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Anew Technique to Obtain Analytical Solutions for The Fractional Foam Drainage Formula

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ABSTRACT

In the present study, an innovative approach is proposed to solve a special case of fractional foam drainage using the Laplace residual power series technique (LRPS) in conjunction with the Caputo operator for determining the fractional derivative. The study provides extensive guidelines for utilizing this approach to solve time-fractional nonlinear formulas. The effectiveness and validity of the proposed method are investigated and established by comparing the obtained results with the accurate responses using graphs. The study also confirms that the accuracy of the proposed technique increases with the number of items in the combined solution of the problems, as demonstrated by the convergence of the correlation between the obtained solutions and the actual solutions for the special case of fractional foam drainage formula. The research findings suggest that the proposed technique is not only accurate and uncomplicated but also highly adaptable, making it suitable for addressing both linear and nonlinear situations.

MSC..

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1. Introduction

In recent years, great progress has been made in the field of partial differential equations as pioneers in the field, such as [1-7] were given a comprehensive introduction to fractional of differential equations concepts and gained an organized comprehension of partial calculus. Considering the existence and originality of solutions. In 2010 [8] a paper was published on recent achievements in the concept of differential equation with fractional derivatives. Furthermore, as demonstrated by the studies of [9-10], scientists have investigated multiple uses of calculus in multidisciplinary domains including the processing of images and control theories. These contributions have laid a strong foundation for continued research and development in the field of fractional calculus and its applications.

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There are no approaches in the research literature that generate precise answers to nonlinear differential equations with fractions. Just approximate solutions can be obtained through linearization, consecutive or perturbation approaches. These approaches include the technique of variational iteration [11], the technique of Adomian decomposition [12-15], the Homotopy method of analysis [16] and the Iteration Laplace transformation technique [17].

The primary goal of this research is to investigate the results for the special case of fractional Foam drainage concept stated by using a new unique approach known as the Laplace residual power series approach.

$$D_\eta^\alpha u(x,\eta)=\rho u_{xx}(x,\eta),\ \ x\in\mathbb{R},\eta>0,\rho>0,0<\alpha\le 1, \tag{1}$$

And initial condition as:

$u(x,\eta)=\varsigma(x)$,

The present article follows the following pattern: Section 2 contains descriptions and results about Caputo's derivatives and fractions power series. We develop a LRPS approach to the special case of fractional foam drainage issue in Section 3. Section 4 presents graphical findings for the special case of the foam drainage concept.

2. Preliminaries

In this section, we also go through the Laplace transformation's outcomes and the essential concept of fractions calculus:

Definition 1 [18]. The fractional derivative in the Caputo meaning defined as:

$$^{C}D^\alpha\varphi(\mathrm{x},\eta)=\mathrm{J}^{\gamma-\alpha}\varphi^{\gamma}(\mathrm{x},\eta),\ \gamma-1<\alpha\le\gamma,\mathrm{x}>0 \tag{2}$$

where J^α represents the Riemann-Liouville (RL) integrating operations as:

$$\mathrm{J}^\alpha\varphi(\mathrm{x},\eta)=\frac{1}{\Gamma(\alpha)}\int_0^\kappa(\kappa-\eta)^{\alpha-1}\varphi(x,\eta)d\eta \tag{3}$$

and $\gamma\in\mathbb{N}$.

Definition 2. [18] The transform of Laplace (LT) given on the function $\varphi(\mathrm{x},\eta)$ is

$$\mathcal{L}\{\varphi(\mathrm{x},\eta)\}=\int_0^\infty e^{-s\eta}\varphi(\mathrm{x},\eta)d\eta,\ s>\alpha \tag{4}$$

And the inverse LT defined as:

$$\mathcal{L}^{-1}\{\Omega(\mathrm{x},s)\}=\int_{\upsilon-i\infty}^{\upsilon+i\infty}e^{s\eta}\Omega(\mathrm{x},s)ds,\ \upsilon=Re(s)>\upsilon_0 \tag{5}$$

Lemma 3. [19] Suppose $\varphi(x,\eta)$ is a piecewise continuous function having $\Omega(\mathrm{x},s)=\mathcal{L}\{\varphi(\mathrm{x},\eta)\}$. Then, the following criteria are valid:

