Journal of Al-Qadisiyah for Computer Science and Mathematics Vol. 15(2) 2023 , pp Math. 152–160

University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Application of $\lambda-$ Fuzzy Measure and Fuzzy Integrals in The Admission Process for Doctoral Study in Iraqi Universities

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ARTICLE INFO

Article history:
Received: 15 /03/2023
Rrevised form: 30 /04/2023
Accepted: 02 /05/2023
Available online: 30 /06/2023

Keywords:

Fuzzy measure -making using Multicriteria, λ-Fuzzy measures, Choquet integral, Shilkret integral, PAN integral

ABSTRACT

One of the most crucial issues in higher education is the controls for applying and admission to postgraduate studies for the selection of applicants. The present paper discusses the application of λ–fuzzy measures and fuzzy integrals (Choquet integral, Shilkret integral, and PAN integral) to review the mechanisms of postgraduate admissions decision-making.

MSC.

https://doi.org/10.29304/jqcm.2023.15.2.1257

1. Introduction

In 1965, Lotfi Zadeh issued his fundamental article "Fuzzy Sets" which introduced the fuzzy set theory and fuzzy logic in general [1]. Crisp sets are defined in classical set theory by characteristic functions with values of zero or one, whereas membership grades with values between 0 and 1 have taken the place of sharp borders in fuzzy set theory [2]. Furthermore, Choquet introduced non-additive measures and their related integrals in 1953 [3] and Sugeno separately defined them in [4] to expand the traditional measure by substituting the non-additive characteristic for the additivity characteristic. The capacity to express the significance of many sources of information and ways of interacting with them is a key characteristic of fuzzy integrals and non-additive measures.

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Non-additive measures and fuzzy integrals have a wide range of uses, including in multi-criteria decision-making, image processing, information fusion, pattern recognition, and classification (see [9-12]).

This paper provides an application of a $\lambda-$fuzzy measure and a fuzzy integral such as the Choquet integral, Shilkret Integral, and PAN to rank doctoral applicants in the admission process and also gives the comparison between fuzzy integrals. The remaining parts of the paper are organized as described as follows: Section 2 gives a brief introduction to postgraduate application and admission requirements. Some definitions related to non-additive measures and integrals are given in section 3. Section 4 demonstrates the application of λ -fuzzy measures and the corresponding integrals in admission controls for doctoral study, and a case study. Section 5 contains conclusions.

2. An Overview of The Application and Admission Controls for Doctoral Studies

The controls for applying and admission to doctoral studies are controls issued by the Ministry of Higher Education and Scientific Research / Department of Research and Development. The task of the research and development department and the committee on study leaves is to supervise and monitor the results of nomination for study leave.
The admission mechanism for postgraduate studies is based on the weights for comparison between applicants for study leave inside Iraq and in accordance with the instructions for study leave. These weights include average, age, years of service, published research, and the scientific title as shown in table 1. below.

Table1- Comparison weights for leave applicants to study a PhD in Iraq

Average	**Maximum degree**	**Maximum degree**
99-90	30	
89-80	20	30
79-70	10	
Age	**Maximum degree**	**Maximum degree**
29-35	25	
36-40	20	
41-45	15	25
46-50	10	
Years of service	**Maximum degree**	**Maximum degree**
One degree for all year	20	20
Published researchs	**Maximum degree**	**Maximum degree**
Outside Iraq	Two marks for each research	10
Inside Iraq	One marks for each research	
Scientific title	**Maximum degree**	**Maximum degree**
Professor	15	
Assistant Professor	10	15
Teacher	8	
Assistant teacher	5	

3.Basic Concepts

The definitions of the integrals (Choquet, Shilkret, and PAN integral) needed for this research's use have been provided.

