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On Triple g-Transformation and Its Properties

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ABSTRACT

In this paper, we introduce a new transformation , namely , triple g-transformation and denote it as T_{3sg}. This transformation consider as generalized to some types of triple transformations. We defined it as the following form :

$$T_{3sg}(f(x,y,z)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{q_1(s)x-q_2(s)y-q_3(s)z} f(x,y,z)\, dxdydz$$

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1. Introduction:

The integral transformations are one of import methods for solving many of problems. Such that we can convert ordinary differential equation to algebraic equation and then we back by inverse of this integral transformation, but the case partial differential equations we use the integral transformation to convert partial differential equations to ordinary differential equations [5]. By using double integral transformation we can convert partial differential equations of two independent variables to algebraic equations , thus we need to triple transformation to solve partial differential equations with three independent variables. the triple integral transformation is has import applications [1,2]. This transformation is distinguished by the generalities of the most known integral transformations and the possibility of finding new integral transformations from it.

In [4] , H.Jaferi presented general integral transformation and he called it g-transformation also he studied properties of g-transformation and its applications in differential equations. By taking g-transformation with one parameter s , T_{3sg}-transformation is constructed. In this paper , theorems and examples related with T_{3sg}-transformation are presented.

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2.Triple g-Transformation:

2.1 Definition [4]:

g-transformation $g(f(x))$ for a piecewise function f(x) where $x \in [\ 0, \infty\ [$ and $|f(x)| \leq Me^{kx}$ is defined by the following integral

$$g(f(x)) = p(s)\int_0^\infty e^{-q(s)x}\ f(x)dx \quad = \overline{f}, p(s) \neq 0 \qquad (1)$$

such that the integral is convergent for some q(s), s is positive constant, and

$$\|g(f(x))\| \leq \frac{p(s)M}{k - q(s)}$$

2.2 Definition [4]:

Double T_{2sg}-transformation T_{2sg} (f(x,y) for a piecewise function f(x,y) where $x \in [0, \infty]$, y$\in [0, \infty]$ and $|f(x,y)| \leq Me^{k(x+y)}$ is defind by the follwoing integral:

$$T_{2sg}(f(x,y)) = P_1(s)P_2(s)\int_0^\infty\int_0^\infty e^{-q_1(s)x-q_2(s)y}\ f(x,y)dxdy = p(s)\int_0^\infty\int_0^\infty e^{-q_1(s)x-q_2(s)y}\ f(x,y)dxdy = \overline{\overline{f}} \qquad (2)$$

Where $p(s) = P_1(s)P_2(s)$

such that the integral is convergent for some $q_1(s), q_2(s)$ are positive functions, and $\|gsD(f(x,y))\| \leq \frac{P_1P_2M}{k-q_1q_2}$ (3)

2.3 Remark:

Let f(x,y,z) be a function of three variables then we use the following symbols in this paper:

$$\overline{\overline{f_{s12}}} = p(s)\int_0^\infty\int_0^\infty e^{-q_1(s)x-q_2(s)y}\ f(x,y,z)dxdy$$
$$\overline{\overline{f_{s13}}} = p(s)\int_0^\infty\int_0^\infty e^{-q_1(s)x-q_3(s)z}\ f(x,y,z)dxdz$$
$$\overline{\overline{f_{s23}}} = p(s)\int_0^\infty\int_0^\infty e^{-q_2(s)x-q_3(s)y}\ f(x,y,z)dydz$$

2.4 Definition:

Let f be a continuous function of three variables then the triple g-transformation

of $f(x,y,z)$ is defined as following :

$$T_{3sg}(f(x,y,z)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{q_1(s)x-q_2(s)y-q_3(s)z}\ f(x,y,z)\ dxdydz$$

Where x,y,z > 0 and s is positive constant , and

$$sup\left|\frac{f(x,y,z)}{e^{ax+by+cz}}\right| < 0$$

For some $a, b, c \in R$

The inverse of T_{3sg} – transform is defined as following :

$f(x,y,z) = \frac{1}{2\pi i}\int_{\propto-i\infty}^{\propto+i\infty} e^{q_1(s)x+q_2(s)y+q_3(s)z}\ F(s)\ ds$

2.5 Example :

1 – T_{3sg} (1) = $p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{q_1(s)x-q_2(s)y-q_3(s)z}$ (1)dx dy dz

$= p(s)\int_0^\infty\int_0^\infty [\frac{1}{-q_1(s)}\ e^{q_1(s)x}\ ]_0^\infty\ \ e^{-q_2(s)y-q_3(s)z}$ dy dz

