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On *ms*-Convergence of Nets and Filters

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Abstract:
In this paper I'm introduce and study another type of convergence in a minimal structures namely Minimal Structure Convergence (*ms*-convergence) of nets and filters by using the concept of *ms*-open sets. Also I'm investigate some properties of these concepts. As well as I'm used two functions on the basis of Minimal structure in various forms, one check transmission character compacting minimal structure and other check transmission two characters compacting minimal structure and compact from one side depending on the requirements of research.
Keywords: minimal structure, *ms*-open, *ms*-closed, *ms*-convergent, *ms*-limit, *ms*-cluster.

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Introduction:

The notion of converge is one of difference converge theorems used in general the basicnotion inanalysis. There're two topologies that lead to equivalent results. One of them base on the notion of anet in 1922 due to Moore and Smith [1], another one, which going back to work of Cartan [2] in 1937 is based on the notion of a filter. Al-Janabi S. H. and Al-Ysaary F. J. Provided a new concepts of minimal continuous, minimal open and minimal closed function also concepts of minimal proper function in [3] . Alimohammady M. and Roohi M. in [4] give the definition of minimal closed set (*m*-closed set) and give the definition of minimal continuous function (*m*-continuous function) and study the properties of it. Ravi O., Ganesan S., Tharmar S. and Balamukugan in [5] give the definition of minimal closed function (*m*-closed function) and study some properties of it. Also Al-Janabi S. H. and Al-Ysaary F. J. in [3] used it to construct a definition of minimal proper function and certain types of it (m_*-proper, $_*m$-proper and $_*m_*$-proper functions) . For asubset E of X, the minimal structure closure and the minimal structure interior of E in X are denoted by $\overline{E}^{ms}$ and $E^{\circ ms}$ respectively [4],[6],[7] . Now, In this paper (S, M_S) represent minimal structure space on which not separation axiom are assumed unless otherwise mentioned. For a non empty set S the T_{M_S} I'm denoted the topology on S induced by minimal structure. Finally, we give some properties of the *ms*-proper, ms_*-proper functions by using the concept of minimal structure exceptional (msE_f) set. I'm use T_U to denote the usual topology. In this paper every word (minimal) is mean (minimal structure).

1. Basic definitions and notations:

We introduce some elementary concept which we need in our work.

Definition 1.1 [4]: Let S be a non-empty setand $P(S)$ the power set of X. A subfamily M_S of $P(S)$ is called aminimal structure (briefly *m*-structure) on S if $\phi, S \in M_S$. In this case (S, M_S) is said to be minimal structure space (briefly *ms*-space) . Aset $E \in P(S)$ is said *ms*-open set if $E \in M_S$. $C \in P(S)$ is an *ms*-closed set if $C^c \in M_S$.

Remark 1.2 [3]: If (S, M_S) is *ms*-space then there's always asubfamilies T_{M_S} of M_S satisfies the conditions of topological spaces (at least the family $\{\phi, S\}$) and the intersection of these families represent the indiscrete topology on S. T_{M_S} called induced topology from minimal structure M_S. If E is open set in S is mean $E \in T_{M_S}$. Also if C is closed set in S mean that $C^c \in T_{M_S}$.

Remark 1.3 [3]:

i. For all open set is an*ms*-open set;
ii. For all closed set is an*ms*-closed set.

The converse isn't true in general and we show that from the following next example.

Example 1.4: Let $S=\{a,b,c\}$ such that $M_S=\{\phi,S,\{a\},\{b\},\{b,c\}\}$ is m-structure on S and $T_{1M_S}=\{\phi,S,\{b\}\}$ be one of the topological space induced by the minimal structure, then the set $\{a\}$ is *ms*-open (also *ms*-closed) set, but $\{a\},\{a\}^c=\{b,c\}\notin T_{1M_S}$.

Definition 1.5 [8]: Let S be a non-empty set and M_S an m-structure on S. For asubset E of S, the minimal closure of E (briefly $\overline{E}^{ms}$) and the minimal interior of E (briefly $E^{\circ ms}$), defined by:

$$\overline{E}^{ms}=\bigcap\{C:E\subseteq C,C^c\in M_S\}$$

$$E^{\circ ms}=\bigcup\{O:O\subseteq E,E\in M_S\}$$

Definition 1.6 [3]: An *ms*-space (S,M_S) is said to be an:

(i) *ums*-space if the arbitrary union of *ms*-open sets is an *ms*-open set.

