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# Novel Transform for Mathematical Systems of Second Order Whether Under or Not Initial Conditions

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ABSTRACT

In this work, the formulas of set solution for linear mathematical systems of the second order are derived whether subject to or not subject to initial conditions. In addition to a set of supporting examples have been resolved.

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## 1. Introduction

Many applied mathematical systems, whether or not subject to initial conditions, have wide applications in other sciences. Recently, linear mathematical systems have been used to calculate the costs of activities, which facilitate

The work of accountants and auditors, as well as predicting future costs for projects and preparing a budget for different institutions[2].

Moreover, integral transformations are practical and helpful strategies for solving linear system ofequations. These systems difficult to solve in traditional ways which integral transformations are converting them into algebraic equations that are easy to solve [6,7].

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The most well-known transformations used to solve systems of equations are the Laplace and Fourier transforms [8,9]. Kilicman published a Novel transformation for solving various differential equations as constant heat transfer in 2016 [10].Many authors [1,3,4,5] employed Novel transformation to solve many types of differential equations, which have more applications in other areas.

## 2. Basic definitions and properties of Novel(Abboodh) transform

*The following definition of Novel(Aboodh) transform for the function Ḩ (t), t > 0:*

$$N(Ḩ(t)) = \frac{1}{q}\int_0^{\infty} e^{-qt}\, Ḩ(t)\,dt \quad t > 0,$$

*Ḩ(t) is a real function,* $\frac{e^{-qt}}{q}$ *is the kernel function, and* $N$ *is the operator of Novel transform.* $N^{-1}$ *is the inverse of Novel transform*

$$N^{-1}\{N(Ḩ(t))\} = Ḩ(t) \quad for \quad t > 0,$$

***Property (Linearity)** : If* $Ḩ_1(t), Ḩ_2(t), \dots, Ḩ_n(t)$ *have Novel transform then :*

$N(b_1Ḩ_1(t) \pm b_2Ḩ_2(t) \pm \cdots b_nḨ_n(t)) = b_1N(Ḩ_1(t)) \pm b_2N(Ḩ_2(t)) \pm \cdots \pm b_nN(Ḩ_n(t))$ $\quad b_1, b_2, \cdots, b_n$ *are constants*

***Theorem (1) [10]:** If the function* $Ḩ^n(t)$ *is the derivative of the function Ḩ(t) then its Novel transform is defined by :*

$$N(Ḩ'(t)) = \frac{1}{q} N(Ḩ(t)) - \frac{Ḩ(0)}{q},$$

$$\vdots \qquad \vdots$$

$$N(Ḩ^{(n)}(t)) = q^n N_i(Ḩ(t)) - q^{n-2}Ḩ(0) - q^{n-3}Ḩ'(0) - \cdots - Ḩ^{(n-2)}(0) - \frac{1}{q}Ḩ^{(n-1)}.$$

***Table 1 . The Novel transformation for several functions***

| ID | Function Ḩ (t) | $N(Ḩ(t)) = \frac{1}{q}$ L( Ḩ (t)) | |
|---|---|---|---|
| 1 | $b$ | $\frac{b}{q^2}$ | $b \in R, q > 0$ |
| 2 | $t^n$ | $\frac{n!}{q(q^{n+1})}$ | $n \in N, q > 0$ |
| 3 | $e^{bt}$ | $\frac{1}{q(q-b)}$ | $b \neq q, q > 0$ |
| 4 | $\sin bt$ | $\frac{a}{q(q^2+b)}$ | $q > 0$ |
| 5 | $\cos bt$ | $\frac{1}{(q^2-b^2)}$ | $b \neq q$ |
| 6 | $\sinh bt$ | $\frac{a}{q(q^2-b^2)}$ | $b \neq q, q \neq 0$ |
| 7 | $\cosh bt$ | $\frac{1}{(q^2-b^2)}$ | $b \neq q$ |

