On Triple g-Transformation and Its Properties

Rabab Jasim Hadi AL-Owaidia, Metaq Hamza Geemb

aAl-Qadisiyah university, Al-Diwaniyah, 58001, Iraq, Email: Rabab594@qu.edu.iq
bAl-Qadisiyah university, Al-Diwaniyah, 58001, Iraq, Email: methaq.geem@qu.edu.iq

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ABSTRACT

In this paper, we introduce a new transformation, namely, triple g-transformation and denote it as \( T_{3g} \). This transformation considers generalization to some types of triple transformations. We defined it as the following form:

\[
T_{3g}(f(x,y,z)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(s)x+q_2(s)y+q_3(s)z} f(x,y,z) \, dx \, dy \, dz
\]

MSC. 44-XX

1. Introduction:

The integral transformations are one of the important methods for solving many of problems. Such that we can convert ordinary differential equation to algebraic equation and then we back by inverse of this integral transformation, but the case partial differential equations we use the integral transformation to convert partial differential equations to ordinary differential equations [5]. By using double integral transformation we can convert partial differential equations of two independent variables to algebraic equations, thus we need to triple transformation to solve partial differential equations with three independent variables. The triple integral transformation is has important applications [1,2]. This transformation is distinguished by the generalities of the most known integral transformations and the possibility of finding new integral transformations from it.

In [4], H.Jaferi presented general integral transformation and he called it g-transformation also he studied properties of g-transformation and its applications in differential equations. By taking g-transformation with one parameter \( s \), \( T_{3g} \)-transformation is constructed. In this paper, theorems and examples related with \( T_{3g} \)-transformation are presented.
2. Triple g-Transformation:

2.1 Definition [4]:

g-transformation $g(f(x))$ for a piecewise function $f(x)$ where $x \in [0, \infty]$ and $|f(x)| \leq M^{ks}$ is defined by the following integral

$$g(f(x)) = p(s) \int_{0}^{\infty} e^{-q(s)x} f(x) dx = \mathcal{F}_p(s)$$

such that the integral is convergent for some $q(s)$, $s$ is a positive constant, and

$$\|g(f(x))\| \leq p(s)M^{k-q(s)}$$

2.2 Definition [4]:

Double $T_{2g}$-transformation $T_{2g}(f(x,y))$ for a piecewise function $f(x,y)$ where $x \in [0, \infty]$, $y \in [0, \infty]$ and $|f(x,y)| \leq M^{e^{(x+y)}}$ is defined by the following integral:

$$T_{2g}(f(x,y)) = p_1(s)p_2(s) \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y} f(x,y) dx dy = p(s) \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y} f(x,y) dx dy = \mathcal{F}_{p_1p_2M^{k-q_1q_2}}$$

where $p(s) = p_1(s)p_2(s)$ such that the integral is convergent for some $q_1(s), q_2(s)$ are positive functions, and $\|gD(f(x,y))\| \leq p_1p_2M^{k-q_1q_2}$

2.3 Remark:

Let $f(x,y,z)$ be a function of three variables then we use the following symbols in this paper:

$$\mathcal{F}_{111} = p(s) \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y-q_3(s)z} f(x,y,z) dx dy dz$$

$$\mathcal{F}_{112} = p(s) \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y} f(x,y,z) dx dy$$

$$\mathcal{F}_{122} = p(s) \int_{0}^{\infty} e^{-q_1(s)x-q_3(s)z} f(x,y,z) dx dz$$

2.4 Definition:

Let $f$ be a continuous function of three variables then the triple g-transformation

of $f(x,y,z)$ is defined as following:

$$T_{3g}(f(x,y,z)) = p(s) \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y-q_3(s)z} f(x,y,z) dx dy dz$$

where $x,y,z > 0$ and $s$ is positive constant, and

$$\sup_{x,y,z} |f(x,y,z)| < 0$$

For some $a,b,c \in R$

The inverse of $T_{3g}$-transform is defined as following:

$$f(x,y,z) = \frac{1}{2\pi e} \int_{0}^{\infty} e^{q_1(s)x+q_2(s)y+q_3(s)z} F(s) ds$$