Definition 1 [5] Consider the finite set $\mathbb{X}=\{y_1, y_2, \dots, y_n\}$. A function $\mu_\lambda: 2^{\mathbb{X}} \to [0,\infty]$ is termed as λ-fuzzy measure if it meets the criteria below:

1- $\mu(\mathbb{X})=1$

2-If $\mathbb{U}, \mathbb{V} \in 2^{\mathbb{X}}$, then $\mu_\lambda(\mathbb{U} \cup \mathbb{V}) = \mu_\lambda(\mathbb{U}) + \mu_\lambda(\mathbb{V}) + \lambda\mu_\lambda(\mathbb{U})\mu_\lambda(\mathbb{V})$ with $\mathbb{U} \cap \mathbb{V} = \emptyset$.

Generally, it may be demonstrated that

$$\mu_\lambda(\{y_1, y_2, \dots, y_n\}) = \frac{1}{\lambda}\left[\prod_{i=1}^{n}(1+\lambda\mu_\lambda(\{y_i\}) - 1\right] \quad ,\text{if } \lambda \neq 0 \qquad (1)$$

λ could be calculated as follows:

$$1+\lambda = \prod_{i=1}^{n}(1+\lambda\mu_\lambda(\{y_i\}) \qquad (2)$$

Definition 2 [3] The Choquet integral (briefly, Choq-Int) of a function $f: \mathbb{X} \to [0,\infty]$ with regard to μ_λ (where μ_λ is λ-fuzzy measure on $\mathbb{X}$) is specified by:

$$\int C_{\mu_\lambda}(f) = \sum_{i=1}^{n}\left(f\left(y_{(i)}\right) - f\left(y_{(i-1)}\right)\right) \mu_\lambda\left(Z_{(i)}\right) \qquad (3)$$

Where, $f\left(y_{(i)}\right)$ is permuted to $0 \le f\left(y_{(1)}\right) \le f\left(y_{(2)}\right) \le \cdots f\left(y_{(n)}\right) \le 1$,
$Z_{(i)} = \left\{y_{(i)}, \dots, y_{(n)}\right\}$.

Definition 3 (The Shilkret integral) [6]

Let μ be a normalized fuzzy measure on $\mathbb{X}$ and f a function on $\mathbb{X}$ with range $=\{y_1, y_2, \dots, y_n\}$ the shilkret integral(briefly, Shil-Int) of a measurable function $f: \mathbb{X} \to R^+$ is given by

$$Sh_\mu(f) = \underset{y \in [0,\infty]}{Max}\left(y_i . \mu\left(\left\{x \mid f(x) \ge y_i\right\}\right)\right) \qquad (4)$$

Where $0 \le y_1 \le y_2 \le \cdots \le y_n \le 1$, and $\mu : \mathbb{U} \to [0,\infty]$ is a fuzzy measure

Definition 4(The PAN- integral) [7], [8]

The PAN integral(briefly, PAN-Int) of a measurable function $f: \mathbb{X} \to [0,\infty]$ is

$$(p)\int_{\mathbb{U}} f\, d\mu = \max\left\{\sum_{i=1}^{n} k_i\, \mu(\mathbb{U}_i) \middle| \sum_{i=1}^{n} b(k_i, \mathbb{U}_i) \le f,\ \text{where} (\mathbb{U}_i)_{i=1}^{n} \text{ is a partition of } \mathbb{X}\right\} \qquad (5)$$

$$\text{where } b(k,\mathbb{U})(\mathbb{X}) = \begin{cases} k & \text{if } c \in \mathbb{U}. \\ 0 & \text{elsewhere.} \end{cases}$$

4. Application of λ-Fuzzy Measures and Fuzzy Integrals in Admission Controls for Doctoral Studies

In this section, fuzzy integrals like Choq-Int, Shil-Int and PAN-Int are used to determine the ranking of for doctoral study applicants according to the weights mentioned in the table1

Case study

In this case study, data of 6 applicants for doctoral study/mathematics major at an Iraqi university were taken, according to comparison weights for doctoral study: average, age, years of service, published research. The calculation of the final score for each applicant according to the criteria is shown in the last column of table 2.