## 3. The formula of general solution for system of nth-order for dimension n.

*Linear system of n-order in dimension n has the form.*

$$Ḥ_1^{(r)} = b_{11}Ḥ_1(t) + b_{12}Ḥ_2(t) + \cdots + b_{1j}Ḥ_n(t) + v_1(t)$$

$$Ḥ_2^{(r)} = b_{21}Ḥ_1(t) + b_{22}Ḥ_2(t) + \cdots + b_{2j}Ḥ_n(t) + v_2(t)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$Ḥ_n^{(r)} = b_{i1}Ḥ_1(t) + b_{i2}Ḥ_2(t) + \cdots + b_{ij}Ḥ_n(t) + v_n(t), \text{ where}$$

$$Ḥ^{(r)} = \begin{pmatrix} \frac{d^nḤ_1(t)}{dt^r} \\ \frac{d^nḤ_2(t)}{dt^r} \\ \vdots \\ \frac{d^nḤ_m(t)}{dt^r} \end{pmatrix}, \quad B = (bij) = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \dots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} \end{pmatrix},$$

$$Ḥ = \begin{pmatrix} Ḥ_1(t) \\ Ḥ_2(t) \\ \vdots \\ Ḥ_n(t) \end{pmatrix}, \quad v = \begin{pmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \end{pmatrix},$$

*which can be stated as the following formula:*

$$\begin{pmatrix} \frac{d^rḤ_1(t)}{dt^r} \\ \frac{d^rḤ_2(t)}{dt^r} \\ \vdots \\ \frac{d^rḤ_n(t)}{dt^r} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1j} \\ b_{21} & b_{22} & \dots & b_{2j} \\ \vdots & \vdots & \dots & \vdots \\ b_{i1} & b_{i2} & \dots & b_{ij} \end{pmatrix} \begin{pmatrix} Ḥ_1(t) \\ Ḥ_2(t) \\ \vdots \\ Ḥ_n(t) \end{pmatrix} + \begin{pmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \end{pmatrix}$$

*or*

$$Ḥ^{(r)}(t) = BḤ(t) + v(t) \qquad (1)$$

*If* $v(t) = 0$, *then the system* $(1)$ *is called homogenous system,* $b_{ij}$, *,i,j =1,2…,n real constants.*

### 3.1 Solutions of second order non-homogeneous systems under initial conditions.

*A non-homogeneous system of second order has the form .*

$$Ḥ''(t) = BḤ(t) + v(t) \qquad (2)$$

*Novel transformation for both sides*

$$q^2\mathcal{N}(Ḥ_1) - Ḥ_1(0) - \frac{Ḥ_1'(0)}{q} = b_{11}\mathcal{N}(Ḥ_1) + b_{12}\mathcal{N}(Ḥ_2) + \cdots + b_{1n}\mathcal{N}(Ḥ_n) + \mathcal{N}(v_1)$$

$$q^2\mathcal{N}(Ḥ_2) - Ḥ_2(0) - \frac{Ḥ_2'(0)}{q} = b_{21}\mathcal{N}(Ḥ_1) + b_{22}\mathcal{N}(Ḥ_2) + \dots + b_{2n}\mathcal{N}(Ḥ_n) + \mathcal{N}(v_2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$q^2\mathcal{N}(Ḥ_n) - Ḥ_n(0) - \frac{Ḥ_n'(0)}{q} = b_{n1}\mathcal{N}(Ḥ_1) + b_{n2}\mathcal{N}(Ḥ_2) + \cdots + b_{nn}\mathcal{N}(Ḥ_n) + \mathcal{N}(v_n),$$

*where* $Ḥ_1(0), Ḥ_2(0), \cdots, Ḥ_n(0)$ *and* $Ḥ_1'(0), Ḥ_2'(0), \cdots, Ḥ_n'$ *(0) are known initial conditions.*

$$(q^2 - b_{11})\mathcal{N}(Ḥ_1) - b_{12}\mathcal{N}(Ḥ_2) - \cdots - b_{1n}\mathcal{N}(Ḥ_n) = Ḥ_1(0) + \frac{Ḥ_1'(0)}{q} - b_{11}\frac{Ḥ_1(0)}{q} - b_{12}\frac{Ḥ_2(0)}{q} - \cdots - b_{1n}\frac{Ḥ_n(0)}{q} + \mathcal{N}(v_1)$$