$= \frac{p(s)}{q_1(s)}\int_0^\infty\int_0^\infty e^{-q_2(s)y-q_3(s)z}$ dy dz

$T_{3sg}(1) = \frac{p(s)}{q_1(s)q_2(s)q_3(s)}$

2 - $T_{3sg}(e^{ax+by+cz}) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{ax+by+cz}\ e^{q_1x-q_2y-q_3z}$ dx dy dz

$= p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{-(q_1-a)x}\ e^{-(q_2-b)y}\ e^{-(q_3-c)z}$ dx dy dz

$T_{3sg}\left(e^{ax+by+cz}\right) = \frac{p(s)}{(q_1-a)(q_2-b)(q_3-c)}$

3 – $T_{3sg}(\cosh(ax+by+cz)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty \cosh(ax+by+cz)\, e^{-q_1x}\, e^{-q_3z} dx\, dy\, dz$

$= \frac{p(s)}{2}\int_0^\infty\int_0^\infty\int_0^\infty [e^{(ax+by+cz)} + e^{-(ax+by+cz)}] e^{-q_1x-q_2y-q_3z}\, dx\, dy\, dz$

$= \frac{p(s)}{2}\left[\frac{1}{(q_1-a)(q_2-b)(q_3-c)} + \frac{1}{(q_1+a)(q_2+b)(q_3+c)}\right]$

$= \frac{p(s)}{2}\left[\frac{(q_1+a)(q_2+b)(q_3+c)+(q_1-a)(q_2-b)(q_3-c)}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}\right]$

$= \frac{p(s)}{2}\left[\frac{2\, q_1\, q_2\, q_3 + 2bc\, q_1 + 2\, ac\, q_2 + 2\, ab\, q_3}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}\right]$

$T_{3sg}(\cosh(ax+by+cz)) = \frac{p(s)(q_1\, q_2\, q_3 + bc\, q_1 + ac\, q_2 + ab\, q_3)}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}$

4 –$T_{3sg}(\sinh(ax+by+cz) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty \cosh(ax+by+cz)\, e^{-q_1x} e^{-q_2y}\ e^{-q_3z}\, dx\, dydz$

$= \frac{p(s)}{2}\int_0^\infty\int_0^\infty\int_0^\infty [e^{(ax+by+cz)} + e^{-(ax+by+cz)}]\, e^{-q_1x-q_2y-q_3z}\, dx\, dy\, dz$

$= \frac{p(s)}{2}\left[\frac{1}{(q_1-a)(q_2-b)(q_3-c)} - \frac{1}{(q_1+a)(q_2+b)(q_3+c)}\right]$

$= \frac{p(s)}{2}\left[\frac{(q_1+a)(q_2+b)(q_3+c)-(q_1-a)(q_2-b)(q_3-c)}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}\right]$

$= \frac{p(s)}{2}\left[\frac{2\, q_1\, q_2\, c + 2b\, q_1\, q_3 + 2\, aq_2q_3 + 2\, ab\, c}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}\right]$

$T_{3sg}(\sinh(ax+by+cz) = \frac{p(s)(q_1\, q_2 c + bq_1\, q_3 + aq_2\, q_3 + ab\, c)}{(q_1^2-a^2)(q_2^2-b^2)(q_3^2-c^2)}$

2.6 Proposition:

T_{sg} – transformation satisfies the linear property , i.e.

$T_{3sg}(a\, f(x,y,z) + b\, h(x,y,z)) = aT_{3sg}(f(,y\, x,z) + b\, T_{3sg}(h(x,y,z))$

Where $a, b \in R$

Proof:

$$T_{3sg}\left(a\, f(x,y,z) + b\, h(x,y,z)\right) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty [a\, f(x,y,z) + bh(x,y,z)]\, e^{q_1x-q_2y-q_3z}\, dx\, dy\, dz$$

$= ap(s)\int_0^\infty\int_0^\infty\int_0^\infty f(x,y,z)e^{-q_1x-q_2y-q_3z}\, dx\, dy\, dz + bp(s)\int_0^\infty\int_0^\infty\int_0^\infty h(x,y,z)e^{-q_1x-q_2y-q_3z} dx\, dy\, dz$