2.5 Example:

1. $T_{3g}(1) = p(s) \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} e^{-q_1(s)x-q_2(s)y-q_3(s)z} (1) dx dy dz$

$$= p(s) \int_{0}^{\infty} \int_{0}^{\infty} \left[ \frac{1}{q_1(s)} \right] e^{-q_2(s)y-q_3(s)z} dy dz$$

$$= \frac{p(s)}{q_1(s)} \int_{0}^{\infty} \int_{0}^{\infty} e^{-q_2(s)y-q_3(s)z} dy dz$$

$$T_{3g}(1) = \frac{p(s)}{q_1(s)q_2(s)q_3(s)}$$

2. $T_{3g}(e^{ax+by+cz}) = p(s) \int_{0}^{\infty} \int_{0}^{\infty} e^{ax+by+cz} e^{q_1(s)x+q_2(s)y+q_3(s)z} dx dy dz$

$$= p(s) \int_{0}^{\infty} \int_{0}^{\infty} e^{-(q_1(s)x+q_2(s)y+q_3(s)z)} dx dy dz$$
3. Proposition:

Where $T$ is a function such that $|f(x,y,z)| \leq K$, and $T_{3sg}(f(x,y,z)) = f(s)$ exists, then

$$T_{3sg} \left( e^{ax+by+cz} \right) = \frac{p(s)}{(q_1-a)(q_2-b)(q_3-c)}$$

$$3 - T_{3sg} \left( \cosh(ax + by + cz) \right) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{ax+by+cz} \cosh(ax + by + cz) e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= \frac{p(s)}{2} \int_0^\infty \int_0^\infty \int_0^\infty \left[ e^{ax+by+cz} - e^{-(ax+by+cz)} \right] e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= \frac{p(s)}{2} \left\{ \left[ (q_1-a)(q_2-b)(q_3-c) + (q_1+a)(q_2+b)(q_3+c) \right] - \left[ (q_1-a)(q_2-b)(q_3-c) + (q_1+a)(q_2+b)(q_3+c) \right] \right\}$$

$$= \frac{p(s)}{2} \int_0^\infty \int_0^\infty \int_0^\infty \left[ 2 q_1 q_2 q_3 + 2bhq_1 + 2ahq_2 + 2abq_3 \right] e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$T_{3sg} \left( \cosh(ax + by + cz) \right) = \frac{p(s)(q_1 q_2 q_3 + bc q_1 + ac q_2 + ab q_3)}{(q_1-a)(q_2-b)(q_3-c)}$$

$$4 - T_{3sg} \left( \sinh(ax + by + cz) \right) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{ax+by+cz} \sinh(ax + by + cz) e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= \frac{p(s)}{2} \int_0^\infty \int_0^\infty \int_0^\infty \left[ e^{ax+by+cz} + e^{-(ax+by+cz)} \right] e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= \frac{p(s)}{2} \left\{ \left[ (q_1-a)(q_2-b)(q_3-c) + (q_1+a)(q_2+b)(q_3+c) \right] - \left[ (q_1-a)(q_2-b)(q_3-c) + (q_1+a)(q_2+b)(q_3+c) \right] \right\}$$

$$= \frac{p(s)}{2} \int_0^\infty \int_0^\infty \int_0^\infty \left[ 2 q_1 q_2 q_3 + 2bhq_1 + 2ahq_2 + 2abq_3 \right] e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$T_{3sg} \left( \sinh(ax + by + cz) \right) = \frac{p(s)(q_1 q_2 q_3 + bc q_1 + ac q_2 + ab q_3)}{(q_1-a)(q_2-b)(q_3-c)}$$

2.6 Proposition:

$T_{3sg}$ transformation satisfies the linear property, i.e.

$$T_{3sg}(a f(x,y,z) + b h(x,y,z)) = a T_{3sg}(f(x,y,z)) + b T_{3sg}(h(x,y,z))$$

Where $a, b \in R$

Proof:

$$T_{3sg}(a f(x,y,z) + b h(x,y,z)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty [a f(x,y,z) + b h(x,y,z)] e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= a p(s) \int_0^\infty \int_0^\infty \int_0^\infty f(x,y,z) e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz + b p(s) \int_0^\infty \int_0^\infty \int_0^\infty h(x,y,z) e^{-q_1x-q_2y-q_3z} \, dx \, dy \, dz$$

$$= a T_{3sg}(f(x,y,z)) + b T_{3sg}(h(x,y,z))$$

2.7 Table of $T_{3sg}$ transformation of selected functions

<table>
<thead>
<tr>
<th>ID</th>
<th>$f(x)$</th>
<th>$T_{3sg}(f(x,y,z)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{q_1(x)+q_2(y)+q_3(z)} f(x,y,z) , dx , dy , dz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K, K$ constant</td>
<td>$K q_1 q_2 q_3$</td>
</tr>
<tr>
<td>2</td>
<td>$\sin(ax + by + cz)$</td>
<td>$p(s)(q_1 + a^2 q_1^2 + b^2 q_1^2 + c^2 q_1^2)$</td>
</tr>
<tr>
<td>3</td>
<td>$\cos(ax + by + cz)$</td>
<td>$p(s)(q_1 + a^2 q_1^2 + b^2 q_1^2 + c^2 q_1^2)$</td>
</tr>
<tr>
<td>4</td>
<td>$x^m y^n z^n$</td>
<td>$p(s)(q_1)^{m+n} (q_2)^{m+1} (q_3)$</td>
</tr>
</tbody>
</table>

