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Theoretical Study for Adjusting ANOVA Procedures For The One-Way Random Effects Model Under Correlated Errors Within Class And Across Classes

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ABSTRACT

In this study, a method was developed to adjust the procedures of analysis of variance for the one-way balanced random effects model when error limits across classes are not independent, while the assumption of independence between the error terms among the different classes is essential in the analysis of variance. The variance-covariance structure and the correlation coefficients for the error limits were defined. The ρ_1 was assumed as the correlation coefficient of error terms within the class and the ρ_2 is the correlation coefficient for error terms in different classes. The work performed by deriving correction factor and calculating the expectation of the mean sum of squares of errors and treatments as well as correcting the f-statistic.

MSC..

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1. Introduction

The analysis of variance (ANOVA) is a popular parametric statistical technique used to analyze data in a variety of fields such as biostatistics and ecology Sahai and Ageel (2012) [1]. It is used to study the effect of an independent linear variable (factor) on the dependent variable (response variable) by testing hypotheses about the equivalence of two or more populations (or treatments) through analysis of observed sample variance [2]. Analysis of variance offers many advantages including being a simple statistical analysis that can be performed and interpreted, robust to some violations of certain assumptions, and produce the desired results [3-5]. As with any parametric test, some assumptions must be satisfied in order to arrive at a reasonable and acceptable conclusion. The usual assumption of the analysis of variance approach is that error measurements in the model are independent normal random variables with zero means and homogeneity variances. However, the assumption of independence of observations and error terms is rarely confirmed, and ignoring correlations between

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observations can lead to unreliable statistical conclusions [6-8]. Some studies have discussed the correlation between observations within the same class, and others have investigated the effect of the correlation between random errors within the same group. Fisher et al. (1950) [9] presented the intraclass correlation coefficient, which measures the similarity of observations of the same class. Since then, many researchers have studied intraclass correlation coefficients in the analysis of variance in several research areas, including epidemiology, genetics, psychology, and sensitivity analysis [10]. Smith and Lewis (1980) [11] provided a technique for correcting the usual F-tests to construct unbiased F-tests by assuming the existence of a correlation between observations in the same class in a k-way factorial experiment. The study showed that ignoring the effect of intraclass correlation can strongly affect a type I error probability at a significance level of 0.05. Pavur and Levis (1983) [12] found a general form of the correlation matrix to produce unbiased F tests in a k-way factorial experiment. Scariano and Davenport (1984) [13] introduced a theoretical study of corrected F-tests in the general linear model to examine the role of these statistics in the hypotheses test when a known, specified form of covariance structure is provided in the presence of correlated errors. Pavur and Davenport (1985) [7] provided a technique for adjusting the usual F-test when observations are correlated. The researchers found that by increasing the number of observations per cell in a factorial experiment, small correlations can affect the F statistic and significantly amplifies the type I error. Consequently, the uncorrected F statistics may abolish the outcomes of the analysis of variance (ANOVA) because it is not a true representation of the sample. AL-KAABAWI (2007) [14] studied the effect of correlations between observations on the type I error rates for multiple comparison procedures for 3-way crossed balanced model. Pavur (1988) [15] showed that the number of repeats in one-way ANOVA model can amplify small correlations and thus ignoring these correlations can easily inflate type I error. Mallick et al. (2020) [16] studied the effect of the first-order autoregressive correlated errors in the one-way and two-way models with repeated measurements. The modified F-test statistics is the classical F statistic multiplied by a constant that is a function of a maximum likelihood estimate of the correlation coefficient. The modified procedure led to obtaining a better result of analysis of variance.

Analysis of variance problems for autocorrelated data were addressed using nonparametric and parametric approaches. The parametric methods consider more robust than non-parametric methods. The parametric methods modify the quadratic forms and their degrees of freedom in the numerator and denominator of the F ratio. Andersen et al. (1981) [17] corrected the degrees of freedom of the distribution of the F statistic based on the number of groups, time points, and autocorrelation coefficient of random errors in the two-way ANOVA model. Lund et al. (2016) [18] replaced autocorrelated data in the F ratio by estimated prediction errors where errors are stationary time series. This approach allows for retaining the classical distribution of the null hypothesis with the usual degrees of freedom.

The previous studies addressed the problem of correlations between data in the same group for various models but have not handled the issues of correlations between errors in the different groups in the one-way random effect model. The present study provides the theory side for modification of the analysis of variance for the one-way random effect model in the presence of correlated errors in the same group and also between groups. The covariance and correlation structures of errors were proposed such that random errors in the same group have the same correlation coefficient $\rho_1$and in the different groups have the same correlation coefficient ρ_2. The test statistic was corrected by modifying the distributions of the two sums of squares the between and within groups according to the proposed correlation structure and under null and alternative hypotheses. The expected values of sums of squares were derived under suggested correlation conditions