Table 2- The data of applicants according to postgraduate admission weight

Applicants	Average	Age	Year of service	Published research	Scientific title	Total
AP_1	20	20	14	10	10	74
AP_2	20	20	13	8	10	71
AP_3	20	15	20	7	10	72
AP_4	20	15	15	10	15	75
AP_5	20	15	20	8	10	73
AP_6	10	10	20	8	10	58

To apply fuzzy integrals, we must first establish the weight interdependencies using the -fuzzy measure. Take into consideration the various levels of significance included within the selected case study. We prioritize average, age, and years of service while placing less emphasis on other weights. As a result, we provide the following λ-fuzzy of various weights.
$\mu_\lambda(\{y_1\})$=μ_λ({Average})=0.4
$\mu_\lambda(\{y_2\})$=μ_λ({Age})=0.3
$\mu_\lambda(\{y_3\})$=μ_λ({Years of service})=0.2
$\mu_\lambda(\{y_4\})$=$\mu_\lambda(P$ublished research)=0.1
=μ_λ({The scientific title})=0.15$\mu_\lambda(\{y_5\})$

The marks of different weights for each applicant in a scale from 0 to 100 are shown in Table 3.

Table3 - The data of applicants from 100

Applicants	Average	Age	Year of service	Published research	Scientific title
AP_1	66	80	70	100	66
AP_2	66	80	65	80	66
AP_3	66	60	100	70	66
AP_4	66	60	75	100	100
AP_5	66	60	100	80	66
AP_6	33	40	100	80	66

From equation (2), λ is computed as

$$.1 + \lambda = (1 + \lambda 0.4)(1 + 0.3\lambda)(1 + 0.2\lambda)(1 + 0.1\lambda)(1 + 0.15\lambda)$$

Thus, $\lambda = \{0, -0.3204, -14.18403, -12.5888 + 7.0542i, -3.6845 + 7.0333i\}$.
Because $\lambda \in (-1, \infty)$, so $\lambda = -0.3204$. Since there are 5 criteria, it is essential to define 2^5 , this means 32 subsets of criteria may be computed by utilizing the equation (1), we have

$\mu_\lambda(\{y_1,y_2\})$=$\mu_\lambda(\{y_1\})$+$\mu_\lambda(\{y_2\})$+$\lambda\mu_\lambda(\{y_1\})\mu_\lambda(\{y_2\})$
=0.4+0.3 −0.3204 × 0.4 × 0.3=0.6616

We may also determine other values via equation (1).

Table 4. The Interdependencies measures between two or more subjects

Sets	***λ**-measure*	*Sets*	***λ**-measure*	*Sets*	***λ**-measure*
y_1, y_2	0.6616	y_4, y_5	0.2452	y_2, y_4, y_5	0.5216
y_1, y_3	0.5744	y_1, y_2, y_3	0.8192	y_3, y_4, y_5	0.4295
y_1, y_4	0.4872	y_1, y_2, y_4	0.7404	y_1, y_2, y_3, y_4	0.8929
y_1, y_5	0.5308	y_1, y_2, y_5	0.7798	y_1, y_2, y_3, y_5	0.9298
y_2, y_3	0.4808	y_1, y_3, y_4	0.6560	y_1, y_2, y_4, y_5	0.8548
y_2, y_4	0.3904	y_1, y_3, y_5	0.6968	y_1, y_3, y_4, y_5	0.7744
y_2, y_5	0.4356	y_1, y_4, y_5	0.6138	y_2, y_3, y_4, y_5	0.6882
y_3, y_4	0.2936	y_2, y_3, y_4	0.5654	y_1, y_2, y_3, y_4, y_5	1
y_3, y_5	0.3404	y_2, y_3, y_5	0.6077		

The algorithm used to calculate the value of the non-additive measure and the interdependencies measure between two or more subjects is as follows.