2.8 Proposition:

Let $f(x,y,z), x,y,z \in [0, \infty]$ be a function such that $|f(x,y,z)| \leq e^{k_1 ax+k_2 by+k_3 cz}$ and $T_{3sg}(f(x,y,z)) = f(s)$ exists, then
\[ T_{3sg}(f(x + a, y + b, z + c)) = e^{\int_{a}^{b} \int_{v}^{w} f(u, v, w) du dv dw} \]

**Proof:**

\[ T_{3sg}(f(x + a, y + b, z + c)) = p(s) \int_{a}^{b} \int_{v}^{w} f(x + a, y + b, z + c) e^{q(x-v)+q(y-w)+q(z-w)} dx dy dz \]

Let \( u = x + a, v = y + b, w = z + c \)

\[ T_{3sg}(f(u, v, w)) = p(s) \int_{u}^{b} \int_{v}^{w} f(u, v, w) e^{-q(u-v)+q(v-w)+q(w-u)} du dv dw \]

**3. The Convolution of \( T_{3sg} \)-transformation**

**3.1 Definition [3,6,7]:**

Let \( f(x, y, z), h(x, y, z) \) be a function defined on \( \mathbb{R}^3 \), then the convolution of the functions \( f(x,y,z) \) and \( h(x,y,z) \) is given as following:

\[ T_{3sg} (f \ast h)(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x-u,y-v,z-w) h(u,v,w) du dv dw \]

**3.2 Theorem:**

Let \( f(x,y,z), h(x,y,z) \) be a function which defined on \( \mathbb{R}^3 \) then \( T_{3sg} \)-transformation of the convolution \( (f \ast h) \) is given as following:

\[ T_{3sg} (f \ast h)(x,y,z) = \frac{1}{p(s)} T_{3sg} (f(x,y,z) \ast h(x,y,z)) \]

**Proof:**

\[ T_{3sg} (f(u,v,w), h(t,r,o)) = \left[ p(s) \int_{a}^{b} \int_{v}^{w} f(u,v,w) e^{-q(u-v)+q(v-w)+q(w-u)} du dv dw \right] \]

**4. The Convolution of \( T_{3sg} \)-transformation**

**4.1 Definition [3,6,7]:**

Let \( f(x,y,z), h(x,y,z) \) be a function defined on \( \mathbb{R}^3 \), then the convolution of the functions \( f(x,y,z) \) and \( h(x,y,z) \) is given as following:

\[ T_{3sg} (f \ast h)(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x-u,y-v,z-w) h(u,v,w) du dv dw \]

**4.2 Theorem:**

Let \( f(x,y,z), h(x,y,z) \) be a function which defined on \( \mathbb{R}^3 \) then \( T_{3sg} \)-transformation of the convolution \( (f \ast h) \) is given as following:

\[ T_{3sg} (f \ast h)(x,y,z) = \frac{1}{p(s)} T_{3sg} (f(x,y,z) \ast h(x,y,z)) \]

**Proof:**

\[ T_{3sg} (f(u,v,w), h(t,r,o)) = \left[ p(s) \int_{a}^{b} \int_{v}^{w} f(u,v,w) e^{-q(u-v)+q(v-w)+q(w-u)} du dv dw \right] \]

\[ = \left[ p(s) \int_{a}^{b} \int_{v}^{w} f(u,v,w) e^{-q(u-v)+q(v-w)+q(w-u)} du dv dw \right] \]
\[x = u + t \rightarrow u = x - t, \ y = v + r \rightarrow v = y - r, \ z = w + o \rightarrow w = z - o\]

\[
\mathcal{p}(s) = \int_0^1 \int_0^1 \int_0^1 e^{-q_1 x - q_4 y - q_{14} z} \mathcal{f}(x - t, y - r, z - o) h(t, r, o) \, dt \, dr \, dz
\]

\[= (p(s))^2 \int_0^1 \int_0^1 \int_0^1 e^{-q_1 x - q_4 y - q_{14} z} [f_{***} h](x, y, z) \, dx \, dy \, dz
\]

\[= \mathcal{P}(s) T_{148}(f_{***} h)(x, y, z)
\]