(i) $\mathcal{L}\{\mathrm{J}^\alpha\varphi(\mathrm{x},\eta)\}=\frac{\Omega(\mathrm{x},s)}{s^\alpha},\ \varrho>0$
(ii) $\mathcal{L}\{\,^{C}D^\alpha\varphi(\mathrm{x},\eta)\}=s^\alpha\Omega(\mathrm{x},s)-\sum_{i=0}^{k-1}s^{\alpha-k-1}\varphi^k(\mathrm{x},0),\ k-1<\alpha\le k;$
(iii) $\mathcal{L}\{\,^{C}D^{k\alpha}\varphi(\mathrm{x},\eta)\}=s^{k\alpha}\Omega(\mathrm{x},s)-\sum_{i=0}^{k-1}s^{\mathrm{k}\alpha-1}\,^{C}D^\alpha\varphi(\mathrm{x},0),\ 0<\alpha\le 1.$

Proposition 4. [18] Note that the function $\varphi(\mathrm{x},\eta)$ is piecewise continuous on the interval $I\times[0,\infty)$, and it has an exponential growth rate of ζ. Given this, the fractional expansions of $\mathbf{\Omega}(\mathrm{x},s)=\mathcal{L}\{\varphi(\mathrm{x},\eta)\}$ can be expressed as follows

$$\mathbf{\Omega}(\mathrm{x},s)=\sum_{m=0}^\infty\frac{\varsigma_m(x)}{s^{1+m\alpha}},\ 0<\alpha\le 1,s>\zeta \tag{6}$$

Hence, $\varsigma_m(x)=\,^{C}D^\alpha\varphi(\mathrm{x},0)$.

Remark 5. [19] Upon applying the inverse Laplace transform to equation (6), we obtain the following expression:

$$\boldsymbol{\varphi}(\mathrm{x},\eta)=\sum_{m=0}^\infty\frac{^{C}\boldsymbol{D}^{\boldsymbol{\alpha}}\boldsymbol{\varphi}(\mathbf{x},\mathbf{0})}{\Gamma(1+m\alpha)}\eta^{m\alpha},\ 0<\alpha\le 1,\eta\ge 0 \tag{7}$$

The fractional Taylor's equation introduced in reference [19] bears resemblance to the equation under consideration.

3. The Proposed Method for Special Case of Fractional Foam Drainage Formula

To exemplify how the LRPS approach can be utilized to generate a series answer to the FPDEs, firstly we take LT of both sides of Eq. (1) we obtain:

$$\mathcal{L}[D_\eta^\alpha u(x,\eta)] = \rho\mathcal{L}[u_{xx}(x,\eta)],\ \ \eta \in I, I \in [0,\infty)\,. \tag{8}$$

The construction of equation (8) can be achieved by utilizing Lemma 3 as follows:

$$s^\alpha U(x,s) - s^{\alpha-1}\,u(x,0) = \rho U_{xx}(x,s),\ s>0. \tag{9}$$

where $U(x,s) = \mathcal{L}[u(x,\eta)]$ and $U_{xx}(x,s) = \mathcal{L}[u_{xx}(x,\eta)]$.

The application of initial conditions from Eq. (9) and the division of the equation by s^αresults in a new form, which is given by:

$$U(x,s) = \frac{\varsigma(x)}{s} + \frac{\rho}{s^\alpha}U_{xx}(x,s),\ s>0. \tag{10}$$

Expanding Eq. (10) results in the following outcome:

$$U(s) = \sum_{j=0}^{\infty}\frac{\varsigma_j(x)}{s^{1+\alpha j}}, s>0 \tag{11}$$

Equation (11) provides the expression for the kth-truncated series as shown below:

$$U_k(s) = \frac{\varsigma(x)}{s} + \sum_{j=1}^{k}\frac{\varsigma_j(x)}{s^{1+\alpha j}}\ , s>0. \tag{12}$$

