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Approach to Grayscale Image Enhancement by Noise Reduction

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ABSTRACT

With the rise in the quantity of digital photos taken each day, there is a rising desire for more accurate and aesthetically appealing images. More precisely, noise is a problem for photos in terms of corruption since certain sounds may be introduced to the image automatically during capture. Additionally, camera sensors and lighting conditions contribute to the noise that might be added to the picture. This is referred to as additive noise. Noise reduction is critical for achieving a beautiful picture. describe a robust approach for decreasing noise in photos based on the heat equation in this study. Additionally, we compare our proposed technique against many existing methods using denoising performance indicators.

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1. INTRODUCTION

Numerous photos captured by cameras or obtained via experiments may have resolution difficulties. For instance, a photographer's sudden movement while shooting photographs might result in some initial distortion or equipment noise connected with such photographs. Additionally, there are other aspects that may alter pictures in the medical profession, including the divergence of x-ray photons, magnification reduction, and the size of the projected item. Generally, one may look at those unpleasant aspects of the picture and attempt to eliminate or at least minimize them in some manner. Numerous writers presented a range of strategies and procedures for picture enhancement, see [1-8]. The primary reason for altering photographs in this way is to reveal information that are otherwise invisible. For instance, "Figure 1" below illustrates a denoised and noisy grayscale picture.

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The majority of the effects seen on photographs are a consequence of altering the angle of view, which is regarded the primary cause of light ray deflection. According to scientists, the deflection of light rays may result in visual blurring and sharpening. Mathematically, this is comparable to heat diffusion, in which heat may move between various degrees. Matrixes or arrays of numbers may be used to mathematically represent digital pictures. Thus, the overall goal of this project is to convert these photos to matrices and then perform the mathematical procedures related with the Heat Equation to them in the next section to improve and then reconvert them to desired images. Various numerical methods have been utilized for more applications [8-17].

a. Denoised image **b. Noised image**

Fig. 1: Noised and denoised Image [18]

2. Involving the Heat Equation

Natural heat diffusion occurs gradually from high to low temperatures, and as seen in the preceding figure, the temperature $u(x,t)$ is entirely dependent on this diffusion over time. This concept may be expressed as a partial differential equation (P.D.E) in the following way:

$$\frac{\partial u}{\partial t} = \alpha \nabla \qquad (1)$$

where α denotes the thermal diffusivity, $\frac{\partial u}{\partial t}$ is the partial derivative of u with respect to t, and ∇u^2 denotes the same variable's second derivative.

The primary reason for altering photographs in this way is to get information that are not visible without altering the images.

Consider Eq. 1 with the thermal diffusivity represented. Depending on the substance, the constant α may have a varied value. The extremes of heat are equivalent to the extremes of light concentration on a picture, which results in a change in resolution. More precisely, we will take a one-dimensional and two-dimensional heat transfer model and use the explicit technique to get an accurate forecast of heat diffusion (where the stability condition is held).

In the following scenario, as the temperature changes, one could perform noising and de-noising the photos. In other expression, applying the forward difference steps may produce noisy photos, but backward time steps may provide de-noising photos. To extend the proposed model to a two-dimensional photo, one must first create a simulation for heat dispersion in the x and y directions. The proposed function is defined by x, y, and t in this example. More exactly, picture denoising is the process of solving the following equation.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\alpha(u-u_0) \qquad (2)$$

where $\frac{\partial u}{\partial t}$ is u's partial derivative with regard to t, $\frac{\partial^2 u}{\partial x^2}$ is second derivative in terms x, and $\frac{\partial^2 u}{\partial y^2}$ is second derivative in terms y.

Indeed, the right-hand side of Eq. 2 contains the real parameter that may be used to adjust the similarity between the original and changed images (denoised image). In a mathematical sense, the concept may be thought of as an optimization problem.

$$k^*=\max_k\{p(k \mid f)\} \qquad (3)$$

where k is the solution and $p(k \mid f)$ denotes the probability of k happening in the presence of the observation f, which may be stated as follows

$$p(k \mid f)=\frac{p(k).p(f|k)}{p(f)} \qquad (4)$$

As a result, we may use the above probability to calculate the solution that was modeled using the same probability, and the following noise reduction can be produced

$$p(f \mid k)=\prod_{(x,y)\in\delta}\frac{1}{\sqrt{2\pi\mu}} \qquad (5)$$

where γ^2 denotes the variance of noise and δ is the collection of components that represent pixels in images.