**3.3 Example:**

Find \(T_{148}^{-1}\left(\frac{u(s^2-1)}{(s^2-a^2+1)}\right)\), \(p(s) = s \), \(q_1(s) = q_2(s) = q_3(s) = s\)

where \(s > 0, \ s \neq 1\)

we note that \(s^2 - a^2 + 1 = (s^2 + 1)(s - 1)\)

\[T_{148}(\frac{u(s^2-1)}{(s^2-a^2+1)}) = \frac{u(s^2-1)}{(s^2+1)/(s-1)}
\]

Also we can write that

\[T_{148}(\frac{u(s^2-1)}{(s^2-a^2+1)}) = \frac{u(s^2-1)}{s} \cdot \frac{s}{s-1}
\]

But

\[T_{148}(\sin(x+y+z)) = \frac{u(s^2-1)}{(s^2+a^2)}
\]

And \(T_{148}(e^{x+y+z}) = \frac{s}{(s-1)^2}\)

Therefore

\[T_{148}(\frac{u(s^2-1)}{(s^2-a^2+1)}) = \sin(x+y+z) e^{x+y+z}
\]

**3.4 Example:**

Find \(T_{148}^{-1}\left(\frac{u(s^2+1)}{(s^2-a^2+1)}\right)\)

Such that \(p(s) = q_1(s) = s^2 \), \(q_2(s) = q_3(s) = s\)

\[T_{148}(\frac{u(s^2+1)}{(s^2-a^2+1)}) = \frac{u(s^2+1)}{s^2} \cdot \frac{s^2}{s^2+1} / \left(s^2-a^2+1\right)
\]

We note that:

\[T_{148}(x^2) = \frac{s^2}{s^2+1}, \ T_{148}(\cosh(y+z)) = \frac{x^2+1}{(s^2+1)}
\]

Therefore

\[T_{148}^{-1}\left(\frac{u(s^2+1)}{(s^2-a^2+1)}\right) = x^2 \cosh(y+z)
\]

**4. Applications of \(T_{148}\) transformation**

**4.1 Proposition:**

1. \(T_{148}(u_1) = q_1 \int_0^1 u_{123}(0,y,z) \)

2. \(T_{148}(u_2) = q_1 \int_0^1 u_{123}(0,y,z) - (u_1)_{123}(0,y,z) \)

3. \(T_{148}(u_{10}) = q_1 \int_0^1 \sum_{i=1}^{n-1} q^{n-i-1} (u_1)_{123}(0,y,z) \)

4. \(T_{148}(u_y) = q_2 u_y \int_0^1 \sum_{i=1}^{n-1} (u_1)_{123}(x,0,z) \)
Proof:

1. \( T_{3\alpha}(u_4) = p(s) \int_0^m \int_0^m e^{-u_4} u_4^m \, du dy dz \)

\[ W = e^{-u_4} \rightarrow dw = -q_4 e^{-u_4} \, dx \]

\[ dv = u_4 \, dx \rightarrow v = u_4 \]

\[ T_{3\alpha}(u_4) = p(s) \int_0^m \int_0^m e^{-u_4} \left[ u_4 e^{-u_4} \right]_0^m + q_4 \int_0^m u_4 e^{-u_4} \, dx dy dz \]

\[ = q_4 p(s) \int_0^m \int_0^m e^{-u_4} \, dx dy dz - p(s) \int_0^m \int_0^m e^{-u_4} u_4(0, y, z) dy dz \]

\[ T_{3\alpha}(u_4) = q_4 \frac{W}{W_{as2}(0, y, z)} \]

2. \( T_{3\alpha}(u_{xx}) = p(s) \int_0^m \int_0^m e^{-u_{xx}} u_{xx}^m \, du dy dz \)

\[ W = e^{-u_{xx}} \rightarrow dw = -q_{xx} e^{-u_{xx}} \, dx \]

\[ dv = u_{xx} \, dx \rightarrow v = u_{xx} \]

\[ T_{3\alpha}(u_{xx}) = p(s) \int_0^m \int_0^m e^{-u_{xx}} \left[ u_{xx} e^{-u_{xx}} \right]_0^m + q_{xx} \int_0^m u_{xx} e^{-u_{xx}} \, dx dy dz \]

\[ = q_{xx} p(s) \int_0^m \int_0^m e^{-u_{xx}} \, dx dy dz - p(s) \int_0^m \int_0^m e^{-u_{xx}} u_{xx}(0, y, z) dy dz \]

\[ T_{3\alpha}(u_{xx}) = q_{xx} \frac{W}{W_{as2}(0, y, z)} \]