The utilization of primary LRPS approaches, including the LRF of Eq. (10), can facilitate the identification of the unknown parameter value $\varsigma_j(x)$, which is denoted by the following expression:

$$\mathrm{LRes}(x,s) = \mathrm{U}(x,s) - \frac{\varsigma(x)}{s} - \frac{\rho}{s^\alpha}U_{xx}(x,s), s>0. \tag{13}$$

The expression for the definition of the kth-LRF is as follows:

$$\mathrm{LRes}_k(x,s) = \mathrm{U}_k(x,s) - \frac{\varsigma(x)}{s} - \frac{\rho}{s^\alpha}U_{(k)xx}(x,s), s>0. \tag{14}$$

It is obvious that for $s>0$ and $k = 0,1,2,3,\dots$. $\mathrm{Lim}_{s\to\infty}\mathrm{LRes}_k(x,s) = \mathrm{LRes}(x,s)$, $\mathrm{LRes}(x,s) = 0$. As a result, $\mathrm{Lim}_{s\to\infty}\left(s^k\mathrm{LRes}(x,s)\right) = 0$. Additionally, it was established [19,20] and for $s>0$ and $k = 0,1,2,3,\dots$. Moreover, previous studies [19, 20] have demonstrated that...

$$\mathrm{Lim}_{s\to\infty}\left(s^{k+1}\mathrm{LRes}(x,s)\right) = \mathrm{Lim}_{s\to\infty}\left(s^{k+1}\mathrm{LRes}_k(x,s)\right) = 0, k = 1,2,3,.. \tag{15}$$

If we assume that $\mathrm{U}_1(x,s) = \frac{\varsigma(x)}{s} + \frac{\varsigma_1(x)}{s^{1+\alpha}}$, then Eq. (14) can be interpreted as:

$$\mathrm{LRes}_1(x,s) = \frac{\varsigma_1(x)}{s^{1+\alpha}} - \rho\frac{\varsigma^{(2)}(x)}{s^{1+\alpha}} - \rho\frac{\varsigma_1^{(2)}(x)}{s^{1+2\alpha}}, s>0. \tag{16}$$

By multiplying both sides of Eq. (16) with $s^{1+\alpha}$, we obtain:

$$s^{1+\alpha}\mathrm{LRes}_1(x,s) = \varsigma_1(x) - \rho\varsigma^{(2)}(x) - \rho\frac{\varsigma_1^{(2)}(x)}{s^\alpha}, s>0. \tag{17}$$

By solving the formula given for $\varsigma_1(x)$and applying the assumption in Eq. (15), as well as taking the limit as s approaches infinity for both sides of Eq. (17), we can determine the value of $\varsigma_1(x)$:

$$0 = \varsigma_1(x) - \rho\varsigma^{(2)}(x). \tag{18}$$

By plugging in the value of $\varsigma_1(x)$into the algebraic formula (18), we can obtain the following result:

$$\varsigma_1(x) = \rho\varsigma^{(2)}(x). \tag{19}$$

To determine the value of the next unknown parameter $\varsigma_2(x)$, we can substitute the 2nd-truncated series of Eq. (16), $\mathrm{U}_2(x,s) = \frac{\varsigma(x)}{s} + \frac{\varsigma_1(x)}{s^{1+\alpha}} + \frac{\varsigma_2(x)}{s^{1+2\alpha}}$, into the 2nd-LRF and apply the following equation:

$$\mathrm{LRes}_2(\mathrm{x},\mathrm{s}) = \frac{\varsigma_2(x)}{s^{1+2\alpha}} - \rho\frac{\varsigma_1^{(2)}(x)}{s^{1+2\alpha}} - \rho\frac{\varsigma_2^{(2)}(x)}{s^{1+3\alpha}}, s > 0. \quad (20)$$

We obtain the following equation by multiplying both sides of Eq. (20) with $s^{1+2\alpha}$:

$$s^{1+2\alpha}\mathrm{LRes}_2(\mathrm{x},\mathrm{s}) = \varsigma_2(x) - \rho\varsigma_1^{(2)}(x) - \rho\frac{\varsigma_2^{(2)}(x)}{s^{\alpha}}, s > 0. \quad (21)$$