$$(q^2 - b_{22})\mathcal{N}(Ḥ_2) - b_{21}\mathcal{N}(Ḥ_1) - \cdots - b_{2n}\mathcal{N}(Ḥ_n) = Ḥ_2(0) + \frac{Ḥ_2'(0)}{q} - b_{21}\frac{Ḥ_1(0)}{q} - b_{22}\frac{Ḥ_2(0)}{q} - \cdots - b_{2n}\frac{Ḥ_n(0)}{q} + \mathcal{N}(v_2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$(\text{ɋ}^2 - b_{nn})N(\text{Ḩ}_n) - b_{n1}N(\text{Ḩ}_1) - \cdots - b_{n2}N(\text{Ḩ}_2) = \text{Ḩ}_n(0) + \frac{\text{Ḩ}'_n(0)}{\text{ɋ}} - b_{n1}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{n2}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{nn}\frac{\text{Ḩ}_n(0)}{\text{ɋ}} + N(v_n).$

*Additionally, a straightforward calculation to obtain* $N(\text{Ḩ}_1(t)), \cdots, N(\text{Ḩ}_n(t))$.

$$\Delta = \begin{vmatrix} (\text{ɋ}^2 - b_{11}) & -(b_{12}) & \cdots & -(b_{1n}) \\ -(b_{21}) & (\text{ɋ}^2 - b_{22}) & \cdots & -(b_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ -(b_{n1}) & -(b_{n2}) & \cdots & (\text{ɋ}^2 - b_{nn}) \end{vmatrix}$$

*Also*

$$N(\text{Ḩ}_1) = \frac{1}{\Delta}\begin{vmatrix} \delta_1 & -(b_{12}) & \dots & -(b_{1n}) \\ \delta_2 & (\text{ɋ}^2 - b_{22}) & \dots & -(b_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ \delta_n & -(b_{n2}) & \dots & (\text{ɋ}^2 - b_{nn}) \end{vmatrix}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$N(\text{Ḩ}_n) = \frac{1}{\Delta}\begin{vmatrix} (\text{ɋ}^2 - b_{11}) & -(b_{12}) & \dots & \delta_1 \\ -(b_{21}) & (\text{ɋ}^2 - b_{22}) & \dots & \delta_2 \\ & \vdots & & \\ -(b_{n1}) & -(b_{n2}) & \dots & \delta_n \end{vmatrix},$$

*where,*

$\delta_1 = \text{Ḩ}_1(0) + \frac{\text{Ḩ}'_1(0)}{\text{ɋ}} - b_{11}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{12}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{1n}\frac{\text{Ḩ}_n(0)}{\text{ɋ}} + N(v_1)$

$\delta_2 = \text{Ḩ}_2(0) + \frac{\text{Ḩ}'_2(0)}{\text{ɋ}} - b_{21}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{22}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{2n}\frac{\text{Ḩ}_n(0)}{\text{ɋ}} + N(v_2)$

$\vdots$

$\delta_n = \text{Ḩ}_n(0) + \frac{\text{Ḩ}'_n(0)}{\text{ɋ}} - b_{n1}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{n2}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{nn}\frac{\text{Ḩ}_n(0)}{\text{ɋ}} + N(v_n)$

*After taking the inverse of Novel transformation for* $(N(\text{Ḩ}_i))$, $i = 1,2,3, \dots . n$, *obtained the set solution of system (2).*

### 3.2 Solutions of second order homogeneous systems under initial conditions.

*From the previous formula (2), if the vector* $v(t) = 0$, *so the solution has the from :*

$$\Delta = \begin{vmatrix} (\text{ɋ}^2 - b_{11}) & -(b_{12}) & \cdots & -(b_{1n}) \\ -(b_{21}) & (\text{ɋ}^2 - b_{22}) & \cdots & -(b_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ -(b_{n1}) & -(b_{n2}) & \cdots & (\text{ɋ}^2 - b_{nn}) \end{vmatrix}$$