2. Mathematical Model

The mathematical model of the balanced one-way random effects model can be written as follows:

$$Y_{ij} = \mu + \alpha_i + e_{ij} \qquad (i = 1, 2, \ldots, n \ ; \ j = 1, 2, \ldots, \mathrm{k}) \tag{1}$$

where Y_{ij} is the j^{th} observation corresponding to the i^{th} treatment group, μ is the general mean, α_i is the random effect corresponding to the i^{th}treatment group of the factor, e_{ij} is the random error due to the j^{th}observation, at the i^{th}treatment group. For model (1), the α_i and e_{ij} are independent and identical distributed normal random variables with means 0 and variances σ_1^2 and σ_2^2, respectively. We adopted an error covariance structure based on the assumption that the errors are correlated. Errors occurring within the same group have a certain correlation, denoted as ρ_1, and errors occurring between groups have correlation, called ρ_2. The correlation coefficients ρ_1 and ρ_2 are called the intra-class and between-class error correlation coefficients, respectively, and the covariance structure of the errors is given by

$$\operatorname{cov}\left(e_{ij}, e_{i'j'}\right) = \begin{cases} \sigma_2^2 & \text{if } i' = i' \text{ and } j' = j' \\ \rho_1\sigma_2^2 & \text{if } i' = i' \text{ and } j' \neq j' \\ \rho_2\sigma_2^2 & \text{if } i' \neq i' \end{cases} \tag{2}$$

Main effects are combined to find the observed value, as we can see from the mathematical model for each ANOVA design. Therefore, the covariance structure between observations due to the correlation between errors in the model (1) is given by

$$\operatorname{cov}\left(\mathrm{Y}_{ij}, \mathrm{Y}_{i'j'}\right) = \begin{cases} \sigma_1^2 + \sigma_2^2 & \text{if } i' = i' \text{ and } j' = j' \\ \sigma_1^2 + \rho_1\sigma_2^2 & \text{if } i' = i' \text{ and } j' \neq j' \\ \sigma_1^2 + \rho_2\sigma_2^2 & \text{if } i' \neq i' \end{cases} \tag{3}$$

The sums of squares of the total, treatments (between groups), and errors (within groups) are given by

$$SST = \sum_{i=1}^{n}\sum_{j=1}^{k}\left(y_{ij} - \bar{y}_{..}\right)^2, \ \mathrm{SSB} = \sum_{i=1}^{n}\sum_{j=1}^{k}\left(\bar{y}_{i.} - \bar{y}_{..}\right)^2, \text{ and}$$

$$\mathrm{SSW} = \sum_{i=1}^{n}\sum_{j=1}^{k}\left(y_{ij} - \bar{y}_{i.}\right)^2, \tag{4}$$

$$\text{Where } \bar{y}_{..} = \frac{1}{nk}\sum_{i=1}^{n}\sum_{j=1}^{k} y_{ij}, \text{ and } \bar{y}_{i.} = \frac{1}{k}\sum_{i=1}^{n} y_{ij}, \tag{5}$$

Theorem 1. The sufficient statistics $\bar{y}_{..}$ and $\bar{y}_{i.}$ have the following distributions:

1. $\bar{y}_{..} \sim N\left(\mu, \frac{1}{nk}\left[k\sigma_1^2 + \sigma_2^2\left[\left(1-\rho_1\right) + \mathrm{k}\left(\rho_1 + \left(n\text{-}1\right)\rho_2\right)\right]\right]\right)$
2. $\bar{y}_{i.} \sim N\left(\mu, \frac{1}{k}\left[k\sigma_1^2 + \sigma_2^2\left(1-\rho_1\right) + \mathrm{k}\,\rho_1\,\sigma_2^2\right]\right)$

Proof. We begin to prove (1) and (2), respectively. First, from (1), we have

$$\bar{Y}_{..} = \mu + \bar{\alpha}_{.} + \bar{e}_{..}, \tag{6}$$

where

$$\bar{y}_{..} = \sum_{i=1}^{n}\sum_{j=1}^{k}\frac{y_{ij}}{nk}, \ \bar{\alpha}_{.} = \sum_{i=1}^{n}\frac{\alpha_i}{n} \text{ and } \bar{e}_{..} = \sum_{i=1}^{n}\sum_{j=1}^{k}\frac{e_{ij}}{nk}.$$

From covariance structure in equation (2) and (3), then it easily follows that $\bar{y}_{..}$ is a normal random variable with $E\left(\bar{y}_{..}\right) = \mu$ and

$$\mathrm{var}\left(\bar{y}_{..}\right) = \frac{1}{nk}\left[k\sigma_1^2 + \sigma_2^2\left[\left(1-\rho_1\right) + \mathrm{k}\left(\rho_1 + (n\text{-}1)\rho_2\right)\right]\right] \tag{7}$$

Next, we prove (2) in the same way. Again, from model (1), we can get

$$\bar{y}_{\mathrm{i.}} = \mu + \alpha_i + \bar{e}_{\mathrm{i.}} \ , \tag{8}$$

where

$$\bar{y}_{\mathrm{i.}} = \sum_{j=1}^{k}\frac{y_{ij}}{k} \ \text{ and } \ \bar{e}_{\mathrm{i.}} = \sum_{j=1}^{k}\frac{e_{ij}}{k} \ .$$