To derive the following formula, take the limit as s approaches infinity for both sides of Eq. (21) and apply Eq. (15):

$$0 = \varsigma_2(x) - \rho\varsigma_2^{(2)}(x). \quad (22)$$

We can obtain the value of $\varsigma_2(x)$ by solving the resulting algebraic equation:

$$\varsigma_2(x) = \rho\varsigma_2^{(2)}(x)\,. \quad (23)$$

As same way, we can find $\varsigma_3(x)$ *and* $\varsigma_4(x)$ as follows

$$\varsigma_3(x) = \rho\varsigma_2^{(2)}(x),$$

$$\varsigma_4(x) = \rho\varsigma_3^{(2)}(x). \quad (24)$$

The factor $\varsigma_k(x)$ can be determined by examining the pattern of the computed factors, which continues as follows:

$$\varsigma_k(x) = \rho\varsigma_2^{(k-1)}(x). \quad (25)$$

Eq. (12) can be expressed as an infinite series using the following representation:

$$U(x,s) = \frac{\lambda(x)}{\mathrm{s}} + \sum_{j=1}^{\infty}\left(\frac{\lambda_j(x)}{s^{1+\alpha j}}\right), s > 0. \quad (26)$$

By utilizing the inverse Laplace transform of Eq. (26) in the provided simplified format, we can obtain the solution for Eqs. (6) and (7):

$$u(x,\eta) = \lambda(x) + \sum_{j=1}^{\infty}\left(\frac{\lambda_j(x)}{s^{1+\alpha j}}\right)\frac{\eta^{\alpha j}}{\Gamma(1+\alpha j)}. \quad (27)$$

4. Numerical Issues

This section is dedicated to exploring how LRPSM can be utilized to obtain an approximate

solution for the FPDEs:

Issue 1. Consider with regard to the shape's FPDEs:

$$D_\eta^\alpha u(x,\eta) - u_{xx}(x,\eta) = 0\,, \quad \eta > 0, 0 < \alpha \le 1, \quad (28)$$

And initial condition as:

$$u(\mathrm{x},0) = \mathrm{e}^x \quad (29)$$

The Laplace transform of Eq. (29) yields Eq. (28) as the resulting equation.

$$U(x,s) = \frac{\mathrm{e}^x}{s} + \frac{1}{s^\alpha}U_{xx}(x,s), \ \ s > 0. \quad (30)$$

The kth-truncated series is purported to be:

$$U_k(x,s) = \frac{\mathrm{e}^x}{s} + \sum_{j=1}^{k}\frac{\varsigma_j(x)}{s^{1+\alpha j}}, s > 0. \quad (31)$$

Therefore, the kth LRFs can be expressed as:

$$\mathrm{LRes}_k(\mathrm{x},\mathrm{s}) = \mathrm{U}_k(x,s) - \frac{\mathrm{e}^x}{s} - \frac{1}{s^\alpha}\mathrm{U}_{(k)xx}(x,s), s > 0. \quad (32)$$

By inserting the kth-truncated series (Eq. 31) into the kth LRF (Eq. 32), we can derive $\varsigma_j(x)$. We can then derive the relationship by multiplying the resulting expression by $s^{1+\alpha j}$ as

$$\mathrm{Lim}_{s\to\infty}\left(s^{k+1}\mathrm{LRes}_k(x,s)\right)=0, k=1,2,3,..$$

Therefore, a few of the values are:

$\varsigma_1(x)=\mathrm{e}^x$,

$\varsigma_2(x)=\mathrm{e}^x$,

$\varsigma_3(x)=\mathrm{e}^x$,

$\varsigma_4(x)=\mathrm{e}^x$,

$\varsigma_5(x)=\mathrm{e}^x$.