*Also,*

$$N(\text{Ḩ}_1) = \frac{1}{\Delta}\begin{vmatrix} \vartheta_1 & -(b_{12}) & \cdots & -(b_{1n}) \\ \vartheta_2 & (\mathcal{P}^2 - b_{22}) & \cdots & -(b_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ \vartheta_n & -(b_{n2}) & \cdots & (\mathcal{P}^2 - b_{nn}) \end{vmatrix}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$N(\text{Ḩ}_n) = \frac{1}{\Delta}\begin{vmatrix} (\text{ɋ}^2 - b_{11}) & -(b_{12}) & \cdots & \vartheta_1 \\ -(b_{21}) & (\text{ɋ}^2 - b_{22}) & \cdots & \vartheta_2 \\ \vdots & \vdots & \cdots & \vdots \\ -(b_{n1}) & -(b_{n2}) & \cdots & \vartheta_n \end{vmatrix}$$

*Where*

$\vartheta_1 = \text{Ḩ}_1(0) + \frac{\text{Ḩ}'_1(0)}{\text{ɋ}} - b_{11}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{12}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{1n}\frac{\text{Ḩ}_n(0)}{\text{ɋ}}$

$\vartheta_2 = \text{Ḩ}_2(0) + \frac{\text{Ḩ}'_2(0)}{\text{ɋ}} - b_{21}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{22}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{2n}\frac{\text{Ḩ}_n(0)}{\text{ɋ}}$

$\vdots \qquad \vdots \qquad \vdots$

$\vartheta_n = \text{Ḩ}_n(0) + \frac{\text{Ḩ}'_n(0)}{\text{ɋ}} - b_{n1}\frac{\text{Ḩ}_1(0)}{\text{ɋ}} - b_{n2}\frac{\text{Ḩ}_2(0)}{\text{ɋ}} - \cdots - b_{nn}\frac{\text{Ḩ}_n(0)}{\text{ɋ}}$

*In similar way by taking the inverse of the Novel transform for* $N(Ḩ_i), i = 1,2 \ldots, n$ *yields the set solution of system homogeneous.*

***Notation 1 [10 ]:*** *The number of arbitrary constants which can appear in the general solution of linear system , such as:*

$$b_{11}(D)Ḩ_{11} + b_{12}(D)Ḩ_{13} + \cdots + b_{1n}(D)Ḩ_{1n} = v_1(t)\,,$$
$$b_{21}(D)Ḩ_{21} + b_{22}(D)Ḩ_{22} + \cdots + b_{2n}(D)Ḩ_{2n} = v(t),$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$b_{m1}(D)Ḩ_{m1} + b_{m2}(D)Ḩ_{\ 2} + \cdots, b_{mn}(D)Ḩ_{mn} = v_n(t),$$

*where D is an operator which is represent* $D = \frac{d}{dt}$ , $b_{11}(D) \ldots b_{mn}(D)$ *are functions of D,* $x_{ij}$ *and t,* $i, j = 1,2, \ldots, n$ *represent dependent and independent variable in the system, respectively.*

*Can be equal to the degree of D in the determent.*

$$\begin{vmatrix} b_{11}(D) & \cdots & b_{1n}(D) \\ \vdots & \cdots & \vdots \\ b_{m1}(D) & \cdots & b_{mn}(D) \end{vmatrix} = \Delta$$

*If Δ ≡ 0 then the set of the solutions is not independent and it is out of our studying so assume Δ is not zero.*

## 4. Solutions of second order systems without subjected to initial conditions.

*In this section , general formula for the homogeneous and non-homogeneous system of second order in dimension n× n without subjected to any initial conditions are derived .*