3. \( T_{3\alpha}(u_{zi}) = q_{zi} \frac{W}{W_{as2}(0, y, z)} \)

We will prove by using mathematical induction

1) when \( n = 1 \) we get that (3) is true by (1)

2) suppose that (3) is true when \( n = k \)

That is:

\[ T_{3\alpha}(u_{k}) = q_k \frac{W}{W_{as2}(0, y, z)} \]

III) let \( n = k + 1 \) and let \( w = u_{k+1} \)

\[ T_{3\alpha}(u_{k+1}) = T_{3\alpha}(w) = q_{k+1} \frac{W}{W_{as2}(0, y, z)} \]

\[ = q_{k+1} \frac{W}{W_{as2}(0, y, z)} - p \sum_{i=0}^{k+1} q_i \int_0^m u_{k+1} e^{-u_{k+1}} \, dy dz \]

\[ = q_{k+1} \frac{W}{W_{as2}(0, y, z)} - p \sum_{i=0}^{k+1} q_i \int_0^m u_{k+1} e^{-u_{k+1}} \, dy dz \]

There of the fact is true for all \( n \in \mathbb{Z}^+ \)

There of the fact is true for all \( n \in \mathbb{Z}^+ \)

4. \( T_{3\alpha}(u_p) = T_{3\alpha}(u_p) = p(s) \int_0^m \int_0^m e^{-u_p} u_p \, dy dz \)
Therefore (7) is true for all $r \in \mathbb{Z}^+$.

5- We will prove by using mathematical induction

I) when $m=1$, we get that (5) is true by (4).

II) suppose that (5) is true when $m=k$.

That is:

$$T_{34g}(u_k^{(k)}) = q_k \frac{3}{4} \pi y - \sum_{i=0}^{k-1} q_k^{i+1} (u_k^{(i)})_{s13}(x,0,z)$$

III) When $m=k+1$ and let $w = u_k^{(k)}$, $w_x = u_k^{(k+1)}$

$$T_{34g}(u_x^{(k+1)}) = T_{34g}(w_x) = q_k \frac{3}{4} \pi y - w_{s13}(x,0,z)$$

$$= q_k \left[ q_k \frac{3}{4} \pi y - \sum_{i=0}^{k-1} q_k^{i+1} \left( u_k^{(i)} \right)_{s13}(x,0,z) \right] - (u_k^{(k)})_{s13}(x,0,z)$$

Therefore (5) is true for all $m \in \mathbb{Z}^+$.

6- $T_{34g}(u_x) = p(s) \int_0^m \int_0^m e^{-q_x x - q_y y} \left[ \int_0^m e^{-q_z z} u_x \, dz \right] \, dx \, dy$

$$W = e^{-q_x} \rightarrow dw = -q_x e^{-q_y} dy$$

$$dv = u_x dy \rightarrow v = u$$

$$T_{34g}(u_x) = p(s) \int_0^m \int_0^m e^{-q_x x - q_y y} \left[ u e^{-q_z z} + q_x \int_0^m e^{-q_z z} \, dz \right] \, dx \, dy$$

$$= q_x p(s) \int_0^m \int_0^m e^{-q_x x - q_y y} \left\{ u(x,y,0) - (u_x)_{s12}(x,y,0) \right\} dx \, dy$$

$$T_{34g}(u_x) = q_x \left[ q_x \frac{3}{4} \pi y - u_{s12}(x,y,0) \right]$$

7- We will prove by using mathematical induction

I) when $r=1$, we get that (7) is true by

II) suppose that (7) is true when $r=k$.

That is:

$$T_{34g}(u_k^{(k)}) = q_k \frac{3}{4} \pi y - \sum_{i=0}^{k-1} q_k^{i+1} (u_k^{(i)})_{s12}(x,y,0)$$

III) Let $r=k+1$ and let $w = u_k^{(k)}$, $w_x = u_k^{(k+1)}$

$$T_{34g}(u_x^{(k+1)}) = T_{34g}(w_x) = q_k \frac{3}{4} \pi y - w_{s12}(x,y,0)$$

$$= q_k \left[ q_k \frac{3}{4} \pi y - \sum_{i=0}^{k-1} q_k^{i+1} \left( u_k^{(i)} \right)_{s12}(x,y,0) \right] - (u_k^{(k)})_{s12}(x,y,0)$$

Therefore (7) is true for all $r \in \mathbb{Z}^+$. 