By entering the values of $\varsigma_j(x)$.into (31) for $j=1,2,3,\ldots$, we can derive their specific expressions.

$$U(x,s)=\frac{\mathrm{e}^x}{s}+\frac{\mathrm{e}^x}{s^{1+\alpha}}+\frac{\mathrm{e}^x}{s^{1+2\alpha}}+\frac{\mathrm{e}^x}{s^{1+3\alpha}}+\frac{\mathrm{e}^x}{s^{1+4\alpha}}+\frac{\mathrm{e}^x}{s^{1+5\alpha}} \quad (33)$$

The result can be obtained by taking the inverse Laplace transform of the equation.

$$u(x,\eta)=\mathrm{e}^x+\frac{\mathrm{e}^x\eta^\alpha}{\Gamma(1+\alpha)}+\frac{\mathrm{e}^x\eta^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{\mathrm{e}^x\eta^{2\alpha}}{\Gamma(1+3\alpha)}+\frac{\mathrm{e}^x\eta^{4\alpha}}{\Gamma(1+4\alpha)}+\frac{\mathrm{e}^x\eta^{5\alpha}}{\Gamma(1+5\alpha)} \quad (34)$$

The accurate solution of Eq. (34) is:

$$u(\eta)=e^{x+\eta} \quad (35)$$

Figure 1 provides visual evidence of the precision of the proposed method in generating accurate outcomes for the given problem. The figure showcases the graphs of the exact solution and the 5th-order approximation of Eqs. (28) and (29) over the interval [0,2]. Based on the results, it can be inferred that the proposed approach is a dependable analytical and numerical technique for obtaining exact solutions to fractional partial differential equations.

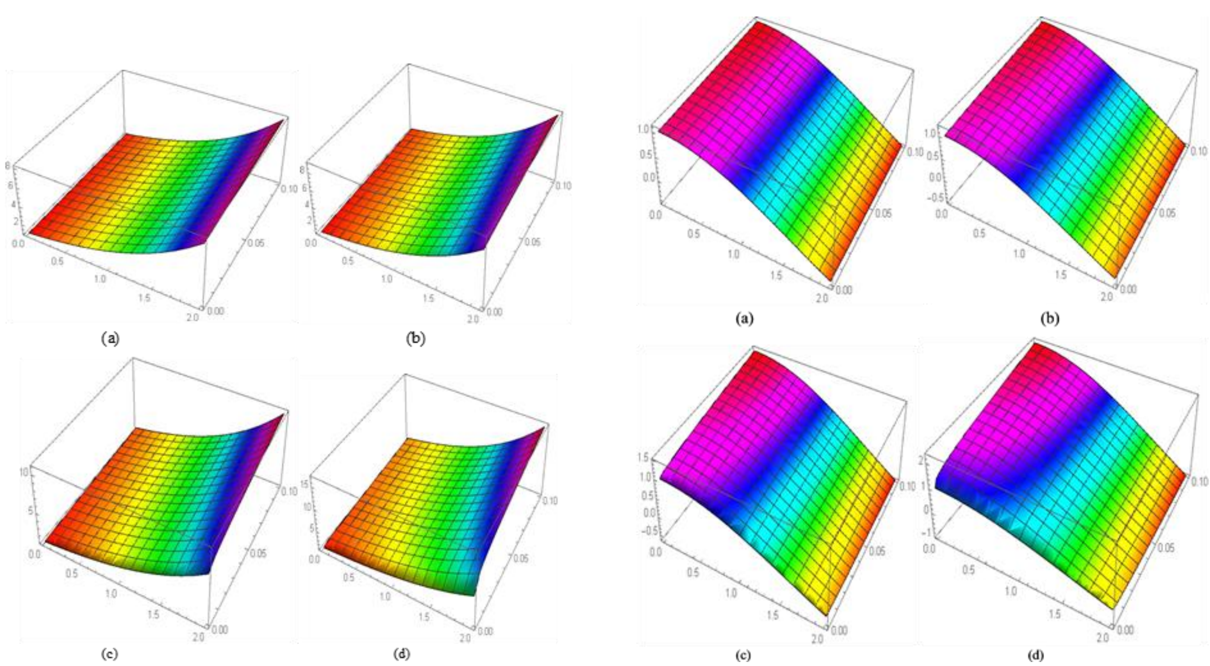

Fig. 1. The fifth-order analytical solution for Issue 1 and 2 is obtained using the proposed method at various values of α, namely: (a) $\alpha=1$, (b) $\alpha=0.75$, (c) $\alpha=0.75$, (d) $\alpha=0.25$.