*A non-homogeneous system has the form*

$$Ḩ''(t) = BḨ(t) + v(t) \qquad (3)$$

*without subjected to any initial conditions.*

*Where* $Ḩ'' = \begin{pmatrix} \frac{d^2Ḩ_1}{dt^2} \\ \frac{d^2Ḩ_2}{dt^2} \\ \vdots \\ \frac{d^2Ḩ_n}{dt^2} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{22} & \ldots & b_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ b_{n1} & b_{n2} & \ldots & b_{nn} \end{pmatrix}$, $Ḩ(t) = \begin{pmatrix} Ḩ_1(t) \\ Ḩ_2(t) \\ \vdots \\ Ḩ_n(t) \end{pmatrix}$, $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$ *so,*

$$\begin{pmatrix} \frac{d^2Ḩ_1}{dt^2} \\ \frac{d^2Ḩ_2}{dt^2} \\ \vdots \\ \frac{d^2Ḩ_n}{dt^2} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{22} & \ldots & b_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ b_{n1} & b_{n2} & \ldots & b_{nn} \end{pmatrix} \begin{pmatrix} Ḩ_1(t) \\ Ḩ_2(t) \\ \vdots \\ Ḩ_n(t) \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

*Taking Novel transformation of both sides, it has*

$$q^2 N(Ḩ_1) - Ḩ_1(0) - \frac{Ḩ_1'(0)}{q} = b_{11}N(Ḩ_1) + b_{12}N(Ḩ_2) + \cdots + b_{1n}N(w_n) + N(v_1)$$
$$q^2 N(Ḩ_2) - Ḩ_2(0) - \frac{Ḩ_2'(0)}{q} = b_{21}N(Ḩ_1) + b_{22}N(Ḩ_2) + \cdots + b_{2n}N(Ḩ_n) + N(v_2)$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$q^2 N(Ḩ_n) - Ḩ_n(0) - \frac{Ḩ_n'(0)}{q} = b_{n1}N(Ḩ_1) + b_{n2}N(Ḩ_2) + \cdots + b_{nn}N(Ḩ_n) + N(v_n),$$

$Ḩ_1(0), Ḩ_2(0), \cdots, Ḩ_n(0)$ *and* $Ḩ_1'(0), Ḩ_2'(0), \cdots, Ḩ_n'$ *(0) are unknown initial conditions, after simplification the terms :*

$$(q^2 - b_{11})N(Ḩ_1) - (b_{12})N(Ḩ_2) - \cdots - (b_{1n})N(Ḩ_n) = Ḩ_1(0) + \frac{Ḩ_1'(0)}{q} - N(v_1)$$
$$(q^2 - b_{22})N(Ḩ_2) - (b_{21})N(Ḩ_1) - \cdots - (b_{2n})N(Ḩ_n) = Ḩ_2(0) + \frac{Ḩ_2'(0)}{q} - N(v_2)$$
$$\vdots \qquad \vdots \qquad \vdots$$

$(q^2 - b_{nn})N(H_n) - (b_{n1})N(H_1) - \cdots - (b_{n2})N(H_2) = H_n(0) + \frac{H_n'(0)}{q} - N(v_n)$ .

*We can solve the above system and the solution is*

$$N\{H_j(t)\} = \frac{X_j(q)}{Y_j(q)}, j = 1,2, \dots n \quad Y_j(q) \neq 0 \qquad (4)$$

*where* $Y_j(q)$ *is matrix of dimension n×n of* q *represents denominator of Novel transformation of the function* $H_j(t)$ *and* $N(v_j)$. $X_j(q)$ *is also matrix of dimension n×n of H which represents extend of Novel transformation for* $N(v_j)$ *and unknown initial conditions* $H_j(0)$.

*The solution of system(3) can be obtained by taking inverse Novel transformation for(4),as:*

$$H_j(t) = N^{-1}\left\{\frac{X_j(q)}{Y_j(q)}\right\} \quad j = 1,2, \dots, n \qquad Y_j(q) \neq 0 \qquad (5)$$

*From the previous formula (3), if the vector* $v(t) = 0$*, so*

$$(q^2 - b_{11})N(H_1) - (b_{12})N(H_2) - \cdots - (b_{1n})N(H_n) = H_1(0) + \frac{H_1'(0)}{q}$$
$$(q^2 - b_{22})N(H_2) - (b_{21})N(H_1) - \cdots - (b_{2n})N(H_n) = H_2(0) + \frac{H_2'(0)}{q}$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$(q^2 - b_{nn})N(H_n) - (b_{n1})N(H_1) - \cdots - (b_{n2})N(H_2) = H_n(0) + \frac{H_n'(0)}{q}$$