(ii) *ims*-space if any finite intersection of *ms*-open sets is an *ms*-open set.

Definition 1.7: Let S be an *ms*-space and $H\subseteq S$. An *ms*-neighborhood of H is any subset of S which contains *ms*-open set containing H . The *ms*-neighborhod of asubset $\{s\}$ is also called *ms*-neighborhod of the point s .

Definition 1.8 [9]: An *ms*-space S is called *ms*-Hausdorff (*ms*-T_2) if for any two points $s,t\in S$ distinction between it's there are two *ms*-open sets K,L of S distinction between it such that $s\in K$ and $t\in L$.

Definition 1.9: If $f:S\to V$ be afunction of aspace S into aspace V then f is called:

i. *ms*-continuous if $f^{-1}(H)\in M_S$, forever $H\in M_V$ [8].

ii. ms_*-continuous if there is non-indiscrete topology T_{M_V} so that $f^{-1}(H)\in M_S, \forall H\in T_{M_V}$ [3].

Now, we review some basic definitions, theorems and remarks about a net.

Definition 1.10 [10]: Aset X is said to be adirected if there's arelation $\leq$ on X satisfy:

i. $x\leq x$ for all $x\in X$.

ii. If $x_1\leq x_2$ and $x_2\leq x_3$ then $x_1\leq x_3$.

iii. If $x_1,x_2\in X$, there's some $x_3\in X$ with $x_1\leq x_3$ and $x_2\leq x_3$.

Definition 1.11 [10]: Anet in aset S is afunction $\chi:X\to S$, where X is directe set. The point $\chi(x)$ is denoted by χ_x .

Definition 1.12 [10]: Asubnet of anet $\chi:X\to S$ is the composition $\chi\circ\varphi$, where $\varphi:M\to X$ and M is directed set, sothat:

(i) $\varphi(m_1)\leq\varphi(m_2)$, where $m_1\leq m_2$.

(ii) For ever $x\in X$ there's some $m\in M$ suchthat $x\leq\varphi(m)$. For $m\in M$ the point $\chi\circ\varphi(m)$ is often written χ_{xm} .

Definition 1.13 [11]: Let $(\chi_x)_{x\in X}$ be anet in atopological space S and $E\subseteq S, s\in S$ thus :

(i) $(\chi_x)_{x\in X}$ is called eventual in E if there's $x_o\in X$ sothat $\chi_x\in E$ for ever $x\geq x_o$.

(ii) $(\chi_x)_{x\in X}$ is called frequent in E if for ever $x\in X$ there's $x_o\in X$ with $x_o\geq x$ suchthat $\chi_{x_o}\in E$.

(iii) $(\chi_x)_{x\in X}$ be said to be convergence to s if $(\chi_x)_{x\in X}$ eventually in each neighborhood of s (written $\chi_x\to X$). The point x is said tobe alimit point of $(\chi_x)_{x\in X}$.

(iv) $(\chi_x)_{x\in X}$ be said tobe has s as acluster point if $(\chi_x)_{x\in X}$ is frequent in each neighborhod of s (written $\chi_x\propto s$).

2. *ms*-Converge of Nets:

In this part, I'm introduce other types of converge namely minimal structure convergence (*ms*-convergence) of net and study some properties of the concept of *ms*-limit point and *ms*-cluster point of the net in a given space. Also, we give some properties, remarks and examples about this subject.

Definition 2.1 [12]: Let $(\chi_x)_{x\in X}$ is anet in *ms*-space S , $s\in S$, then $(\chi_x)_{x\in X}$ is:

i. an*ms*-converge to s. If $(\chi_x)_{x\in X}$ is eventual for ever *ms*-neighborhod of S (written $\chi_x\xrightarrow{ms}s$). The point s is said *ms*-limitpoint of $(\chi_x)_{x\in X}$.

ii. said to have s as *ms*-cluster point if $(\chi_x)_{x\in X}$ it's frequent in each *ms*-neighborhod of s (written $\chi_x\overset{ms}{\propto}s$).