From covariance structure in equation (2) and (3), it is easily seen that $\bar{y}_{i.}$ is a normal random variable with $E\left(\bar{y}_{\mathrm{i.}}\right) = \mu$ and

$$\mathrm{var}\left(\bar{y}_{\mathrm{i.}}\right) = \frac{1}{k}\left[k\sigma_1^2 + \sigma_2^2\left(1-\rho_1\right) + \mathrm{k}\rho_1\sigma_2^2\right] \tag{9}$$

□

Theorem 2. Whether $\mathrm{H}_0 : \sigma_1^2 = 0$ is true or not, $\dfrac{SSW}{\sigma_2^2\left(1-\rho_1\right)} \sim \chi^2\left(n\left(k\text{-}1\right)\right)$ and

when $\mathrm{H}_0 : \sigma_1^2 = 0$ is true, $\dfrac{SSB}{\sigma_2^2\left[\left(1-\rho_1\right) + \mathrm{k}\left(\rho_1 - \rho_2\right)\right]} \sim \mathrm{X}^2\left(n\text{-}1\right)$.

Proof. Both the total sum of squares and the degrees of freedom have the addition property, so according to Cochrane's theorem [18], and under the null hypothesis, $SSW/\sigma_2^2\left(1-\rho_1\right)$ has $\mathrm{X}^2\left(n\left(k\text{-}1\right)\right)$ and $SSB/\sigma_2^2\left[\left(1-\rho_1\right) + \mathrm{k}\left(\rho_1 - \rho_2\right)\right]$ has $\mathrm{X}^2\left(n-1\right)$.

□

Theorem 3. Under the null hypothesis $\mathrm{H}_0 : \sigma_1^2 = 0$, the expected sums squares are given by

1. $E\left(\dfrac{SSW}{\left(1-\rho_1\right)\sigma_2^2}\right) = \mathrm{n}\left(k - 1\right)$
2. $E\left(\dfrac{SSB}{\sigma_2^2\left[\left(1-\rho_1\right) + \mathrm{k}\left(\rho_1 - \rho_2\right)\right]}\right) = \left(\mathrm{n} - 1\right)$

Proof. From equation (4),

$$\mathrm{E}\left(\mathrm{SSW}\right) = \sum_{i=1}^{n}\sum_{j=1}^{k}E\left[\left(y_{ij} - \bar{y}_{\mathrm{i.}}\right)^2\right] = \sum_{i=1}^{n}\sum_{j=1}^{k}\left[E\left(y_{ij}^{\ 2}\right) - 2\mathrm{E}\left(y_{ij}\bar{y}_{\mathrm{i.}}\right) + \mathrm{E}\left(\bar{y}_{\mathrm{i.}}^2\right)\right] \tag{10}$$

From equation (3), it is easy to show that

$$E\left(y_{ij}\bar{y}_{i.}\right) = \frac{1}{k}\left[k\sigma_1^2 + \sigma_2^2\left(1-\rho_1\right) + k\,\rho_1\,\sigma_2^2\right] + \mu^2.$$

Since

$$E\left(\bar{y}_{i.}^2\right) = \operatorname{var}\left(\bar{y}_{i.}\right) + \left[E\left(\bar{y}_{i.}\right)\right]^2,$$

and using theorem 1 point 2, we obtain

$$E\left(\bar{y}_{i.}^2\right) = \frac{1}{k}\left[k\sigma_1^2 + \sigma_2^2\left(1-\rho_1\right) + k\,\rho_1\,\sigma_2^2\right] + \mu^2, \tag{11}$$

And

$$E\left(y^2_{ij}\right) = \operatorname{var}\left(y_{ij}\right) + \left[E\left(y_{ij}\right)\right]^2 = \sigma_1^2 + \sigma_2^2 + \mu^2.$$

By Substituting the values of $E\left(y_{ij}\bar{y}_{i.}\right)$, $E\left(\bar{y}_{i.}^2\right)$ and $E\left(y^2_{ij}\right)$ into equation (10), we have

$$E(SSW) = n\left(k-1\right)\,\left(1-\rho_1\right)\sigma_2^2 \tag{12}$$

Therefore, it follows that

$$E\left(\frac{SSW}{\left(1-\rho_1\right)\sigma_2^2}\right) = n\left(k-1\right).$$

The proof of part (2) is similar to part (1)

$$E(SSB) = \sum_{i=1}^{n}\sum_{j=1}^{k} E\left[\left(\bar{y}_{i.} - \bar{y}_{..}\right)^2\right] = \sum_{i=1}^{n}\sum_{j=1}^{k}\left[E\left(\bar{y}_{i.}^{\,2}\right) - 2E\left(\bar{y}_{i.}\bar{y}_{..}\right) + E\left(\bar{y}_{..}^{\,2}\right)\right] \tag{13}$$

From theorem 1 we can obtain that

$$E\left(\bar{y}_{..}^{\,2}\right) = \frac{1}{nk}\left[k\sigma_1^2 + \sigma_2^2\left[\left(1-\rho_1\right) + k\left(\rho_1 + (n-1)\rho_2\right)\right]\right] + \mu^2 \tag{14}$$