ALGORITHM 1

Step1:input n number of subjects individual values $\mu_1, \mu_2,\ldots,\mu_n$ for n $\lambda -$ measure for each subject

Step2:compute roots λ_n from the equation(1)

Step3:choose $\lambda \in (-1, \infty)$

for $\mathrm{i} = 2{:}\, \mathrm{n}$

Compute μ_k for each set n choose i from equation (2)

Whene $i = n$ then $\mu_{\mathrm{i=1}}$

End

Step4:out put

μ_k values corresponding to the interdependencies measures between two or more subjects

For instance, applying Choq-Int for student S_1, we obtain

$S_1 = ch\int f d\mu$ =$f(y_1).\,\mu_\lambda(\{y_1, y_2, y_3\,, y_4, y_5\})$+$(f(y_3) - f(y_5)).\,\mu_\lambda(\{y_3, y_2\,, a_4\}) + f(y_2) -$
$f(y_3)).\,\mu_\lambda(\{y_2{,}y_4\})$+ $f(y_4) - f(y_2).\,\mu_\lambda(\{y_4\})$

= 66×1+4×0.5654+10×0.3904+20× 0.1
=74.1656

To apply shilkrit integral, we will permute $f(y_i)$. That is

$$f(y_1) \le f(y_5) \le f(y_3) \le f(y_2) \le f(y_4) \qquad (66 \le 66 \le 70 \le 80 \le 100).$$

Utilizing the following formula, we can determine μ_f.

$\mu_f(\{y_i\}) = \mu(\{x \,/\, f(x) \ge f(y_i)\})$:

$\mu_f(\{y_1\}) = \mu(\{x \,/\, f(x) \ge f(y_1)\}) = 1$

$\mu_f(\{y_5\}) = \mu(\{x \,/\, f(x) \ge f(y_5)\}) = \mu\{y_5, y_3, y_2, y_4\} = 0.682$

$\mu_f(\{y_3\}) = \mu(\{x \,/\, f(x) \ge f(y_3)\}) = \mu\{y_3, y_2, y_4\} = 0.5654$

$\mu_f(\{y_2\}) = \mu(\{x \,/\, f(x) \ge f(y_2)\}) = \mu\{y_2, y_4\} = 0.3904$

$\mu_f(\{y_4\}) = \mu(\{x \,/\, f(x) \ge f(y_4)\}) = \mu\{y_4\} = 0.1$

$$Sh_\mu(f) = Max\{f(y_1).\,\mu f(\{y_1\}), f(y_2).\,\mu f(\{y_2\}), f(y_3).\,\mu f(\{y_3\}), f(y_4).\,\mu f(\{y_4\}), f(y_5).\,\mu f(\{y_5\})\}$$