3. Methodology

The following equation has been obtained by using an Forward time steps FTS at the fixed time t_n and a second-order central time steps CTS for the space derivative at the point x_j in this approach.

$$\frac{u_j^{n+1}-u_j^n}{k}=\frac{u_{j+1}^n-2u_j^n+u_{j-1}^n}{h^2} \qquad (6)$$

This provides an explicit solution technique for the one-dimensional heat equation.

Indeed, we can derive the value of u_j^{n+1} from the other values we know as follows:

$$u_j^{n+1} = \left(1 - 2\frac{k}{h^2}\right) u_j^n + \frac{k}{h^2} u_{j-1}^n + \frac{k}{h^2} u_{j+1}^n \tag{7}$$

(7)

Take the vector u below, which has 100 non-negative components, and put it into the following finite differences equation using fixed values for dt and dx to demonstrate how this technique functions.

$$u_{(j)}^{n+1} = u_{(j)}^n + \frac{dt}{dx^2}\left(u_{(j-1)}^n - 2u_{(j)}^n + u_{(j+1)}^n\right) \tag{8}$$

The following findings is obtained by applying 21 forward time steps (FTS) on the vector

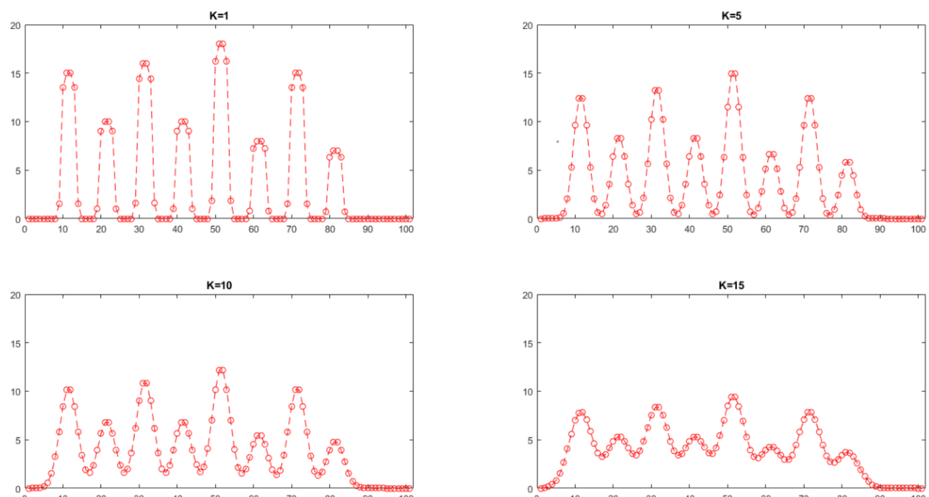

Fig. 1: Four stages of FTS of the vector u. [19] .

On the other hand, the following equation is obtained by using a backward difference at time t_{n+1} and a second-order central difference for the space derivative at point x_j.

$$\frac{u_j^{n+1} - u_j^n}{k} = \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2} \tag{9}$$

This technique is meant to find the solution of the one-dimensional heat equation implicitly. Indeed, one may acquire the value of u_j^{n+1} by finding the solution of a linear system.

$$\left(1 + 2\frac{k}{h^2}\right) u_j^{n+1} - \frac{k}{h^2} u_{j-1}^{n+1} - \frac{k}{h^2} u_{j+1}^{n+1} = u_j^n \tag{10}$$

Take the vector u below, which has 100 non-negative components, and put it into the following finite differences equation using fixed values for dt and dx to understand how the implicit approach functions.

$$u_{(j)}^n = u_{(j)}^{n+1} - \frac{dt}{dx^2}\left(u_{(j-1)}^{n+1} - 2u_{(j)}^{n+1} + u_{(j+1)}^{n+1}\right) \tag{11}$$

The following findings is obtained by doing 21 backward time steps (BTS) on the vector .