Again, using covariance structure in equation (3) it is obtained

$$E\left(\bar{y}_{i.}\bar{y}_{..}\right) = \frac{1}{nk}\left\{\left[k\sigma_1^2 + \sigma_2^2\left(1-\rho_1\right) + k\,\rho_1\,\sigma_2^2 + k\left(n-1\right)\rho_2\sigma_2^2\right]\right\} + \mu^2 \tag{15}$$

Substituting equations (11), (14) and (15) into equation (13) we get that

$$E(SSB) = \sigma_2^2\left[\left(1-\rho_1\right) + k\left(\rho_1 - \rho_2\right)\right]\left[(n-1) + \frac{k\left(n-1\right)\sigma_1^2}{\sigma_2^2\left[\left(1-\rho_1\right) + k\left(\rho_1-\rho_2\right)\right]}\right] \tag{16}$$

When the null hypothesis is true, $\sigma_1^2 = 0$, then we can get,

$$E\left(\frac{SSB}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]}\right)=\left(n\text{-}1\right) \tag{17}$$

□

Theorem 4. Under $\mathrm{H}_A:\ \sigma_1^2\neq 0$ the expected sum squares of treatments is given by

$$E\left(\frac{SSB}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]}\right)=\left(n\text{-}1\right)+\frac{\mathrm{k}\left(n\text{-}1\right)\sigma_1^2}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]}$$

Proof. Since

$$\bar{y}_{i.}\sim N\left(\mu,\frac{1}{k}\left[k\sigma_1^2+\sigma_2^2\left(1-\rho_1\right)+\mathrm{k}\rho_1\sigma_2^2\right]\right),$$

$$\bar{y}_{..}\sim N\left(\mu,\frac{1}{nk}\left[k\sigma_1^2+\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1+\left(n\text{-}1\right)\rho_2\right)\right]\right]\right)\text{ and}$$

$$\mathrm{E}\left(\bar{y}_{i.}\bar{y}_{..}\right)=\frac{1}{nk}\left\{\left[k\sigma_1^2+\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1+\left(n\text{-}1\right)\rho_2\right)\right]\right]\right\}+\mu^2\quad\text{then}$$

$$\mathrm{E}\left(\mathrm{SSB}\right)=\sum_{i=1}^{n}\sum_{j=1}^{k}E\left[\left(\bar{y}_{i.}-\bar{y}_{..}\right)^2\right]=\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]\left[\left(n\text{-}1\right)+\frac{\mathrm{k}\left(n\text{-}1\right)\sigma_1^2}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]}\right]$$

$$E\left(\frac{SSB}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]}\right)=\left(n\text{-}1\right)+\frac{\mathrm{k}\left(n\text{-}1\right)\sigma_1^2}{\sigma_2^2\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]} \tag{18}$$

□

Theorem 5. Let each observation in a set of n random samples is normally distributed with the same variance, $\sigma_1^2+\sigma_2^2$, where σ_1^2 is the variance of random effects and σ_2^2 is the variance of random errors. Then, under covariance structure in equation (2) and (3),

1. If $\mathrm{H}_0:\ \sigma_1^2=0$ is true, the corrected statistics

$$F^*=\frac{SSB\Big/\left\{\sigma_2^2\left(n-1\right)\left[\left(1-\rho_1\right)+\mathrm{k}\left(\rho_1-\rho_2\right)\right]\right\}}{SSW\Big/\left\{\sigma_2^2\,n\left(k-1\right)\left(1-\rho_1\right)\right\}}$$

has a central F-distribution with $n-1$ and $n\left(k-1\right)$ degrees of freedom.

2. If $\mathrm{H}_A:\ \sigma_1^2=0$ is true, the corrected statistics

$$F^* = \frac{SSB \Big/ \left\{\sigma_2^2 (n-1)\left[(1-\rho_1) + \mathrm{k}\,(\rho_1 - \rho_2)\right]\right\}}{SSW \Big/ \left\{\sigma_2^2\; n(k-1)\;(1-\rho_1)\right\}}$$

has a noncentral F-distribution with degrees of freedom $n-1$ and $n(k-1)$ respectively and a non-central parameter η, where $\eta = \frac{k(\mathrm{n}-1)\;\sigma_1^2}{\sigma_2^2\left[(1-\rho_1) + \mathrm{k}\,(\rho_1 - \rho_2)\right]}$.

proof. We know that SSB and SSW are independent. By theorem 2, the $\frac{SSB}{\sigma_2^2\left[(1-\rho_1) + \mathrm{k}(\rho_1 - \rho_2)\right]}$ and $\frac{SSW}{(1-\rho_1)\sigma_2^2}$ have central Chi-square distributions with $n-1$ and $n(k-1)$ degrees of freedom, respectively.

Then from the definition of the F distribution, the corrected F^*-statistic has a central F-distribution with $n-1$ and $n(k-1)$ degrees of freedom under H_0 is true.

