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Fuzzy Base and Local Base in Fuzzy Bitopological Space

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Abstract :

The aim of this work to define the fuzzy base and fuzzy local base in fuzzy bitopological space and so to compare them with fuzzy base and fuzzy local base in fuzzy topological space.

Keywords: Fuzzy Bitopology, Fuzzy Base and Fuzzy Local Base.

Mathematics subject classification : 54A40

Introduction:

The theory of fuzzy bitopology introduced by Kandil A., and his colleagues [3]. Many authors have generalized some concepts of topology in fuzzy bitopology, for example, Kandil A., and his colleagues [4], Hong Wang [5], Ahmed Abd El-Kader and his colleagues [1] and others. In this work, we introduced some examples to support some definitions and reverses in this subject. Throughout this paper, I^X will denote the set of all fuzzy sets in X.

1- Preliminaries

Definition 1.1 [6]: Let X be a nonempty set, a fuzzy point p_x^t in X is fuzzy set with support $x \in X$ and value $t \in [0,1]$.

Definition 1.2 [2]: Let (X, τ) be a fuzzy topological space (fts). A subfamily $\mathcal{B} \subseteq \tau$ is called base for τ iff each member of τ can be expressed as the union of some members of $\mathcal{B}$.

Example 1.1: Let $X = \{a, b\}$ and $\tau = \{X, \emptyset, \{p_a^1\}, \{p_b^1\}, \{p_a^{0.3}\}, \{p_a^{0.3}, p_b^1\}\}$ is fuzzy topology on X, then the collection $\mathcal{B} = \{\{p_a^1\}, \{p_b^1\}, \{p_a^{0.3}\}\}$ is base for fuzzy topology τ.

Definition 1.3 [6]: Let (X,τ) be a fts and p_x^t is a fuzzy point in X. A subfamily $\mathcal{B}_{p_x^t}$ of fuzzy sets in τ which containing p_x^t is called a local base of p_x^t iff for each member A of τ containing p_x^t there exists a member $B_{p_x^t} \in \mathcal{B}_{p_x^t}$ such that $p_x^t \in B_{p_x^t} \subseteq A$.

Example 1.2: Let X be a non empty set and τ be the collection of all fuzzy sets then (X,τ) is fts which called discrete fts, [6]. For any fuzzy point p_x^t in X, the set $\{p_x^t\}$ is local base at p_x^t.

Definition 1.4 [3]: A fuzzy bitopological space (fbts) is a triple (X,τ_1,τ_2), where τ_1 and τ_2 are arbitrary fuzzy topologies on X.

2- Base in Fuzzy Bitopological Space

Definition 2.1: Let (X,τ_1,τ_2) be a fbts. Then the collection $\mathcal{B} \subseteq \tau_1 \cup \tau_2$ is base for fbts (X,τ_1,τ_2) if for each $G \in \tau_i$, $i = 1$ *or* 2 , is union elements of $\mathcal{B}$.

Example 2.1: Let $X = \{a,b\}$ and τ_1 and τ_2 are fuzzy topologies on X defined as follows:

$\tau_1 = \{X,\emptyset,\{p_a^1\},\{p_b^1\},\{p_a^{0.5}\},\{p_a^{0.5},p_b^1\}\}$ and

$\tau_2 = \{X,\emptyset,\{p_a^{0.7}\}\}$. Then the collection $\mathcal{B} = \{\{p_a^1\},\{p_b^1\},\{p_a^{0.5}\},\{p_a^{0.7}\}\}$ is basis for fbts (X,τ_1,τ_2).

Remark 2.1: If we put $\tau_1 = \tau_2 = \tau$, then the above basis reduce to the corresponding basis for fuzzy topological space (X,τ).

Theorem 2.1: Let (X,τ_1,τ_2) be a fbts and $\mathcal{B} \subseteq \tau_1 \cup \tau_2$ then $\mathcal{B}$ is base for fbts (X,τ_1,τ_2) iff for any $G \in \tau_i$, $i = 1$ *or* 2, and $p_x^t \in G$ there exist $B \in \mathcal{B}$ such that $p_x^t \in B \subseteq G$.

Proof: Let (X,τ_1,τ_2) be a fbts. Let $\mathcal{B} \subseteq \tau_1 \cup \tau_2$ is base for fbts (X,τ_1,τ_2), $G \in \tau_i$, $i = 1$ *or* 2 , and $p_x^t \in G$. Since $G = \cup_{j\in J} U_j$ and $U_j \in \mathcal{B}, \forall j \in J$, then there exist $k \in J$ such that $p_x^t \in U_k \subseteq G$.

Conversely: Let $\mathcal{B} \subseteq \tau_1 \cup \tau_2$ be a collection with the following property: for any $G \in \tau_i$, $i = 1$ *or* 2 , if $p_x^t \in G$ there exist $B \in \mathcal{B}$ such that $p_x^t \in B \subseteq G$. Since $G = \cup_{p_x^t \in G} p_x^t$ and by hypothesis: for any $p_x^t \in G$ there exist $B_{p_x^t} \in \mathcal{B}$ such that $p_x^t \in B_{p_x^t} \subseteq G \Rightarrow G = \cup_{p_x^t \in G} B_{p_x^t}$, therefore $\mathcal{B}$ is base for fbts (X,τ_1,τ_2).