$= a\, T_{3sg}(f(x,y,z)) + b\, T_{3sg}(h(x,y,z))$

2.7 Table of $\boldsymbol{T_{3sg}}$-transformation of selected functions

ID	$f(x)$	$T_{3sg}\left(f(x,y,z)\right) = p(s)\int_0\int_0\int_0 e^{q_1(s)x-q_2(s)y-q_3(s)z}\, f(x,y,z)\, dxdydz$
1	K , K constant	$K\frac{p(s)}{q_1(s)q_2(s)q_3(s)}$
2	$\sin(ax+by+cz)$	$p(s)\left[\frac{aq_2q_3 - abc + bq_1q_3 + cq_1q_2}{(q_1^2+a^{2)}(q_2^2+b^{2)(}q_3^2+c^{2)}}\right]$
3	$\cos(ax+by+cz)$	$p(s)\left[\frac{q_1q_2q_3 - bcq_1 - abq_3 - acq_2}{(q_1^2+a^{2)}(q_2^2+b^{2)(}q_3^2+c^{2)}}\right]$
4	$x^ky^mz^n$	$\frac{p(s)k!\ m!\ \ n!}{(q_1(s))^{k+1}(q_2(s))^{m+1}(q_3(s))^{n+1}}$

2.8 Proposition:

Let $f(x,y,z)$, $x,y,z \in [0, \infty]$ be a function such that $|f(x,y,z)| \le \mu\, e^{k_1x+k_2y+k_3z}$ and $T_{3sg}(f(x,y,z)) = F(s)$ exists , then

$$T_{3sg}\,(f(x+a\,,y+b\,,z+c)) = e^{aq_1+bq_2+cq_3}\Big[T_{3sg}\big(f(u,v,w)\big) - T_{2sg}\Big(\int_0^c f(u,v,w)e^{-q_3w}\,dw\Big) - T_{2sg}\Big(\int_0^b f(u,v,w)e^{-q_2v}\,dv\Big) + T_{sg}\Big(\int_0^c\int_0^b f(u,v,w)e^{-q_2v}e^{-q_3w}\Big)dvdw - T_{2sg}\Big(\int_0^a f(u,v,w)e^{-q_1u}\,du\Big) + T_{sg}\Big(\int_0^c\int_0^a f(u,v,w)e^{-q_1u}e^{-q_3w}\Big)dudw + T_{sg}\Big(\int_0^b\int_0^a f(u,v,w)e^{-q_1u}e^{-q_3v}\Big)dudv - p(s)\int_0^c\int_0^b\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw\,\big(f(u,v,w)\big)\Big]$$

Proof :

$$T_{3sg}\,(f(x+a\,,y+b\,,z+c)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty f(x+a\,,y+b\,,z+c\,)e^{q_1x-q_2y-q_3z}dxdydz$$

Let $u = x + a\ ,v = y + b\ ,w = z + c$

$$T_{3sg}\big(f(\,u\,,v\,,w)\big) = p(s)\int_c^\infty\int_b^\infty\int_a^\infty f(u,v,w\,)e^{-q_1(u-a)}e^{-q_2(v-b)}\,e^{-q_3(w-c)}dudvdw$$

$$= p(s)\,e^{aq_1+bq_2+cq_3}\int_c^\infty\int_b^\infty\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}dudvdw$$

$$= p(s)e^{aq_1+bq_2+cq_3}\Big[\int_c^\infty\int_b^\infty\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w} - \int_c^\infty\int_b^\infty\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw\Big]$$

$$= p(s)e^{aq_1+bq_2+cq_3}\Big[\int_c^\infty\int_0^\infty\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w} - \int_c^\infty\int_0^b\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w} - \int_c^\infty\int_0^\infty\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw + \int_c^\infty\int_0^b\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw\Big]$$

$$= p(s)e^{aq_1+bq_2+cq_3}\Big[\int_0^\infty\int_0^\infty\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}du\,dv\,dw - \int_0^c\int_0^\infty\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}du\,dv\,dw - \int_0^\infty\int_0^b\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}du\,dv\,dw + \int_0^c\int_0^b\int_0^\infty f(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}du\,dv\,dw - \int_0^\infty\int_0^\infty\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw + \int_0^c\int_0^\infty\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw + \int_0^\infty\int_0^b\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw - \int_0^c\int_0^b\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw\Big]$$

$$T_{3sg}\,(f(x+a\,,y+b\,,z+c)) = e^{aq_1+bq_2+cq_3}\Big[T_{3sg}\big(f(u,v,w)\big) - T_{2sg}\Big(\int_0^c f(u,v,w)e^{-q_3w}\,dw\Big) - T_{2sg}\Big(\int_0^b f(u,v,w)e^{-q_2v}\,dv\Big) + T_{sg}\Big(\int_0^c\int_0^b f(u,v,w)e^{-q_2v}e^{-q_3w}\Big)dvdw - T_{2sg}\Big(\int_0^a f(u,v,w)e^{-q_1u}\,du\Big) + T_{sg}\Big(\int_0^c\int_0^a f(u,v,w)e^{-q_1u}e^{-q_3w}\Big)dudw + T_{sg}\Big(\int_0^b\int_0^a f(u,v,w)e^{-q_1u}e^{-q_3v}\Big)dudv - p(s)\int_0^c\int_0^b\int_0^a f(u,v,w)e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw\,\big(f(u,v,w)\big)\Big]$$