The proof of point (2), under H_A is true, the numerator $\frac{SSB}{\sigma_2^2\left[(1-\rho_1) + \mathrm{k}(\rho_1 - \rho_2)\right]}$ has a non-central Chi-square distribution $\chi^2_{(n-1),\eta}$ with non-central parameter $\eta = \frac{k(\mathrm{n}-1)\;\sigma_1^2}{\sigma_2^2\left[(1-\rho_1) + \mathrm{k}\,(\rho_1 - \rho_2)\right]}$. From theorem 2 the denominator $\frac{SSW}{(1-\rho_1)\sigma_2^2}$ has a central Chi-square distribution regardless of whether H_0 or H_A is true . Then we can shown that

$$F^* = \frac{SSB \Big/ \left\{\sigma_2^2 (n-1)\left[(1-\rho_1) + \mathrm{k}\,(\rho_1 - \rho_2)\right]\right\}}{SSW \Big/ \left\{\sigma_2^2\; n(k-1)\;(1-\rho_1)\right\}}$$

$$= \frac{(1-\rho_1)}{\left[(1-\rho_1) + \mathrm{k}\,(\rho_1 - \rho_2)\right]} \times \frac{SSB/(n-1)}{SSW/n(k-1)} = \mathrm{CF} \tag{19}$$

, where C is correction factor. This implies that under H_A, the corrected statistic, F^*has a noncentral F-distribution with degrees of freedom $(n-1)$, $n(k-1)$, and non-centrality parameter η.

□

3. Correction Factor

This section discusses the values of the correction factor C to make sure that F tests are correctly performed under certain correlation patterns for random errors. It has been demonstrated that the modified test statistic F^* is the usual F statistic multiplied by the correction factor,

$$F^{*} = CF = \frac{(1 - \rho_1)}{\left[(1 - \rho_1) + \text{k}\,(\rho_1 - \rho_2)\right]} \times \text{F}$$

It is important to note that in the case $\rho_1 = \rho_2$, the corrected statistic is identical to the classical F statistic, consequently, no correction is required. In the case the factor of correction is not equivalent to one, then there is a need for correction [14] [20-21]. The values of the correction factor rely on the values of the correlation coefficients that include three cases. In case one, the values of both ρ_1 and ρ_2 are positive ; In case two are both negative, and in case three ρ_1 is positive and ρ_2 negative. In all three cases, values $\rho_1 > \rho_2$ produce a positive correlation coefficient. Tables 1, 2 and 3 show that the values of C increase as ρ_2 increase and decrease as ρ_1 increase for the three studied cases of correlation coefficients. As result, the values of the correction factor are restricted between 0 and 1. In addition, tables 1, 2 and 3 show that the increase in the size of the replications minimizes the value of the correlation factor where 2, 8, 40, and 80 were specified to be the replication size.

Table 1: Correction factor for $k = 2$ and 8 and for negative values of ρ_1 and ρ_2.

K=2										
ρ_2	ρ_1									
	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
-0.9	1.000	0.900	0.810	0.727	0.652	0.583	0.520	0.462	0.407	0.357
-0.8	*	1.000	0.895	0.800	0.714	0.636	0.565	0.500	0.440	0.385
-0.7	*	*	1.000	0.889	0.790	0.700	0.619	0.546	0.478	0.417
-0.6	*	*	*	1.000	0.882	0.778	0.684	0.600	0.524	0.455
-0.5	*	*	*	*	1.000	0.875	0.765	0.667	0.579	0.500
-0.4	*	*	*	*	*	1.000	0.867	0.750	0.647	0.556
-0.3	*	*	*	*	*	*	1.000	0.857	0.733	0.625
-0.2	*	*	*	*	*	*	*	1.000	0.846	0.714
-0.1	*	*	*	*	*	*	*	*	1.000	0.833
0.00	*	*	*	*	*	*	*	*	*	1.000
K=8										
ρ_2	ρ_1									
	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
-0.9	1.000	0.692	0.515	0.400	0.319	0.259	0.213	0.177	0.147	0.122
-0.8	*	1.000	0.680	0.500	0.385	0.304	0.245	0.200	0.164	0.135
-0.7	*	*	1.000	0.667	0.484	0.368	0.289	0.231	0.186	0.152
-0.6	*	*	*	1.000	0.652	0.467	0.351	0.273	0.216	0.172
-0.5	*	*	*	*	1.000	0.636	0.448	0.333	0.256	0.200
-0.4	*	*	*	*	*	1.000	0.619	0.429	0.314	0.238
-0.3	*	*	*	*	*	*	1.000	0.600	0.407	0.294
-0.2	*	*	*	*	*	*	*	1.000	0.579	0.385
-0.1	*	*	*	*	*	*	*	*	1.000	0.556
0.0	*	*	*	*	*	*	*	*	*	1.000

The *mentions that this factor could not be calculated according to condition $\rho_1 > \rho_2$ which guarantees a positive value of the correction factor.

Table 2: Correction factor for $k = 40$ and 80 and for negative values of ρ_1 and ρ_2.