Remark 2.2: In definition 2.1, for every element $G \in \tau_i$, $i = 1$ *or* 2, G is union elements of $\mathcal{B}$ and this doesn't mean the union elements of $\mathcal{B}$ is element in τ_i, $i = 1$ *or* 2, as in example 2.1, such that $\{p_a^{0.7}\}$ and $\{p_b^1\}$ belong to $\mathcal{B}$, but $\{p_a^{0.7}\} \cup \{p_b^1\} = \{p_a^{0.7}, p_b^1\} \notin \tau_1 and \tau_2$. Therefore, we can give the following theorem.

Theorem 2.2: Let X be a nonempty set and $\mathcal{B} \subseteq I^X$. Let $X = \cup \{B : B \in \mathcal{B}\}$ then $\mathcal{B}$ is base for some fbts (X,τ_1,τ_2).

Proof:

We can building in easy fuzzy topologies $\tau_1 and \tau_2$ on X from $\mathcal{B}$ as following:

1- $\tau_1 = \tau_2 = \{\emptyset, X\}$
2- $\tau_1 = \{\emptyset, X\}$ and $\tau_2 = \{\emptyset, X, B\}$, such that $B \in \mathcal{B}$or any
3- $\tau_1 and \tau_2$ satisfy the conditions of fuzzy topology on X, such that the elements of $\tau_1 and \tau_2$ are union elements of $\mathcal{B}$.

But we will take the generalized case. We will building fuzzy topology τ_1 on X and take the other fuzzy topology τ_2 as in 1,2 or 3.

Now, $U \in \tau_1$ if U is union elements $B \in \mathcal{B}$ and $B \cap B_j$ is union elements of $\mathcal{B}$, $\forall B_j \in \mathcal{B}$, or $U = X$.

1- It's clear $X \in \tau_1$ and since $\emptyset \cap B_j$ is union elements of $\mathcal{B}$, $\forall B_j \in \mathcal{B}$, $\emptyset \in \tau_1$.
2- Let $U_1, U_2, \ldots \in \tau_1$, then U_i is union elements of $B_j \in \mathcal{B}$ and $\forall B_j$ then $B_j \cap B$ is union elements of $\mathcal{B}$, $\forall B \in \mathcal{B}$. Then $\cup_i U_i = \cup_{i,j} B_j$ and this mean $\cup_i U_i$ is union same elements B_j, therefore $\cup_i U_i \in \tau_1$.
3- Let $U, V \in \tau_1 \Longrightarrow U = \cup_i B_i$ and $B_i \cap B$ is union elements of $\mathcal{B}$, $\forall B \in \mathcal{B}$, and so $V = \cup_j B_j$ and $B_j \cap B$ is union elements of $\mathcal{B}$, $\forall B \in \mathcal{B}$. Then $U \cap V = \cup_i B_i \cap \cup_j B_j = \cup_{i,j} (B_i \cap B_j)$, but $B_i \cap B_j$ is union elements of $\mathcal{B} \Longrightarrow U \cap V \in \tau_1$.

Therefore τ_1 is fuzzy topology on X, hence (X, τ_1, τ_2) is fbts.

Corollary 2.1: Let X be a nonempty set and $\mathcal{B} \subseteq I^X$ is base for fbts (X, τ_1, τ_2), such that $\tau_1$and τ_2 are some fuzzy topologies on X, then there exist base $\delta \subseteq \mathcal{B}$ for some fuzzy topology τ on X.

Proof:

It's obvious by proof of theorem 2.2.

Theorem 2.3: Let X be a nonempty set and τ_1 and τ_2 are fuzzy topologies on X. If $\mathcal{B}_1$ and $\mathcal{B}_2$ are fuzzy bases for $\tau_1$and τ_2, respectively, then $\mathcal{B}_1 \cup \mathcal{B}_2$ is base for fbts (X, τ_1, τ_2).

Proof: It's obvious

The converse of theorem 2.3 is not true in general, i.e., if we have $\mathcal{B}$ is base for some fbts (X, τ_1, τ_2) then $\mathcal{B}$ is not necessary union two bases $\mathcal{B}_1$ (for τ_1) and $\mathcal{B}_2$ (for τ_2) on X as following example.