$H_1(0), H_2(0), \cdots, H_n(0)$ *and* $H_1'(0), H_2'(0), \cdots, H_n'(0)$ *are unknown initial conditions, similar with pervious steps can be obtained :*

$$N\{H_j(t)\} = \left\{\frac{X_j(q)}{Y_j(q)}\right\}, \quad j = 1,2, \dots n, Y_j(q) \neq 0 \qquad (6)$$

*The solution of homogenous system can be obtained by taking inverse Novel transformation for (6), as:*

$$H(t) = N^{-1}\left\{\frac{X_j(q)}{Y_j(q)}\right\}, \quad j = 1,2, \dots n, Y_j(q) \neq 0$$

## 5. Applications

*In this section, some of the supported systems are resolved depending on the previously derived formats*

***Example(1):*** *To solve the system*

$$H_1''(t) = -3H_1(t) - 2H_2(t) \qquad (7)$$

$$H_2''(t) = 4H_1(t) + 3H_2(t)$$

$H_1(0) = 1, H_2(0) = 0$ *and* $H_1'(0) = 0, H_2'(0) = 1$

***Solution:*** *Form formula in section 3.1 , yields:*

$$N(H_1(t)) = \frac{2q}{q(q^2+1)} + \frac{1}{q(q^2+1)} - \frac{q}{q(q^2-1)} - \frac{1}{q(q^2-1)}$$

*Now, taking the inverse of Novel transformation for both sides of the above equation*

$H_1(t) = 2\cos(t) + \sin(t) - \cosh(t) - \sinh(t)$.

*In similar way,* $N(H_2(t))$ *can be obtained by :*

$$N(H_2(t)) = \frac{-2q}{q(q^2+1)} - \frac{1}{q(q^2+1)} + \frac{2q}{q(q^2-1)} + \frac{2}{q(q^2-1)}.$$

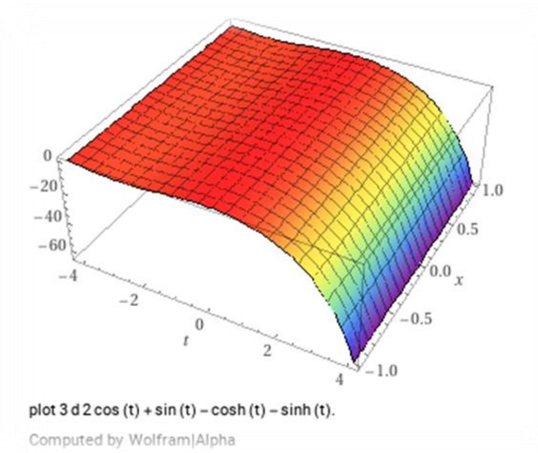
*Taking the inverse of Novel transformation.*

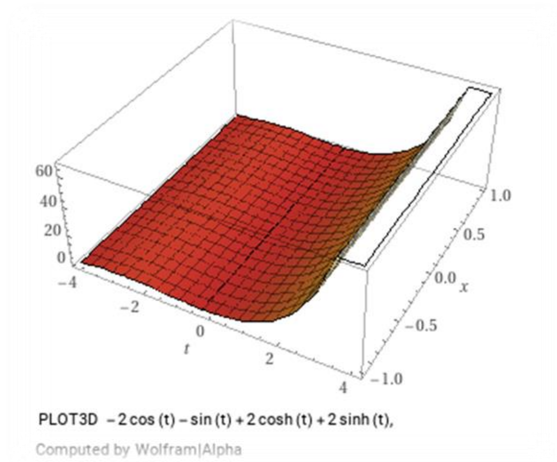
$H_2(t) = -2\cos(t) - \sin(t) + 2\cosh(t) + 2\sinh(t)$,

*Table (2) The solution of system (7) under the value of t .*

| *t* | $H_1(t)$ | $H_2(t)$ |
|---|---|---|
| *t=1* | *-0.7011340317* | *3.41941586* |
| *t=2* | *-5.355374948* | *12.74443105* |
| *t=3* | *-18.0359419* | *38.12147882* |