$= p(s)\,e^{aq_1+bq_2+cq_3}[\,\int_c^\infty\int_0^\infty\int_0^a f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}du\,dv\,dw - \int_c^\infty\int_0^b\int_0^a\ e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$

$= p(s)\,e^{aq_1+bq_2+cq_3}[\,\int_c^\infty\int_0^\infty\int_0^a f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}du\,dv\,dw - \int_c^c\int_0^\infty\int_0^\infty f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$ - $\int_c^\infty\int_0^b\int_0^\infty f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$ $+\int_c^c\int_0^b\int_0^\infty f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$ - $\int_0^\infty\int_0^\infty\int_0^a f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}du\,dv\,dw + \int_c^c\int_0^\infty\int_0^a f\,e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$ $+\int_0^\infty\int_0^b\int_0^a\ e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$ - $\int_0^c\int_0^b\int_0^a\ e^{-q_1u}\,e^{-q_2v}\,e^{-q_3w}\,du\,dv\,dw$

[T_3sg (f) -T_2sg ($\int_0^c e^{-q_3w}$ f dw)] - T_2sg ($\int_0^b e^{-q_2v}$ f dv)] + T_2sg ($\int_0^c\int_0^b e^{-q_2v-q_3w}$ f dv dw)

$-$ T_2sg ($\int_0^a e^{-q_1u}$ f du) + T_2sg ($\int_0^c\int_0^a e^{-q_1u-q_3w}$ f du dw)

+ T_2sg ($\int_0^b\int_0^a e^{-q_1u-q_2v}$ f du dv) - p(s) $\int_0^c\int_0^b\int_0^a$ f $e^{-q_1u-q_2v-q_3w}$ f du dv dw

3.The Convolution of T_{3sg}-transformation

3.1 Definition [3,6,7]:

Let $f(x,y,z), h(x,y,z)$ be a function defined on R^+ , then the convolution of the functions f(x,y,z) and h(x,y,z) is given as following :

T_{3sg} (f∗∗∗ h)(x,y,z) = $\int_0^x\int_0^y\int_0^z f\,(x-u, y-v, z-w\,)$ h (u,v,w) du dv dw

3.2 Theorem :

Let f(x,y,z),h(x,y,z) be a function which defined on R^+ then T_{3sg}-transformation of the convolution (f∗∗∗ h)) is given as following:

T_{3sg} (f∗∗∗ h) (x,y,z) = $\frac{1}{p(s)}$ T_{3sg} (f(x, y,z) . T_{3sg}(h(t,r,o))

Proof:

T_{3sg} (f(u,v,w) . T_{3sg} (h(t,r,o))

$$= \Big[p(s)\int_0^\infty\int_0^\infty\int_0^\infty f\,(u,v,w\,)e^{-q_1u}e^{-q_2v}\,e^{-q_3w}dudvdw\Big].\Big[p(s)\int_0^\infty\int_0^\infty\int_0^\infty h\,(t,r,o\,)e^{-q_1t}e^{-q_2r}\,e^{-q_3o}dtdrdo\Big]$$

$$= (p(s))^2\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1(u+t)-q_2(v+r)-q_3(w+o)}\ f(u,v,w)h(t,r,o)\,du\,dv\,dwdt\,dr\,do$$

x=u+t → u=x-t , y=v+r → v=y-r , z=w+o → w=z-o

$$= (p(s))^2 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1x-q_2y-q_3z}\ f(x-t, y-r, z-o)h(t,r,o)dx\,dy\,dzdt\,dr\,do$$

$$= (p(s))^2 \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1x-q_2y-q_3z}\ [f *** h](x,y,z)\ dx\,dy\,dz$$

$= P(s)\ T_{3sg}((f *** h)\ (x,y,z))$

$T_{3sg}(f *** h)(x,y,z) = \frac{1}{p(s)} T_{3sg}\big(f(x,y,z)\big).T_{3sg}\big(h(x,y,z)\big)$

3.3 Example :

Find $T_{3sg}^{-1}(\frac{s(3s^2-1)}{(s^3-s^2+s-1)^3})$, $p(s) = s$, $q_1(s) = q_2(s) = q_3(s) = s$

where $s > 0$, $s \neq 1$

we note that $s^3 - s^2 + s - 1 = (s^2+1)(s-1)$

$T_{3sg}^{-1}(\frac{s(3s^2-1)}{(s^3-s^2+s-1)^3}) = T_{3sg}^{-1}(\frac{s(3s^2-1)}{(s^2+1)^3(s-1)^3})$