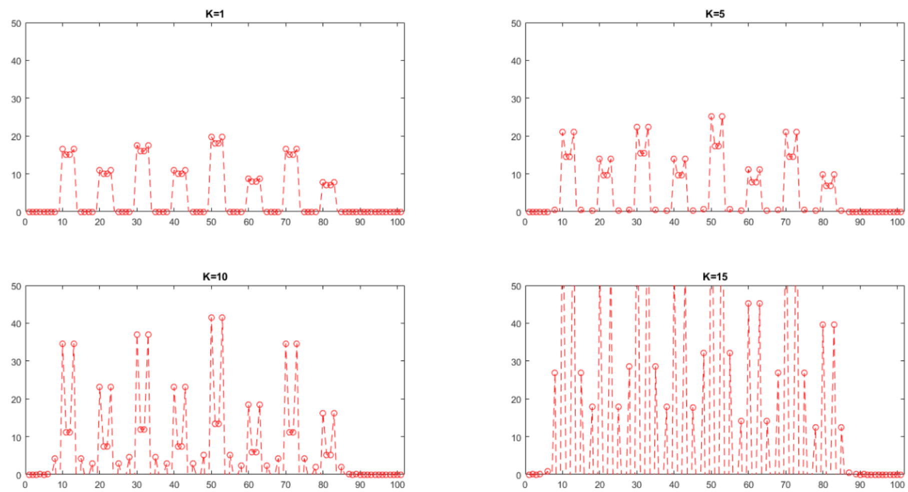

Fig. 2: Four stages of BTS of the vector u. [20].

4. Examples and Results

We consider here two examples of grayscale images using the standard images of Lena and the Cameraman.

Example 1: We demonstrate here the original image of Lena with some initial noise and calculate the gray level per pixel using a histogram. Further, we show the corresponding results of the denoised image and the gray level per pixel

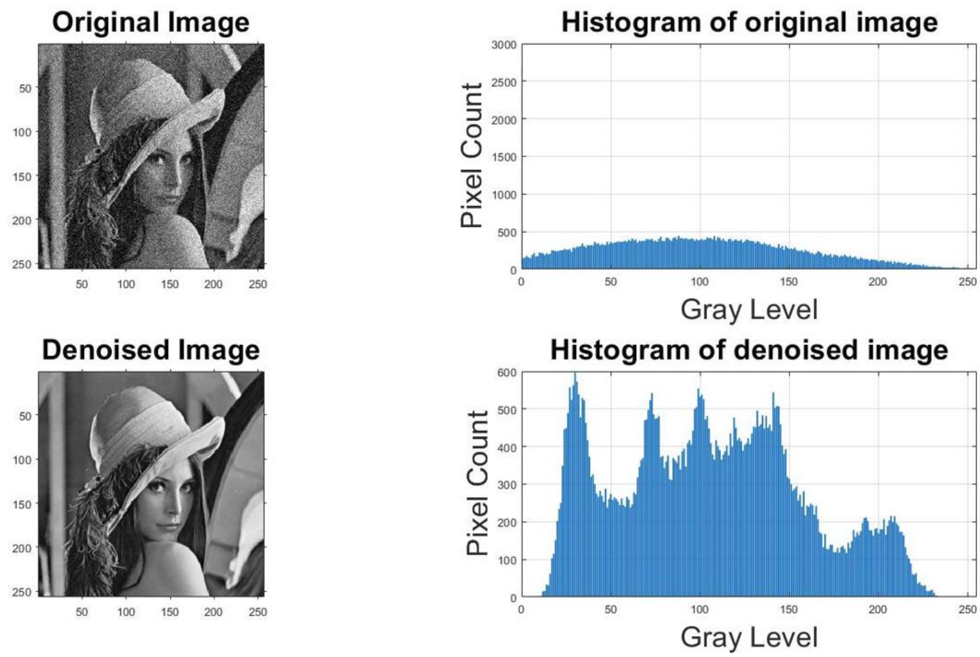

Fig. 3: Denoising Lena's image using a histogram of gray level values per pixel [21].