$$= Max\{(1)(66), (80)(0.3904), (70)(0.5654), (100)(0.1), (66)(0.6882)\}$$

$$= 66$$

Also we can apply PAN-Int using equation (5), we get

$$(p)\int_A f d\mu = Max \left\{ \begin{array}{c}
f(y_1).\mu(\{y_1, y_2\}) + f(y_3).\mu(\{y_3\}) + f(y_4).\mu(\{y_4\}) + f(y_5).\mu(\{y_5\}), \\
f(y_1).\mu(\{y_1, y_3\}) + f(y_2).\mu(\{y_2\}) + f(y_4).\mu(\{y_4\}) + f(y_5).\mu(\{y_5\}), \\
f(y_1).\mu(\{y_1, y_4\}) + f(y_2).\mu(\{y_2\}) + f(y_3).\mu(\{y_3\}) + f(y_5).\mu(\{y_5\}), \\
f(y_1).\mu(\{y_1, y_5\}) + f(y_2).\mu(\{y_2\}) + f(y_3).\mu(\{y_3\}) + f(y_4).\mu(\{y_4\}), \\
f(y_3).\mu(\{y_2, y_3\}) + f(y_1).\mu(\{y_1\}) + f(y_4).\mu(\{y_4\}) + f(y_5).\mu(\{y_5\}), \\
f(y_2).\mu(\{y_2, y_4\}) + f(y_1).\mu(\{y_1\}) + f(y_3).\mu(\{y_3\}) + f(y_5).\mu(\{y_5\}), \\
f(y_5).\mu(\{y_2, y_5\}) + f(y_1).\mu(\{y_1\}) + f(y_3).\mu(\{y_3\}) + f(y_4).\mu(\{y_4\}), \\
f(y_3).\mu(\{y_3, y_4\}) + f(y_1).\mu(\{y_1\}) + f(y_2).\mu(\{y_2\}) + f(y_5).\mu(\{y_5\}), \\
f(y_5).\mu(\{y_3, y_5\}) + f(y_1).\mu(\{y_1\}) + f(y_2).\mu(\{y_2\}) + f(y_4).\mu(\{y_4\}), \\
f(y_5).\mu(\{y_4, y_5\}) + f(y_1).\mu(\{y_1\}) + f(y_2).\mu(\{y_2\}) + f(y_3).\mu(\{y_3\}), \\
f(y_1).\mu(\{y_1, y_2, y_3\}) + f(y_4).\mu(\{y_4\}) + f(y_5).\mu(\{y_5\}), f(y_1).\mu(\{y_1, \\
y_2, y_4\}) + f(y_3).\mu(\{y_3\}) + f(y_5).\mu(\{y_5\}), f(y_1).\mu(\{y_1, y_2, y_5\}) + f(y_3) \\
.\mu(\{y_3\}) + f(y_4).\mu(\{y_4\}), f(y_1).\mu(\{y_1, y_3, y_4\}) + f(y_2).\mu(\{y_2\}) + f(y_5) \\
.\mu(\{y_5\}), f(y_1).\mu(\{y_1, y_3, y_5\}) + f(y_2).\mu(\{y_2\}) + f(y_4).\mu(\{y_4\}), f(y_1). \\
\mu(\{y_1, y_4, y_5\}) + f(y_2).\mu(\{y_2\}) + f(y_3).\mu(\{y_3\}), f(y_3).\mu(\{y_2, y_3, y_4\}) \\
+f(y_1).\mu(\{y_1\}) + f(y_5).\mu(\{y_5\}), f(y_5).\mu(\{y_2, y_3, y_5\}) + f(y_1).\mu(\{y_1\}) + \\
f(y_4).\mu(\{y_4\}), f(y_5).\mu(\{y_2, y_4, y_5\}) + f(y_1).\mu(\{y_1\}) + f(y_3).\mu(\{y_3\}), \\
f(y_5).\mu(\{y_3, y_4, y_5\}) + f(y_1).\mu(\{y_1\}) + f(y_2).\mu(\{y_2\}), f(y_1).\mu(\{y_1, y_2, y_3, y_4\}) \\
+f(y_5).\mu(\{y_5\}), f(y_1).\mu(\{y_1, y_2, y_3, y_5\}) + f(y_5).\mu(\{y_5\}), f(y_1).\mu(\{y_1, y_2, y_4, y_5\}) \\
+f(y_3).\mu(\{y_3\}), f(y_1).\mu(\{y_1, y_3, y_4, y_5\}) + f(y_2).\mu(\{y_2\}), f(y_5).\mu(\{y_2, y_3, y_4, y_5\}) \\
+f(y_1).\mu(\{y_1\}), f(y_1).\mu(\{\mathbb{X}\}), f(y_1).\mu(\{y_1\}) + f(y_2).\mu(\{y_2\}) + f(y_3).\mu(\{y_3\}) + \\
f(y_4).\mu(\{y_4\}) + f(y_5).\mu(\{y_5\})
\end{array} \right\}$$

$= Max\{77.5656, 81.8104, 80.0552, 83.0328, 79.9560, 81.5320, 79.1496, 80.8520, 82.8664, 80.5832, 73.9672, 72.7664, 75.4668, 77.1960, 79.9888, 78.5108, 75.8780, 76.5082, 74.8256, 78.7470, 68.8314, 71.3668, 70.4168, 75.1104, 71.8212, 66, 84.3\}$

=84.3

Similarly, the other results for students AP_2, AP_3, AP_4, AP_5 and AP_6 are shown in table 5

Table 5-Case studies

Applicants	Choq-Int	Shil-Int	PAN-Int	**Total**
AP_1	74.1656	66	84.3	74
AP_2	71.3204	65	81.3	71
AP_3	71.8208	60	81.3	72
AP_4	75.5009	60	84.4	75
AP_5	72.7568	60	82.3	73
AP_6	59.0948	33	63.1	58

We obtain the following ranking of candidates by sorting Choq-Int values as follows.