Also we can write that

$T_{3sg}^{-1}(\frac{s(3s^2-1)}{(s^3-s^2+s-1)^3}) = T_{3sg}^{-1}(\frac{1}{s}\ \frac{s^2(3s^2-1)}{(s^2+1)^3(s-1)^3}) = T_{3sg}^{-1}(\frac{1}{s}\ \frac{s(3s^2-1)}{(s^2+1)^3}\ \frac{s}{(s-1)^3})$

But

$T_{3sg}(\sin(x+y+z)) = \frac{s(3s^2-1)}{(s^2+1)^3}$

And $T_{3sg}(e^{x+y+z}) = \frac{s}{(s-1)^3}$

Therefore

$T_{3sg}^{-1}(\frac{s(3s^2-1)}{(s^3-s^2+s-1)^3}) = \sin(x+y+z) *** e^{x+y+z}$

3.4 Example :

Find $T_{3sg}^{-1}(\frac{6(s^2+1)}{s^{10}(s^2-1)^2})$

Such that $p(s) = q_1(s) = s^2$, $q_2(s) = q_3(s) = s$

$T_{3sg}^{-1}\left(\frac{6(s^2+1)}{s^{10}(s^2-1)^2}\right) = T_{3sg}^{-1}\left(\frac{1}{s^2} * \frac{6(s^2+1)}{s^8(s^2-1)^2}\right) = T_{3sg}^{-1}\left(\frac{1}{s^2}\ \frac{6}{s^8}\ \frac{s^2+1}{(s^2-1)^2}\right)$

We note that :

$T_{3sg}(x^3) = \frac{6}{s^8}$, $T_{3sg}(\cosh(y+z)) = \frac{s^2+1}{(s^2-1)^2}$

Therefore

$T_{3sg}^{-1}(\frac{6(s^2+1)}{s^{10}(s^2-1)^2} = x^3 *** \cosh(y+z)$

4. Applications of T_{3sg}-transformation

4.1 Proposition :

1- $T_{3sg}(u_x) = q_1 \overline{\overline{\overline{u}}} - \overline{\overline{\overline{u_{s23}(0,y,z)}}}$

2 -$T_{3sg}(u_{xx}) = q_1^2 \overline{\overline{\overline{u}}} - q_1 \overline{\overline{\overline{u_{s23}(0,y,z)}}} - \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

3 - $T_{3sg}(u_x^{(n)}) = q_1^n \overline{\overline{\overline{u}}} - \sum_{i=0}^{n-1} q^{n-i-1} \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

4- $T_{3sg}(u_y) = q_2 \overline{\overline{\overline{u_y}}} - \overline{\overline{\overline{u_{s13}(x,0,z)}}}$

5- $T_{3sg}(u_y^{(m)}) = q_2^m \overline{\overline{\overline{u_y}}} - \sum_{i=0}^{m-1} q_2^{k-i-1} \overline{\overline{\overline{(u_y^{(i)})_{s13}(x,0,z)}}}$

6- $T_{3sg}(u_z) = q_3 \overline{\overline{\overline{u_z}}} - \overline{\overline{\overline{u_{s12}(x,y,0)}}}$

7- $T_{3sg}(u_z^{(r)}) = q_3^r \overline{\overline{\overline{u_z}}} - \sum_{i=0}^{r-1} q_3^{k-i-1} \overline{\overline{\overline{(u_z^{(i)})_{s12}(x,y,0)}}}$

8-$T_{3sg}(u_{xyz}) = q_1q_2q_3\overline{\overline{\overline{u}}} - q_1q_2\overline{\overline{\overline{(u)_{s23}(x,y,0)}}} - q_1\overline{\overline{\overline{(u_z)_{s13}(x,0,z)}}} - \overline{\overline{\overline{(u_{yz})_{s23}(0,y,z)}}}$

Proof:

1 - $T_{3sg}(u_x) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}\, u_x\, dxdydz$

$W = e^{-q_1x} \quad\rightarrow\quad dw = -q_1 e^{-q_1x}dx$

$dv = u_x\, dx \quad\rightarrow\quad v = u$

$T_{3sg}(u_x) = p(s)\int_0^\infty\int_0^\infty e^{-q_2y-q_3z}\left[ue^{-q_1x}|_0^\infty + q_1\int_0^\infty u\, e^{-q_1x}\, dx\right]dydz$

$= q_1\, p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}\, u\, dx\, dy\, dz - p(s)\int_0^\infty\int_0^\infty e^{-q_2y-q_3z}\, u(0,y,z)dydz$