Example 2.2: Let $X = \{a, b\}$ and $\tau_1 = \{\emptyset, X, \{p_a^{0.1}, p_b^{0.2}\}\}$ and

$$\tau_2 = \{\emptyset, X, \{p_a^1\}, \{p_b^1\}, \{p_a^{0.3}\}, \{p_b^{0.3}\}, \{p_a^{0.3}, p_b^{0.3}\}, \{p_a^1, p_b^{0.3}\}, \{p_a^{0.3}, p_b^1\}\}$$

Then $\mathcal{B} = \{\{p_a^1\}, \{p_b^1\}, \{p_a^{0.1}, p_b^{0.2}\}, \{p_a^{0.3}\}, \{p_b^{0.3}\}\}$ is base for fbts (X, τ_1, τ_2), but there exist no $\delta \subseteq \mathcal{B}$ such that $\delta \subseteq \tau_1$ is fuzzy base for fuzzy topology τ_1 since $X = \{p_a^1\} \cup \{p_b^1\}$ but $\{p_a^1\}, \{p_b^1\} \notin \tau_1$.

Remark 2.3: One difference between the base in fts and the base in fbts is the base in fts gives unique fuzzy topology but this is not necessary with respect to base in fbts as following example:

Example 2.3: In example 2.2, $\mathcal{B}$ is base for fbts (X, τ_1, τ_2) and so is base for fbts (X, τ_2, τ_3) when $\tau_3 = \{\emptyset, X, \{p_a^{0.3}, p_b^{0.3}\}, \{p_a^1, p_b^{0.2}\}\}$.

3- Local Base in Fuzzy Bitopological Space

Definition 3.1: Let (X, τ_1, τ_2) be a fbts and p_x^t be any fuzzy point in X. Then the collection $\mathcal{B}_{p_x^t} \subseteq \tau_1 \cup \tau_2$ of fuzzy sets which containing p_x^t is local base on fuzzy point p_x^t if for each $G \in \tau_i$, $i = 1$ *or* 2 , containing p_x^t then there exist $B \in \mathcal{B}_{p_x^t}$ such that $p_x^t \in B \subseteq G$.

Example 3.1: Let $X = \{a, b\}$ and $\tau_1 = \{\emptyset, X, \{p_a^1, p_b^{0.2}\}\}$ and

$\tau_2 = \{\emptyset, X, \{p_a^1\}, \{p_b^1\}, \{p_a^{0.3}\}, \{p_b^{0.3}\}, \{p_a^{0.3}, p_b^{0.3}\}, \{p_a^1, p_b^{0.3}\}, \{p_a^{0.3}, p_b^1\}\}$

For any fuzzy point p_a^t in X has local base $\mathcal{B}_{p_a^t} = \{p_a^1\}$ if $t > 0.3$ and $\mathcal{B}_{p_a^t} = \{p_a^{0.3}\}$ if $t \leq 0.3$.

Theorem 3.1: Let X be a nonempty set and τ_1 and τ_2 are fuzzy topologies on X. Let p_x^t be a fuzzy point in X, if $\mathcal{B}_{p_x^t}$ and $\delta_{p_x^t}$ are local bases at p_x^t with respect to $\tau_1$and τ_2, respectively, then $\mathcal{B}_{p_x^t} \cup \delta_{p_x^t}$ is local base at p_x^t for fbts (X, τ_1, τ_2).

Proof:

It's obvious.

Remark 3.1: The converse of theorem 3.1 is not true in general, i.e., for any fuzzy point p_x^t in X we cannot separate local base $\mathcal{B}_{p_x^t}$ to two local bases such that $\mathcal{B}_{p_x^t} = \ell_{p_x^t} \cup \delta_{p_x^t}$ and $\ell_{p_x^t}, \delta_{p_x^t}$ are local bases at p_x^t with respect to $\tau_1$and τ_2, respectively, as following example:

Example 3.2: Consider example 3.1, the fuzzy point $p_b^{0.3}$ in X has local base $\mathcal{B}_{p_b^{0.3}} = \{\{p_b^{0.3}\}\}$. But $\mathcal{B}_{p_b^{0.3}}$ cannot separate to two local bases with respect to fuzzy topologies $\tau_1$and τ_2, respectively.

Remark 3.2: One difference between the local base in fts and the base in fbts is the local base in fts satisfies the following (If the fuzzy point in a fts has a finite local base then it also has a local base consisting of exactly one fuzzy set). This is not satisfied in fbts as following example:

Example 3.3: Consider example 3.1, the fuzzy point $p_b^{0.2}$ in X has local base $\mathcal{B}_{p_b^{0.2}} = \{\{p_a^1, p_b^{0.2}\}, \{p_b^{0.3}\}\}$. But $\{p_a^1, p_b^{0.2}\} \cap \{p_b^{0.3}\} = \{p_b^{0.2}\} \notin \tau_1 \cup \tau_2$ and this mean the fuzzy point $p_b^{0.2}$ cannot posses local base contains exactly one member.

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القاعدة والقاعدة المحلية الضبابية في الفضاءات البايتبولوجية الضبابية

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المستخلص :

الهدف من هذا العمل هو تعريف القاعدة والقاعدة المحلية في الفضاءات التبولوجية الضبابية الثنائية ومقارنتها مع القاعدة والقاعدة المحلية في الفضاءات التبولوجية الضبابية.

الكلمات المفتاحية: الفضاءات التبولوجية الضبابية الثنائية، القاعدة والقاعدة المحلية الضبابية .