*Figure (1) The set solution of system (7)*




*Figure(a) The solution of equation $H_1(t)$* *Figure(b) The solution of equation $H_2(t)$*

***Example(2):****To solve the non homogeneous system under initial conditions*

$$H_1''(t) - H_1(t) + H_2(t) = 2 \qquad (8)$$

$$H_2''(t) - H_1(t) + H_2(t) = 1$$

$$H_1(0) = 0,\ \ H_2(0) = 1,\ \ H_1'(0) = 1\ , H_2'(0) = 0.$$

***Solution:*** *Form formula in section 3.2 , yields:*

$$N(H_1(t)) = \frac{1}{q^4}\begin{vmatrix} \frac{2}{q^2} & 1 \\ 1+\frac{1}{q^2} & (q^2+1) \end{vmatrix} = \frac{1}{q^4}\left(2+\frac{2}{q^2}\right) - \left(1+\frac{1}{q^2}\right)$$

*Take the inverse of Novel transformation for* $N(H_1(t))$*:*

$$H_1(t) = \frac{1}{2}t^2 + \frac{1}{24}t^4$$

*In similar way,* $N(H_2(t))$ *can be obtained by*

$$N(H_2(t)) = \frac{1}{q^4}\begin{vmatrix} (q^2-1) & \frac{2}{q^2} \\ -1 & 1+\frac{1}{q^2} \end{vmatrix} = \frac{1}{q^4}\left[\frac{(q^2-1)}{q^2} + (q^2-1) + \frac{2}{q^2}\right] = \frac{1}{q^2} + \frac{1}{q^6}$$

*The inverse of Novel transformation gives .*

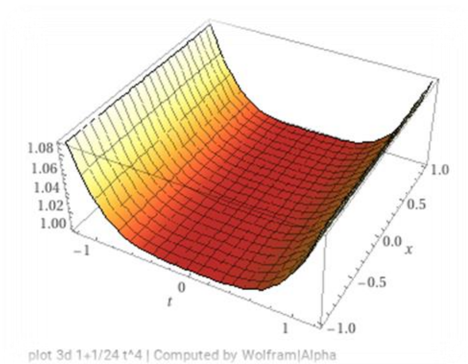
$$H_2(t) = 1 + \frac{1}{24}t^4$$

*Table (3)The solution of system (8) under the value of t .*

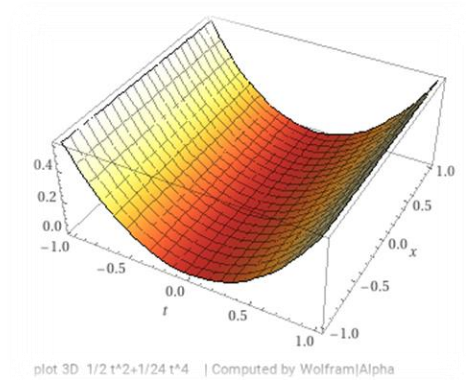
| *t* | $H_1(t)$ | $H_2(t)$ |
|---|---|---|
| *t=1* | *0.5416666667* | *1.041666667* |

| | | |
|---|---|---|
| *t=2* | *2.666666667* | *1.666666667* |
| *t=3* | *7.875* | *4.375* |

*Figure (2) The set solution of system (8).*




*Figure(a) The solution of equation $Ḥ_1(t)$*




*Figure(b) The solution of equation $Ḥ_2(t)$*

***Example(3):*** *To solve the second order system without subjected to initial conditions.*

$$Ḥ_1''(t) = -3Ḥ_1(t) - 2Ḥ_2(t) \qquad (9)$$

$$Ḥ_2''(t) = 4Ḥ_1(t) + 3Ḥ_2(t)$$

***Solution:*** *Compared the system (9) with system in section 4 and formula (6)*

$$Ɲ(Ḥ_1) = \frac{-2}{(q^2+3)}Ɲ(Ḥ_2) + \frac{Ḥ_1(0)}{(q^2+3)} + \frac{Ḥ_1'(0)}{q(q^2+3)}$$

$$Ɲ(Ḥ_2) = \frac{4qḤ_1(0)}{q(q^2-1)(q^2+1)} + \frac{4Ḥ_1'(0)}{q(q^2-1)(q^2+1)} + \frac{(q^3+3q)Ḥ_2(0)}{q(q^2-1)(q^2+1)} + \frac{(q^2+3)Ḥ_2'(0)}{q(q^2-1)(q^2+1)}$$