$T_{3sg}(u_x) = q_1\overline{\overline{\overline{u}}} - \overline{\overline{\overline{u_{s23}(0,y,z)}}}$

2 – $T_{3sg}(u_{xx}) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}\, u_{xx}\, dxdydz$

$W = e^{-q_1x} \quad\rightarrow\quad dw = -q_1 e^{-q_1x}dx$

$dv = u_{xx}\, dx \quad\rightarrow\quad v = u_x$

$T_{3sg}(u_{xx}) = p(s)\int_0^\infty\int_0^\infty e^{-q_2y-q_3z}\left[u_xe^{-q_1x}|_0^\infty + q_1\int_0^\infty u_x\, e^{-q_1x}\, dx\right]dydz$

$= q_1\, p(s)\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1x-q_2y-q_3z}\, u_x\, dx\, dy\, dz - p(s)\int_0^\infty\int_0^\infty e^{-q_2y-q_3z}\, u_x(0,y,z)dydz$

$T_{3sg}(u_x) = q_1\overline{\overline{\overline{u_x}}} - \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

$T_{3sg}(u_{xx}) = q_1\left[q_1\overline{\overline{\overline{u}}} - \overline{\overline{\overline{u_{s23}(0,y,z)}}}\right] - \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

$T_{3sg}(u_{xx}) = q_1^2\overline{\overline{\overline{u}}} - q_1\overline{\overline{\overline{u_{s23}(0,y,z)}}} - \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

3 – $T_{3sg}(u_x^{(n)}) = q_1^n\overline{\overline{\overline{u}}} - \sum_{i=0}^{n-1} q^{n-i-1}\overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

We will proof by using mathematical induction

I) when n=1 we get that (3) is true by (1)

II) suppose that (3) is true when n=k

That is :

$T_{3sg}(u_x^{(k)}) = q_1^k\overline{\overline{\overline{u}}} - \sum_{i=0}^{k-1} q^{k-i-1}\overline{\overline{\overline{(u_x^{(i)})_{s23}(0,y,z)}}}$

III) let n=k+1 , and let $w=u_x^{(k)}$, $w_x=u_x^{(k+1)}$

$T_{3sg}(u_x^{(k+1)}) = T_{3sg}(w_x) = q_1\overline{\overline{\overline{w}}} - \overline{\overline{\overline{w_{s23}(0,y,z)}}}$

$= q_1\left[q_1^k\overline{\overline{\overline{u}}} - \sum_{i=0}^{k-1} q_1^{k-i-1}\overline{\overline{\overline{(u_x^{(i)})_{s23}(0,y,z)}}}\right] - \overline{\overline{\overline{(u_x)_{s23}(0,y,z)}}}$

$= q_1^{k+1}\overline{\overline{\overline{u}}} - p\sum_{i=0}^{k+1-1} q_1^{k+1-i-1}\overline{\overline{\overline{(u_x^{(i)})_{s23}(0,y,z)}}}$

There of the fact is true for all $n \in z^+$

There of the fact is true for all $n \in z^+$

4-$T_{3sg}(u_y)$= - $T_{3sg}(u_y) = p(s)\int_0^\infty\int_0^\infty e^{-q_1x-q_3z}\, u_y\, dy)dx\, dz$

$W = e^{-q_2 y} \quad \rightarrow \quad dw = -\,q_2\, e^{-q_2 y}\, dy$

$dv = u_y\, dy \quad \rightarrow \quad v = u$

$T_{3sg}(u_y) = p(s) \int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_3 z} \left[\, u\, e^{-q_2 y} |_0^{\infty} + q_2 \int_0^{\infty} u\, e^{-q_2 y}\, dy \right] dx\, dz$

$= q_2\, p(s) \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} (e^{-q_1 x - q_2 y - q_3 z}\, u\, dx\, dy\, dz) - p\int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_3 z}\, u(x,0,z)\, dy\, dz$

$T_{3sg}(u_y) = q_2\, T_{3sg}(u) - \overline{\overline{\overline{u_{s13}(x,0,z)}}}$

$T_{3sg}(u_y) = q_2\, \overline{\overline{\overline{u_y}}} - \overline{\overline{\overline{u_{s13}(x,0,z)}}}$

5- We will proof by using mathematical induction

I) when m=1 we get that (5) is true by (4)

II) suppose that (5) is true when m=k

That is :

$T_{3sg}(u_y^{(k)}) = q_2^k\, \overline{\overline{\overline{u_y}}} - \sum_{i=0}^{k-1} q_2^{k-i-1}\, \overline{\overline{\overline{(u_y^{(i)})_{s13}(x,0,z)}}}$