Example 2: We demonstrate here the original image of the Cameraman with some initial noise and calculate the gray level per pixel using a histogram. Further, we show the corresponding results of the denoised image and the gray level per pixel .

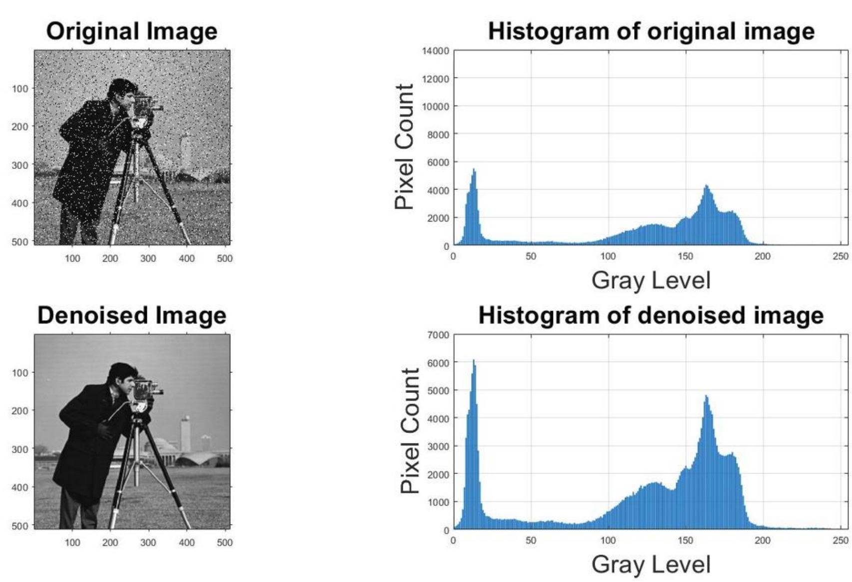

Fig. 4: Denoising the image of the Cameraman with Histogram of gray level per pixel [22].

5. Metrics of denoising performance

The mean squared error is the simplest method to define $PSNR$ (MSE). MSE is defined as the difference between a noise-free $m \times n$ image IM and its noisy approximation K.

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [IM(i,j) - K(i,j)]^2 \tag{12}$$

Therefore, PSNR can be defined as

$$\begin{aligned} PSNR &= 10 \cdot \log_{10}\left(\frac{MAX_{IM}^2}{MSE}\right) \\ &= 20 \cdot \log_{10}\left(\frac{MAX_{IM}}{\sqrt{MSE}}\right) \\ &= 20 \cdot \log_{10}(MAX_{IM}) - 10 \cdot \log_{10}(MSE) \end{aligned} \tag{13}$$

where MAX_{IM} reflects the image's highest potential pixel value.

To verify our suggested technique, perform tests on output photos and generate $PSNR$ values for each image, then compare the findings to those reported in [4] and [8]. Calculating the peak signal-to-noise ratio for our output pictures enables us to verify the suggested technique and demonstrate that it is a competitive approach when compared to other methods.

Table 1: PSNR calculations.

	PSNR Values		
Name of Image	**Fang method [4]**	**Xu method [8]**	**Suggest Method**
Example 1	21.314	22.687	21.191
Example 2	23.732	24.229	23.502
Mean	22.523	23.458	22.3465

6. Conclusion

Photoresolution issues in medical photography can be minimized by converting images to matrices and performing mathematical procedures using the Heat Equation. The Heat Equation describes natural heat diffusion, which changes resolution over time. This project aims to convert photos into matrices and perform mathematical procedures to improve and reconvert them to desired images. The project uses forward time steps and second-order central time steps to solve the one-dimensional heat equation, allowing for the determination of the value of u_j^(n+1) from known values. We developed a novel and robust approach for denoising pictures based on the heat equation in this article. The suggested approach may be implemented by modifying the previously given equation to regulate the intensity of pictures for the purpose of noise reduction. Our approach was tested on grayscale and standard images. Additionally, we compared $PSNR$ values and demonstrated that recommended approach is suitable for this purpose. Additionally, demonstrated the concept by using examples with a range of dimensions and using the discretization of the heat equation to remove certain unwanted elements from an image and lessen the image's contrast.

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