Issue 2. Consider with regard the shape's FPDEs:

$$D_{\eta}^{\alpha}u(x,\eta)+u_{xx}(x,\eta)=0\,,\quad \eta>0, 0<\alpha\leq 1, \tag{36}$$

And initial condition as:

$$u(x,0)=cos(x) \tag{37}$$

The Laplace transform of Eq. (37) yields Eq. (36) as the resulting equation.

$$U(x,s)=\frac{\cos(x)}{s}-\frac{1}{s^{\alpha}}U_{xx}(x,s),\quad s>0. \tag{38}$$

The kth-truncated series is purported to be:

$$U_k(x,s)=\frac{\cos(x)}{s}+\sum_{j=1}^{k}\frac{\varsigma_j(x)}{s^{1+\alpha j}}\,, s>0. \tag{39}$$

Therefore, the kth LRFs can be expressed as:

$$\mathbf{LRes}_k(x,s)=U_k(x,s)-\frac{\cos(x)}{s}+\frac{1}{s^{\alpha}}U_{(k)xx}(x,s), s>0. \tag{40}$$

By inserting the kth-truncated series (Eq. 39) into the kth LRF (Eq. 40), we can derive $\varsigma_j(x)$. We can then derive the relationship by multiplying the resulting expression by $s^{1+\alpha j}$as

$$\mathbf{Lim}_{s\to\infty}\left(s^{k+1}\mathbf{LRes}_k(x,s)\right)=0, k=1,2,3,..$$

Therefore, a few of the values are:

$\varsigma_1(x)=cos(x)$,

$\varsigma_2(x)=cos(x)$,

$\varsigma_3(x)=cos(x)$,

$\varsigma_4(x)=cos(x)$,

$\varsigma_5(x)=cos(x)$.

By entering the values of $\varsigma_j(x)$.into (39) for $j=1,2,3,...$, we can derive their specific expressions.

$$U(x,s)=\frac{cos(x)}{s}+\frac{cos(x)}{s^{1+\alpha}}+\frac{cos(x)}{s^{1+2\alpha}}+\frac{cos(x)}{s^{1+3\alpha}}+\frac{cos(x)}{s^{1+4\alpha}}+\frac{cos(x)}{s^{1+5\alpha}} \tag{41}$$

The result can be obtained by taking the inverse Laplace transform of the equation.

$$u(x,\eta)=1+\frac{\eta^{\alpha}}{\Gamma(\alpha+1)}+\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)}+\frac{\eta^{4\alpha}}{\Gamma(4\alpha+1)}+\frac{\eta^{5\alpha}}{\Gamma(5\alpha+1)} \tag{42}$$

The accurate solution of Eq. (42) is:

$$u(\eta)=cos(x)e^{\eta} \tag{43}$$

Figure 1 provides visual evidence of the precision of the proposed method in generating accurate outcomes for the given problem. The figure showcases the graphs of the exact solution and the 5th-order approximation of Eqs. (36) and (37) over the interval [0,2]. Based on the results, it can be inferred that the proposed approach is a dependable analytical and numerical technique for obtaining exact solutions to fractional partial differential equations.

5. Conclusion

This article presents a novel analytical iterative technique that employs the Laplace residual power series to estimate the solution of a nonlinear special case of fractional foam drainage formula. The study investigates the impact of two distinct initial conditions of the special case Foam model on the physical behavior of the system. The results demonstrate that when α is in proximity to 0, the solutions exhibit a bifurcation phenomenon that generates wave-like patterns, whereas when αapproaches 1, no detectable pattern emerges. This finding provides a new perspective on the relationship between time-fractional derivatives and real-life phenomena. The accuracy of the proposed method is evaluated by analyzing the absolute errors of the approximations of the Foam model, as displayed in Figures 1 and 2. The simplicity and precision of the LRPS approach suggest its potential as a valuable tool for the fractional theory and computations field. In future research, the authors intend to expand the LRPS method's application to linear and nonlinear time-fractional physical problems.

References

[1] K. Oldham and J. Spanier, "The fractional calculus theory and applications of differentiation and integration to arbitrary order," Elsevier, 1974.