$$AP_4 \geq AP_1 \geq AP_5 \geq AP_3 \geq AP_2 \geq AP_6 \quad (6)$$

Shil-Int values are sorted, and the results provide us the following order in which to rank the candidates. $AP_1 \geq AP_2 \geq AP_3 \geq AP_4 \geq AP_5 \geq AP_6$ (7)

The following ranking of candidates is also obtained by sorting PAN-Int values.

$$AP_4 \geq AP_1 \geq AP_5 \geq AP_2 \geq AP_3 \geq AP_6 \quad (8)$$

Finally, by sorting and comparing the weights values of the applicants, we obtain a ranking of the applicants as follows.

$$AP_4 \geq AP_1 \geq AP_5 \geq AP_3 \geq AP_2 \geq AP_6 \quad (9)$$

According to the order shown by the order of the applicants in equations (6), (7), (8), and (9), we conclude that the Choq-Int ranks the candidates in a better order than the Shil-Int or PAN-Int, which is a compatible fair ranking of applicants for Ph.D. study/ major in mathematics in equation (9).

This is because the total values employing Choq-Int are affected by preferred weights. For example, an acceptance index by Choq-Int for an AP4 applicant is greater than that of an AP6 which indicates that an AP4 applicant will have a higher probability of admission than an AP6 applicant.

Considering that Choq-Int is based on a metric - fuzzy and in this metric that depends on the interaction between criteria, the AP4 applicant scores better in average and age than the AP6 applicant. Therefore, the Choq-Int value of the AP4 applicant is greater than the Choq-Int value of the AP6 applicant

5. Conclusion

In this work, non-additive measures and corresponding integrals were applied to a number of postgraduate applicants at an Iraqi university. The results were determined to be consistent with real results on realism. The results of fuzzy integrals were compared, and the results show that the Choquet integration is a good tool for ranking the admission of applicants. This work can be considered as the basis for further research on the applications of the λ fuzzy measures and corresponding integrals in the acceptance mechanism and in other diverse fields of education.

References

[1] Zadeh, L. A, "Fuzzy Sets, Information and control", 8(3):338-353, 1965

[2] Dubois, D. J, "Fuzzy Sets and Systems theory and applications", Vol.144, Academic press,1980.

[3] G. Choquet , "Theory Of Capacities", Ann, Inst, Foureier, Vol. 5, Pp. 131-295, 1954

[4] Sugeno M, "Theory of fuzzy Integrals and it's Applications", Tokyo: Institute of Technology.1974.

[5] M. Grabisch, , T. Murofushi, M. Sugeno, "Fuzzy Measures And Integrals:Theory And Applications'',Physica Verlag, Berlin Heidelberg, 2000.

[6] Mihailovic B., Manzi M., Dapic P, "The Shilkret-like integral on the symmetric interval, *U.P.B. Sci. Bull., Series A*, Vol. 77, Iss. 3, 2015.

[7] Z. Wang, George J.Klir , "*Generalized Measure Theory*", Springer, 2009.

[8] V. Torra, Y. Narukawa,M. Sugeno, "*Non-additive Measures, Theory and applications*"**,** Springer, Studies in Fuzziness and Soft Computing, 2013**.**

[9] R. I. Sabri, M. N. Mohammedali ; J .Abbas, "An Application Of Non-Additive Measures And Corresponding Integrals In Tourism Management'', Baghdad Science Journal, Vol 17 (1), Pp.172-177, 2020

[10**]** Ouda, E. H, "Revaluation of Student Failure Reasons Using Non-Additive Methods", *Baghdad Science Journal*, *14*(4),2017.

[11] Abbas, J., Ouda, E. H., & Fahmi, I. N. A. ,"A Mathematical Model for Iraqi Airways Company about Evaluating Its Objectives and Strategies".,*Iraqi Journal of Science*, 699-706, 2017.

[12] Sadiq, F. A., Mitlif, R. J., & Abbas, J "A computational mechanism for making admission decisions in the centralized admission system". In *AIP Conference Proceedings* (Vol. 2414, No. 1, p. 040005). AIP Publishing LLC. . (2023, February).