III) When m=k+1 and let $w=u_y^{(k)}$, $w_y=u_y^{(k+1)}$

$T_{3sg}(u_y^{(k+1)}) = T_{3sg}(w_y) = q_2\, \overline{\overline{\overline{u_y}}} - \overline{\overline{\overline{w_{s13}(x,0,z)}}}$

$= q_2 \left[q_2^k\, \overline{\overline{\overline{u_y}}} - \sum_{i=0}^{k-1} q_2^{k-i-1}\, \overline{\overline{\overline{(u_y^{(i)})_{s13}(x,0,z)}}} \right] - \overline{\overline{\overline{(u_y)_{s23}(x,0,z)}}}$

$= q_2^{k+1}\, \overline{\overline{\overline{u_y}}} - \sum_{i=0}^{k+1-1} q_2^{k+1-i-1}\, \overline{\overline{\overline{(u_y^{(i)})_{s13}(x,0,z)}}}$

Therefore (5) is true for all $m \in z^+$

6- $T_{3sg}(u_z) = p(s) \int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_2 y} \left[\int_0^{\infty} e^{-q_3 z}\, u_z\, dz\right] dx\, dy$

$W = e^{-q_3 z} \quad \rightarrow \quad dw = -\,q_3\, e^{-q_3 z}\, dz$

$dv = u_z\, dz \quad \rightarrow \quad v = u$

$T_{3sg}(u_z) = p(s) \int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_2 y} \left[u e^{-q_3 z}|_0^{\infty} + q_3 \int_0^{\infty} u\, e^{-q_3 z}\, dz \right] dx\, dy$

$= q_3\, p(s)\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_2 y - q_3 z}\, u\, dx\, dy\, dz - p(s)\int_0^{\infty}\int_0^{\infty} e^{-q_1 x - q_2 y}\, u(x,y,0)\, dx\, dy$

$T_{3sg}(u_z) = q_3\, T_{3sg}(u) - \overline{\overline{\overline{u_{s12}(x,y,0)}}}$

$T_{3sg}(u_z) = q_3\, \overline{\overline{\overline{u_z}}} - \overline{\overline{\overline{u_{s12}(x,y,0)}}}$

7- We will proof by using mathematical induction

I) when r=1 we get that (7) is true by

II) suppose that (7) is true when r=k

That is :

$T_{3sg}(u_z^{(k)}) = q_3^k\, \overline{\overline{\overline{u_z}}} - \sum_{i=0}^{k-1} q_3^{k-i-1}\, \overline{\overline{\overline{(u_z^{(i)})_{s12}(x,y,0)}}}$

III) let r=k+1 , and let $w=u_z^{(k)}$, $w_z=u_z^{(k+1)}$

$T_{3sg}(u_z^{(k+1)}) = T_{3sg}(w_z) = q_3\, \overline{\overline{\overline{u_z}}} - \overline{\overline{\overline{w_{s12}(x,y,0)}}}$

$= q_3 \left[q_3^k\, \overline{\overline{\overline{u_z}}} - \sum_{i=0}^{k-1} q_3^{k-i+1}\, \overline{\overline{\overline{(u_z^{(i)})_{s12}(x,y,0)}}} \right] - \overline{\overline{\overline{(u_z^{(k)})_{s23}(x,y,0)}}}$

$= q_3^{k+1}\, \overline{\overline{\overline{u_z}}} - \sum_{i=0}^{k+1-1} q_3^{k+1-i+1}\, \overline{\overline{\overline{(u_z^{(i)})_{s12}(x,y,0)}}}$

Therefore (7) is true for all $r \in \mathbf{z}^+$

8- $T_{3sg}(u_{xyz}) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty u_{xyz}\, e^{-q_1x-q_2y-q_3z}\, dx\, dy\, dz$

$$= p(s)\int_0^\infty\int_0^\infty e^{-q_2y-q_3z}[[u_{yz}\, e^{-q_1x}]_0^\infty + q_1\int_0^\infty u_{yz}\, e^{-q_1x} dx]\, dydz$$

$$= q_1p(s)\int_0^\infty\int_0^\infty\int_0^\infty u_{yz}\, e^{-q_1x-q_2y-q_3z} dxdydz - \overline{\overline{(u_{yz})_{s23}(0,y,z)}}$$

$$= q_1p(s)[\int_0^\infty\int_0^\infty e^{-q_1x-q_3z}[[u_z\, e^{-q_2y}]_0^\infty + q_2\int_0^\infty u_z\, e^{-q_2y} dy]\, dxdz] - \overline{\overline{(u_{yz})_{s23}(0,y,z)}}$$