8. 

\[ T_{38}(u_{xyz}) = \rho(s) \int_0^\infty \int_0^\infty \int_0^\infty u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx \, dy \, dz \]

\[ = \frac{\rho(s)}{q_1 q_2 q_3} \int_0^\infty \int_0^\infty \int_0^\infty \frac{u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx \, dy \, dz}{u_{xyz}(0, y, z)} \]

\[ = \frac{\rho(p(s))}{q_1 q_2 q_3} \int_0^\infty \int_0^\infty \int_0^\infty u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx \, dy \, dz - \frac{\rho(u_{xyz})}{q_1 q_2 q_3}(0, y, z) \]

\[ = \frac{\rho(q_1 q_2 q_3)}{q_1 q_2 q_3} \int_0^\infty \int_0^\infty \int_0^\infty u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx \, dy \, dz - \frac{\rho(u_{xyz})}{q_1 q_2 q_3}(0, y, z) \]

\[ = \frac{\rho(q_1 q_2 q_3)}{q_1 q_2 q_3} \int_0^\infty \int_0^\infty \int_0^\infty u_{xyz} e^{-q_1 x - q_2 y - q_3 z} dx \, dy \, dz - \frac{\rho(u_{xyz})}{q_1 q_2 q_3}(0, y, z) \]

\[ T_{38}(u_{xyz}) = \rho(q_1 q_2 q_3) - \frac{\rho(u_{xyz})}{q_1 q_2 q_3}(0, y, z) \]

4.2 Example:

Solve the following equation

\[ u_x = 1 \quad u(0, y, z) = 1 \]

Solution:

By taking \( T_{38} \) transformation for the equation we get

\[ q_1 \frac{\partial}{\partial x} u_{xyz}(0, y, z) = \frac{\rho}{q_1 q_2 q_3} \]

We note that:

\[ u_{xyz}(0, y, z) = \rho \int_0^\infty \int_0^\infty u(0, y, z) e^{-q_1 x - q_2 y - q_3 z} dy \, dz = \frac{\rho}{q_1 q_2 q_3} \]

Thus

\[ q_1 \frac{\partial}{\partial x} = \frac{\rho}{q_1 q_2 q_3} \frac{\partial}{\partial x} \]

\[ q_1 \frac{\partial}{\partial x} = \frac{q_1 + 1}{q_1 q_2 q_3} \frac{\partial}{\partial x} \]

We take \( T_{38}^{-1} \) transformation for both sides, we get:

\[ u(x, y, z) = T_{38}^{-1}(\frac{p}{q_1 q_2 q_3} + \frac{p}{q_1 q_2 q_3}) \]

\[ = \frac{p}{q_1 q_2 q_3} + \frac{p}{q_1 q_2 q_3} \]

\[ = e^{x y} y^2 + e^{x y} y^2 = 1 + x \]

4.3 Example:

Solve the following equation

\[ u_{xx} = u_x \quad u_x(0, y, z) = 0 \quad u(0, y, z) = 1 \]

\[ p = q_1 = q_2 = q_3 = s \]

Solution:

By taking \( T_{38} \) transformation for the equation we get

\[ T_{38}(u_{xyz}) = T_{38}(u_x) \]

\[ q_1 \frac{\partial}{\partial x} u_{xyz}(0, y, z) = q_1 \frac{\partial}{\partial x} u_{xyz}(0, y, z) \]

\[ q_1 \frac{\partial}{\partial x} = \frac{p}{q_1 q_2 q_3} \]

\[ q_1 \frac{\partial}{\partial x} = \frac{p}{q_1 q_2 q_3} \]

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\[ q_1 \frac{\partial}{\partial x} = \frac{p}{q_1 q_2 q_3} \]
\[ q_{1} \overline{u} = \frac{p}{q_{2}q_{3}} \]

\[ \overline{u} = \frac{p}{q_{1}q_{2}q_{3}} \]

We take \( T_{3}^{\Delta_{1}} \) transformation for both sides, we get:

\[ u(x,y,z) = T_{3}^{\Delta_{1}} \left( \frac{p}{q_{1}q_{2}q_{3}} \right) = 1 \]

REFERENCES