*using partition fractions for each items, such as :*

$$Ɲ(Ḥ_2) = \left(\frac{2Ḥ_1(0)}{(q^2-1)} - \frac{2Ḥ_1(0)}{(q^2+1)}\right) + \left(\frac{2Ḥ_1'(0)}{q(q^2-1)} - \frac{2Ḥ_1'(0)}{q(q^2+1)}\right) + \left(\frac{2Ḥ_2(0)}{(q^2-1)} - \frac{Ḥ_2(0)}{(q^2+1)}\right)\left(\frac{2Ḥ_2'(0)}{q(q^2-1)} - \frac{Ḥ_2'(0)}{q(q^2+1)}\right)$$

*Now, taking inverse Novel transformation for both side, get :*

$$Ḥ_2(t) = 2b_1\,cosh(t) - 2b_1\,cos(t) + 2b_2\,sinh(t) - 2b_2\,sin(t) + 2b_3 cosh\,(t) - b_3\,cos(t) + 2b_4\,sinh(t) - b_4\,sin(t)$$

*substitute $Ɲ(Ḥ_2)$ in $Ɲ(Ḥ_1)$ and using partition fractions :*

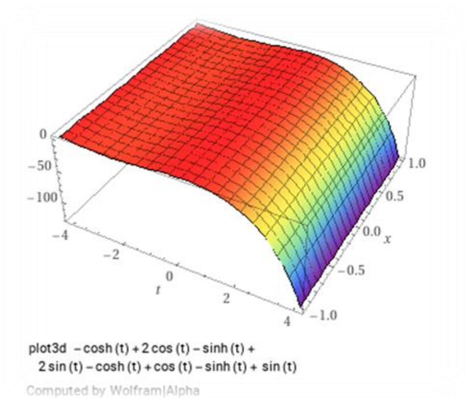
$$Ɲ(Ḥ_1) = \frac{-Ḥ_1(0)}{(q^2-1)} + \frac{2Ḥ_1(0)}{(q^2+1)} - \frac{Ḥ_1'(0)}{q(q^2-1)} + \frac{2Ḥ_1'(0)}{q(q^2+1)} - \frac{Ḥ_2(0)}{(q^2-1)} + \frac{Ḥ_2(0)}{(q^2+1)} - \frac{Ḥ_2'(0)}{q(q^2-1)} + \frac{Ḥ_2'(0)}{(q^2+1)}$$

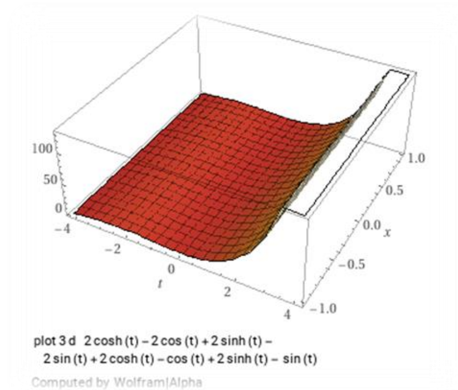
*Taking inverse Novel transformation to both side :*

$$Ḥ_1(t) = -b_1\,cosh(t) + 2b_1\,cos(t) - b_2\,sinh(t) + 2b_2\,sin(t) - b_3\,cosh(t) + b_3\,cos(t) - b_4\,sinh(t) + b_4\,sin(t)$$

*From Remark (1), the number of constants in general solution is equal to the degree of D, so the general solution of (5.17) has four constants $b_1$ , $b_2$ , $b_3$ and $b_4$ which represent the unknown initial conditions.*

*Figure (3) The set solution of system (9).*




*Figure(a) The solution of equation* $Ḫ_1(t)$ *Figure (b) The solution of equation* $Ḫ_2(t)$

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