6]** Bedre O. and Demir I. "Soft Hausdorff spaces and their some properties" , Ann. of fuzzy Math. and Inf. , 2014.
**[7]** Peyghan E. and others "Some results related to soft topological spaces", Iran, Qom Univ., 2014.
**[8]** Janaki C."A new class of homeomorphisms in soft topological spaces" , Math. J. of sci. , 2012.
**[9]** Reddy B. and Sayyed J. "On soft open and closed mapping ", Ann. of fuzzy Math. and Inf. , 2015.
**[10]** Sanjay R. and Samanta T. "Balanced and Absorbing Soft Sets", Jor. of new results in Sci., Page (36-46) , 2015.
**[11]** Irfan D. " Convex and Concave soft sets and some properties " , Kilis Turk., 2013.
**[12]** Chiney S. and Smanta S. "Vector soft topology" , Ann. of fuzzy Math. Inf. , 2015.
**[13]** Sujoy D. , Smanta S. ,"Soft linear functional in soft normed linear spaces", Ann. fuzzy Math., 2013.
**[14]** Sujoy D. , Smanta S. , "Soft linear operators in soft normed linear spaces" , Ann. fuzzy Math, 2013.
**[15]** Chiney S. and Smanta S. "Soft topological vector space" , Ann. of fuzzy Math. Inf. , 2016.

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# بعض النتائج حول شبه المعايير الواهنة

نوري فرحان المياحي خلود محمد حسن

جامعة القادسية
كلية علوم الحاسوب وتكنولوجيا المعلومات
قسم الرياضيات

**المستخلص :**
أن الهدف الرئيس من هذا العمل هو تقديم نوع جديد من الفضاءات التبولوجية الناعمة المتولدة بواسطة عائلة من شبه المعايير الواهنة . وكذلك لقد بينا العلاقة ما بين المعايير شبه الواهنة ودالية منكوفسي الواهنة على الفضاءات الخطية الواهنة.

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