K=40										
ρ_2	ρ_1									
	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
-0.9	1.00	0.31	0.18	0.12	0.09	0.07	0.05	0.04	0.03	0.03
-0.8	*	1	0.30	0.17	0.11	0.08	0.06	0.05	0.04	0.03
-0.7	*	*	1.00	0.29	0.16	0.11	0.08	0.06	0.04	0.04
-0.6	*	*	*	1.00	0.27	0.15	0.10	0.07	0.05	0.04
-0.5	*	*	*	*	1.00	0.26	0.14	0.09	0.06	0.05
-0.4	*	*	*	*	*	1.00	0.25	0.13	0.08	0.06
-0.3	*	*	*	*	*	*	1.00	0.23	0.12	0.08
-0.2	*	*	*	*	*	*	*	1.00	0.22	0.11
-0.1	*	*	*	*	*	*	*	*	1.00	0.20
0.00	*	*	*	*	*	*	*	*	*	1.00
K=80										
ρ_2	ρ_1									
	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
-0.9	1.00	0.18	0.10	0.06	0.05	0.03	0.03	0.02	0.02	0.014
-0.8	*	1.00	0.18	0.09	0.06	0.04	0.03	0.03	0.02	0.015
-0.7	*	*	1.00	0.17	0.09	0.06	0.04	0.04	0.022	0.018
-0.6	*	*	*	1.00	0.16	0.08	0.05	0.05	0.03	0.02
-0.5	*	*	*	*	1.00	0.15	0.08	0.07	0.03	0.024
-0.4	*	*	*	*	*	1.00	0.14	0.13	0.04	0.03
-0.3	*	*	*	*	*	*	1.00	0.13	0.06	0.04
-0.2	*	*	*	*	*	*	*	1.00	0.12	0.06
-0.1	*	*	*	*	*	*	*	*	1.00	0.11
0.00	*	*	*	*	*	*	*	*	*	1.00

The *mentions that this factor could not be calculated according to condition $\rho_1 > \rho_2$ which guarantees a positive value of the correction factor.

Table 3: correction factor for $k = 2$ and 8 and for positive values of ρ_1 and ρ_2.

K=2										
ρ_2	ρ_1									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	1.000	0.818	0.667	0.539	0.429	0.333	0.250	0.177	0.111	0.053
0.1	*	1.000	0.800	0.636	0.500	0.385	0.286	0.200	0.125	0.059
0.2	*	*	1.000	0.778	0.600	0.455	0.333	0.231	0.143	0.067
0.3	*	*	*	1.000	0.750	0.556	0.400	0.273	0.167	0.077
0.4	*	*	*	*	1.000	0.714	0.500	0.333	0.200	0.091
0.5	*	*	*	*	*	1.000	0.667	0.429	0.250	0.111
0.6	*	*	*	*	*	*	1.000	0.600	0.333	0.143
0.7	*	*	*	*	*	*	*	1.000	0.500	0.200
0.8	*	*	*	*	*	*	*	*	1.000	0.333
0.9	*	*	*	*	*	*	*	*	*	1.000
K=8										
ρ_2	ρ_1									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	1.000	0.529	0.333	0.226	0.158	0.111	0.077	0.051	0.030	0.0137
0.1	*	1.000	0.500	0.304	0.200	0.135	0.091	0.059	0.035	0.015
0.2	*	*	1.000	0.467	0.273	0.172	0.111	0.070	0.040	0.018
0.3	*	*	*	1.000	0.428	0.238	0.143	0.086	0.048	0.020
0.4	*	*	*	*	1.000	0.385	0.200	0.111	0.059	0.024
0.5	*	*	*	*	*	1.000	0.333	0.158	0.077	0.030
0.6	*	*	*	*	*	*	1.000	0.273	0.111	0.040
0.7	*	*	*	*	*	*	*	1.000	0.200	0.059
0.8	*	*	*	*	*	*	*	*	1.000	0.111
0.9	*	*	*	*	*	*	*	*	*	1.000

The * mentions that this factor could not be calculated according to condition $\rho_1 > \rho_2$, which guarantees a positive value of the correction factor .

Table 4: correction factor for $k = 40$ and 80 and for positive values of ρ_1 and ρ_2.

K=40										
ρ_2	ρ_1									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	1.000	0.18	0.091	0.06	0.04	0.024	0.016	0.011	0.006	0.003
0.1	*	1.000	0.17	0.081	0.05	0.030	0.02	0.012	0.007	0.003
0.2	*	*	1.000	0.15	0.07	0.040	0.024	0.015	0.008	0.004
0.3	*	*	*	1.000	0.13	0.060	0.032	0.018	0.01	0.004
0.4	*	*	*	*	1.000	0.111	0.05	0.024	0.012	0.005
0.5	*	*	*	*	*	1.000	0.09	0.04	0.016	0.006
0.6	*	*	*	*	*	*	1.000	0.07	0.024	0.008
0.7	*	*	*	*	*	*	*	1.000	0.05	0.012
0.8	*	*	*	*	*	*	*	*	1.000	0.024
0.9	*	*	*	*	*	*	*	*	*	1.000
K=80										
ρ_2	ρ_1									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	1.000	0.101	0.048	0.028	0.018	0.012	0.008	0.005	0.003	0.001
0.1	*	1.000	0.091	0.042	0.024	0.015	0.010	0.006	0.004	0.002
0.2	*	*	1.000	0.081	0.036	0.020	0.012	0.007	0.004	0.002
0.3	*	*	*	1.000	0.070	0.030	0.016	0.009	0.005	0.002
0.4	*	*	*	*	1.000	0.059	0.024	0.012	0.006	0.003
0.5	*	*	*	*	*	1.000	0.048	0.018	0.008	0.003
0.6	*	*	*	*	*	*	1.000	0.036	0.012	0.004
0.7	*	*	*	*	*	*	*	1.000	0.024	0.006
0.8	*	*	*	*	*	*	*	*	1.000	0.012
0.9	*	*	*	*	*	*	*	*	*	1.000

The * mentions that this factor could not be calculated according to condition $\rho_1 > \rho_2$, which guarantees a positive value of the correction factor .