[2] K. S. Miller and B. Ross, "An introduction to the fractional calculus and fractional differential equations," John Wiley & Sons, 1993.

[3] S. S. Ray, "Analytical solution for the space fractional diffusion equation by two-step Adomian decomposition method," Commun. Nonlinear Sci. Numer. Simulat., vol. 14, no. 4, pp. 1295-1306, Apr. 2009.

[4] I. Podlubny, A. Chechkin, T. Skovranek, Y. Chen, and B. M. V. Jara, "Matrix approach to discrete fractional calculus II: partial fractional differential equations," J. Comput. Phys., vol. 228, no. 8, pp. 3137-3153, Apr. 2009.

[5] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, "Theory and applications of fractional differential equations," Elsevier, vol. 204, 2006.

[6] K. Diethelm, "The analysis of fractional differential equations: an application-oriented exposition using differential operators of Caputo type," Springer, vol. 2004, 2010.

[7] R. Caponetto, "Fractional order systems: modeling and control applications," World Scientific, vol. 72, 2010.

[8] E. Hernández, D. O'Regan, and K. Balachandran, "On recent developments in the theory of abstract differential equations with fractional derivatives," Nonlinear Anal. Theory Methods Appl., vol. 73, no. 10, pp. 3462-3471, Nov. 2010.

[9] R. Magin, M. D. Ortigueira, I. Podlubny, and J. Trujillo, "On the fractional signals and systems," Signal Process., vol. 91, no. 3, pp. 350-371, Mar. 2011.

[10] F. Mainardi, "An introduction to mathematical models. Fractional calculus and waves in linear viscoelasticity," Imperial College Press, London, 2010.

[11] V. Turut and N. Güzel, "On solving partial differential equations of fractional order by using the variational iteration method and multivariate Padé approximation," in Proceedings of the 2013 International Conference on Mathematical Sciences and Statistics, Kuala Lumpur, Malaysia, June 2013, pp. 26-34.

[12] K. Al-Khaled and S. Momani, "An approximate solution for a fractional diffusion-wave equation using the decomposition method," Appl. Math. Comput., vol. 165, no. 2, pp. 473-483, Mar. 2005.

[13] Y. Hu, Y. Luo, and Z. Lu, "Analytical solution of the linear fractional differential equation by Adomian decomposition method," J. Comput. Appl. Math., vol. 215, no. 1, pp. 220-229, Jan. 2008.

[14] V. Parthiban and K. Balachandran, "Solutions of system of fractional partial differential equations," Appl. Appl. Math. Int. J. (AAM), vol. 8, no. 1, pp. 17-32, 2013.

[15] K. Al-Khaled and M. Alquran, "An approximate solution for a fractional model of generalized Harry Dym equation," Math. Sci., vol. 9, no. 2, pp. 75-85, 2015.

[16] M. Dehghan, J. Manafian, and A. Saadatmandi, "Solving nonlinear fractional partial differential equations using the homotopy analysis method," Numer. Methods Partial Differ. Equ., vol. 26, no. 2, pp. 448-479, Mar. 2010.

[17] J. Jafaria, M. Nazarib, D. Baleanuc, and C. M. Khalique, "A new approach for solving a system of fractional partial differential equations," Comput. Math. Appl., vol. 66, no. 5, pp. 838-843, Sep. 2013.

[18] A. M. Zidan, A. Khan, R. Shah, M. K. Alaoui, and W. Weera, "Evaluation of time-fractional Fisher's equations with the help of analytical methods," AIMS Math., vol. 7, no. 10, pp. 18746-18766, 2022.

[19] A. El-Ajou, "Adapting the Laplace transform to create solitary solutions for the nonlinear time-fractional dispersive PDEs via a new approach," Eur. Phys. J. Plus, vol. 136, no. 2, 2021.

[20] T. Eriqat, A. El-Ajou, M. N. Oqielat, Z. Al-Zhour, and S. Momani, "A new attractive analytic approach for solutions of linear and nonlinear neutral fractional pantograph equations," Chaos, Solitons Fractals, vol. 138, p. 109957, Jul. 2020.