$$= q_1q_2p(s)\int_0^\infty\int_0^\infty\int_0^\infty u_z\, e^{-q_1x-q_2y-q_3z} dxdydz - q_1\overline{\overline{(u_z)_{s13}(x,0,z)}} - \overline{\overline{(u_{yz})_{s23}(0,y,z)}}$$

$$= q_1q_2p(s)[\int_0^\infty\int_0^\infty e^{-q_1x-q_2y}[[u\, e^{-q_3z}]_0^\infty + q_3\int_0^\infty u e^{-q_3z} dz]\, dxdy] - q_1\overline{\overline{(u_z)_{s13}(x,0,z)}} - \overline{\overline{(u_{yz})_{s23}(0,y,z)}}$$

$$T_{3sg}(u_{xyz}) = q_1q_2q_3\overline{\overline{\overline{u}}} - q_1q_2\overline{\overline{(u)_{s23}(x,y,0)}} - q_1\overline{\overline{(u_z)_{s13}(x,0,z)}} - \overline{\overline{(u_{yz})_{s23}(0,y,z)}}$$

4.2 Example :

Solve the following equation

$$u_x = 1 \quad , \quad u(0,y,z) = 1$$

Solution:

By taking T_{3sg} – transformation for the equation we get

$T_{3sg}(u_x) = T_{3sg}(1)$

$q_1\overline{\overline{\overline{u}}} - \overline{\overline{u_{s23}(0,y,z)}} = \frac{p}{q_1\, q_2\, q_3}$

We note that :

$\overline{\overline{u_{s23}(0,y,z)}} = p\int_0^\infty\int_0^\infty u(0,y,z)e^{-q_2y-q_3z}\, dydz = \frac{p}{q_2\, q_3}$

Thus

$q_1\overline{\overline{\overline{u}}} - \frac{p}{q_2\, q_3} = \frac{p}{q_1\, q_2\, q_3}$

$q_1\overline{\overline{\overline{u}}} = \frac{q_1\, p+p}{q_1\, q_2\, q_3} \rightarrow \overline{\overline{\overline{u}}} = \frac{p(q_1+1)}{q_1^2 q_2\, q_3}$

We take T_{3sg}^{-1} - transformation for both sides , we get :

$u(x,y,z) = T_{3sg}^{-1}\left(\frac{p(q_1+1)}{q_1^2 q_2\, q_3}\right) = T_{3sg}^{-1}\left(\frac{p}{q_1^2 q_2\, q_3} + \frac{p}{q_1\, q_2\, q_3}\right)$

$= T_{3sg}^{-1}\left(\frac{p}{q_1^2 q_2\, q_3}\right) + T_{3sg}^{-1}\left(\frac{p}{q_1\, q_2\, q_3}\right)$

$= x^0y^0z^0 + x^1y^0z^0 = 1 + x$

4.3 Example :

Solve the following equation

$u_{xx} = u_x \quad , \quad u_x\ (0,y,z) = 0 \quad , \quad u(0,y,z) = 1$

$p = q_1 = q_2 = q_3 = s$

Solution :

By taking T_{3sg} – transformation for the equation we get

$T_{3sg}(u_{xx}) = T_{3sg}(u_x)$

$q_1^2\overline{\overline{\overline{u}}} - q_1\overline{\overline{u_{s23}(0,y,z)}} - \overline{\overline{(u_x)_{s23}(0,y,z)}} = q_1\overline{\overline{\overline{u}}} - \overline{\overline{u_{s23}(0,y,z)}}$

$\overline{\overline{u_{s23}(0,y,z)}} = \frac{p}{q_2\, q_3}$ thus $q_1^2\overline{\overline{\overline{u}}} - q_1\frac{p}{q_2\, q_3} = q_1\overline{\overline{\overline{u}}} - \frac{p}{q_2\, q_3}$

$q_1^2\overline{\overline{\overline{u}}} - q_1\overline{\overline{\overline{u}}} = q_1\frac{p}{q_2\, q_3} - \frac{p}{q_2\, q_3} \quad \rightarrow q_1(q_1-1)\overline{\overline{\overline{u}}} = (q_1-1)\frac{p}{q_2\, q_3}$

$q_1 \overline{\overline{\overline{u}}} = \frac{p}{q_2\, q_3}$

$\overline{\overline{\overline{u}}} = \frac{p}{q_1\, q_2\, q_3}$

We take T_{3sg}^{-1} - transformation for both sides , we get :

u (x,y,z) $= T_{3sg}^{-1}\left(\frac{p}{q_1\, q_2\, q_3}\right) = 1$

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