Table 5: correction factor for $k = 2$ and 8 and for positive values of ρ_1 and negative values of ρ_2.

K=2										
ρ_2	ρ_1									
	0.00	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-0.9	0.357	0.310	0.267	0.226	0.188	0.152	0.118	0.086	0.056	0.027
-0.8	0.385	0.333	0.286	0.241	0.200	0.161	0.125	0.091	0.059	0.029
-0.7	0.417	0.360	0.308	0.259	0.214	0.172	0.133	0.097	0.063	0.030
-0.6	0.455	0.391	0.333	0.280	0.231	0.185	0.143	0.103	0.067	0.032
-0.5	0.500	0.429	0.364	0.304	0.250	0.200	0.154	0.111	0.071	0.034
-0.4	0.556	0.474	0.400	0.333	0.273	0.217	0.167	0.120	0.077	0.037
-0.3	0.625	0.529	0.444	0.368	0.300	0.238	0.182	0.130	0.083	0.040
-0.2	0.714	0.600	0.500	0.412	0.333	0.263	0.200	0.143	0.091	0.044
-0.1	0.833	0.692	0.571	0.467	0.375	0.294	0.222	0.158	0.100	0.048
0.00	1.000	0.818	0.667	0.539	0.429	0.333	0.250	0.177	0.111	0.053
K=8										
ρ_2	ρ_1									
	0.00	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-0.9	0.122	0.101	0.083	0.068	0.055	0.043	0.032	0.023	0.015	0.007
-0.8	0.135	0.111	0.091	0.074	0.059	0.046	0.035	0.024	0.0154	0.0072
-0.7	0.152	0.123	0.100	0.081	0.064	0.0495	0.037	0.026	0.016	0.008
-0.6	0.172	0.139	0.111	0.089	0.070	0.054	0.040	0.028	0.018	0.0082
-0.5	0.200	0.158	0.125	0.099	0.077	0.059	0.044	0.030	0.019	0.0089
-0.4	0.238	0.184	0.143	0.111	0.086	0.065	0.048	0.033	0.02	0.0095
-0.3	0.294	0.220	0.167	0.127	0.097	0.073	0.053	0.036	0.022	0.010
-0.2	0.385	0.273	0.200	0.149	0.111	0.082	0.059	0.040	0.024	0.011
-0.1	0.556	0.360	0.250	0.180	0.130	0.094	0.067	0.045	0.027	0.012
0.00	1.000	0.529	0.333	0.226	0.158	0.111	0.077	0.051	0.030	0.014

Table 6: correction factor for $k = 40$ and 80 and for positive values of ρ_1 and negative values of ρ_2.

K=40										
ρ_2	ρ_1									
	0.00	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-0.9	0.027	0.022	0.018	0.014	0.011	0.009	0.0066	0.0047	0.0029	0.0014
-0.8	0.030	0.024	0.020	0.016	0.012	0.0095	0.0071	0.0050	0.0031	0.0015
-0.7	0.035	0.027	0.022	0.017	0.014	0.010	0.0076	0.0053	0.0033	0.0016
-0.6	0.040	0.031	0.024	0.019	0.015	0.011	0.008	0.0057	0.0036	0.0017
-0.5	0.048	0.036	0.028	0.021	0.016	0.012	0.009	0.0062	0.0038	0.0018
-0.4	0.059	0.043	0.032	0.024	0.018	0.014	0.010	0.0068	0.0041	0.0019
-0.3	0.077	0.053	0.039	0.028	0.021	0.015	0.011	0.0074	0.0045	0.0021
-0.2	0.111	0.070	0.048	0.034	0.024	0.018	0.012	0.0083	0.0050	0.0023
-0.1	0.200	0.101	0.063	0.042	0.029	0.020	0.014	0.0093	0.0055	0.0025
0.00	1.000	0.184	0.091	0.055	0.036	0.024	0.016	0.0110	0.0062	0.0028
K=80										
ρ_2	ρ_1									
	0.00	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-0.9	0.014	0.011	0.009	0.007	0.0057	0.004	0.003	0.0023	0.0015	0.0007
-0.8	0.015	0.012	0.010	0.008	0.0062	0.005	0.0036	0.0025	0.0016	0.00073
-0.7	0.018	0.014	0.011	0.009	0.0068	0.0052	0.0038	0.0027	0.0017	0.00078
-0.6	0.020	0.016	0.012	0.010	0.007	0.006	0.004	0.0029	0.0018	0.00083
-0.5	0.024	0.018	0.014	0.011	0.008	0.0062	0.0045	0.0031	0.0019	0.00089
-0.4	0.030	0.022	0.016	0.012	0.010	0.007	0.005	0.0034	0.0021	0.00096
-0.3	0.040	0.027	0.020	0.014	0.011	0.008	0.0055	0.0037	0.0023	0.00104
-0.2	0.059	0.036	0.024	0.017	0.012	0.009	0.0062	0.0041	0.0025	0.0011
-0.1	0.111	0.053	0.032	0.021	0.015	0.010	0.0071	0.0047	0.0028	0.0013
0.00	1.000	0.101	0.048	0.028	0.018	0.012	0.008	0.0053	0.0031	0.0014

3. Conclusions

In this study, we calculated the expected mean squares of random errors and treatments of the one-way random effect model with the correlated random errors within each group and the correlated errors across groups. We discussed the distribution of the test statistic F in the case of a correlation in the limits of random error within each group and between groups. The correction factor was derived and it was used to correct the statistic F. We also presented tables including the value of the correction factor for different values of correlation coefficients. Three correlation coefficient value forms were introduced in these tables, including both correlation coefficients negative, positive or ρ_1 positive, and ρ_2 negative.
A possible future extension is to conduct a simulation study to investigate the effect of correlations existence in the errors between groups on type I and II error probabilities and also introduce an application.

References

[1] H. Sahai and M. Ibrahim Ageel, The analysis of variance: fixed, random and mixed models, Springer Science & Business Media, (2012).

[2] D. Selvamuthu and D. Das, Introduction to statistical methods, design of experiments and statistical quality control, Springer Singapore, (2018).

[3] H. Sahai and M. Miguel Ojeda, Analysis of Variance for Random Models: Volume II: Unbalanced Data Theory, Methods, Applications, and Data Analysis, Springer, (2005).

[4] R. Littell, W. Stroup, and R. Freund, SAS for Linear Models, Fourth Edition (4th ed.), SAS Publishing, (2002).

[5] T. Hill and P. Lewicki, Statistics: Methods and Applications, StatSoft, Inc, (2005).

[6] Z. Abdulhussein Al-kaabawi, Expected mean squares for 4-way crossed model with balanced correlated data," Journal of Basrah Researches ((Sciences)), 36(3), (2010), 45–71.

[7] R. J. Pavur and J. M. Davenport, The (large) effect of (small) correlations in anova, and correction procedures, American Journal of Mathematical and Management Sciences, 5(1-2), (1985), pp: 77–92.

[8] A. S. AL-Mouel, K. J. AL-Abdulla, and Z. A. AL-Rabeaa, Type i error rates for multiple comparison procedures with dependent data for 3-way nested anova model, Journal of Basrah Researches, 20(1), (1999), 39–48.

[9] R. Aylmer Fisher, Statistical methods for research workers. biological monographs and manuals. no. v. Statistical methods for research workers, Biological monographs and manuals. No. V. (11th ed.), (1950).

[10] A. Donner, A review of inference procedures for the intraclass correlation coefficient in the one-way random effects model, International Statistical Review/Revue Internationale de Statistique, 54(1), (1986), 67–82.

[11] J. Smith and T. Lewis, Determining the effects of intraclass correlation on factorial experiments," Communications in Statistics-Theory and Methods, 9(13), (1980), 1353–1364.

[12] R. J. Pavur and T. Levis, Unbiased f tests for factorial experiments for correlated data, Communications in Statistics-Theory and Methods, 12(7), (1983), 829–840.

[13] S. M. Scariano and J. M. Davenport, "Corrected f-tests in the general linear model, Communications in Statistics-Theory and Methods, 13(25), (1984), 3155–3172.

[14] Z. Abdulhussein Al-kaabawi, The effect of dependent data on type i error rates for multiple comparison procedures for 3-way crossed balanced model, basrah journal of science, 25(1), (2007), 73–85.

[15] R. J. Pavur, Type i error rates for multiple comparison procedures with dependent data, The American Statistician, 42(3), (1988), 171–173.

[16] A. Mallick, P. Subbaiah, and G. Xia, Modified testing procedures and multiple comparison methods for anova with ar (1) correlated errors, Computational and Mathematical Methods, 2(2), (2020), e1076.

[17] A. H. Andersen, E. B. Jensen, and G. Schou, Two-way analysis of variance with correlated errors, International Statistical Review/Revue Internationale de Statistique, 49(2), (1981), 153–167.

[18] R. Lund, G. Liu, and Q. Shao, A new approach to anova methods for autocorrelated data, The American Statistician, 70(1), (2016), 55–62.

[19] W. G. Cochran, The distribution of quadratic forms in a normal system, with applications to the analysis of covariance, In Mathematical Proceedings of the Cambridge Philosophical Society, 30(2), (1934), 178–191.

[20] Z. A. AL-Kaabawi and R. D. Abdul-wahab, Expected mean squares for 3-way crossed model with unbalanced correlated data, Basrah Journal of Science, 30(1), (2012), 97–111.

[21] R. D. Abdul-Wahab, Expected mean squares for 4-way crossed model with unbalanced correlated data, Basrah Journal of Science, 31(1), (2013), 1–27.