Remark 2.2: Let S be an *ms*-space and let $E\subseteq S$, χ_x is a net in S , $s\in S$ then the following holds If:

i. $\chi_x\xrightarrow{ms}s$ in (S,T_{M_S}) then $\chi_x\to s$ in (S,M_S) .

ii. $\chi_x\xrightarrow{ms}s$ then $\chi_x\longrightarrow s$ in (S,M_S) .

iii. $\chi_x\overset{ms}{\propto}s$ in (S,T_{M_S}) then $\chi_x\propto s$ in (S,M_S) .

iv. $\chi_x\propto s$ in (S,T_{M_S}) forever T_{M_S} in S then $\chi_x\overset{ms}{\propto}s$ in (S,M_S) .

Note that If χ_x is anet in S, $s \in S$ so that $\chi: X \to S$ in (S, M_S) then χ_x is not necessary be $\chi_x \xrightarrow{ms} S$ in (S, T_{M_S}). And we see that from next example.

Example 2.3: If $S = \{1, -1\}$, $M_S = \{\phi, S, \{1\}, \{-1\}\}$ be an *ms*-structure on S, and let $T_{1M_S} = \{\phi, S\}$ (indiscrete topology on S), and let $\{(-1)^n\}$ be a net in S then $\{(-1)^n\}$ eventually in every neighborhood of 1 (supposing that the open set is only ϕ, S) but $\{(-1)^n\}$ is not eventually in every *ms*-neighborhood of 1, Since $\{1\}$ is an *ms*-neighborhood of 1 but $\{(-1)^n\}$ is not eventually in $\{1\}$

Theorem 2.4: An *ms*-space S is *ms*-T_2 *ms*-space iff each *ms*-convergent net in S has aunique *ms*-limit point.

Proof: Let S is *ms*-T_2 *ms*-space and $(\chi_x)_{x \in X}$ be anet in S suchthat $\chi_x \xrightarrow{ms} s$, $\chi_x \xrightarrow{ms} t$ and $s \neq t$. Since S be an *ms*-T_2 *ms*-space. There're $K \in N_{ms}(s)$ and $L \in N_{ms}(t)$ suchthat $K \cap L = \phi$. Since $\chi_x \xrightarrow{ms} s$, there's $x_0 \in X$ suchthat $\chi_x \in K$ for all $x \geq x_0$. Since $\chi_x \xrightarrow{ms} t$, there is $x_1 \in X$ such that $\chi_{x_1} \in L$ for all $x \geq x_1$. Since X is directed set and $x_0, x_1 \in X$, then there's $x_2 \in X$ suchthat $x_2 \geq x_0$ and $x_2 \geq x_1$. Then $\chi_x \in K$ for ever $x \geq x_2$ and $\chi_x \in L$ for all $x \geq x_2$, thus $K \cap L \neq \phi$, this is a contradiction. So $s = t$.

Conversely: Suppose that S is not $ms - T_2$ *ms*-space, there are $s, t \in S$ and $s \neq t$, for ever $K \in N_{ms}(s)$, $L \in N_{ms}(t)$ so that $K \cap L \neq \phi$. Put $N_S^t = \{K \cap L : K \in N_{ms}(s) \text{and } L \in N_{ms}(t)\}$, where N_S^t is directed set. Thus for all $X \in N_S^t$,there's $\chi_x \in X$ then $(\chi_x)_{x \in N_S^t}$ is anet in S. To prove $\chi_x \xrightarrow{ms} s$ and $\chi_x \xrightarrow{ms} t$, let $G \in N_{ms}(s)$ thus $G \in N_S^t, G \cap S \neq \emptyset$. Thus $\chi_x \in G$ for all $X \geq G$, so $\chi_x \xrightarrow{ms} s$. Also, let $B \in N_{ms}(t)$ then $B \in N_S^t, B \cap S \neq \emptyset$. Thus $\chi_x \in Ꮍ$ for all $X \geq G$, so $\chi_x \xrightarrow{ms} t$. This is a contradiction.

Theorem2.5: If S be an *ims*-space and $E \subseteq S$, then:

i. Apoint $s \in S$ is *ms*-limit point of E iff there's anet in $E - \{s\}$ *ms*-convergence to s.

ii. Aset E is *ms*-closed in S iff no net in E *ms*-convergence to apoint in E^c.

iii. Aset E is *ms*-open in S iff no net in E^c *ms*-convergence to apoint in E.

Proof: (i) Let s is *ms*-limit point of E. To prove that there is anet $(\chi_x)_{x \in X}$ in $E - \{s\}$ so that $\chi_x \xrightarrow{ms} s$. Since s is *ms*-limit point of E, for each $K \in \mathcal{N}_{ms}(s)$, $K \cap E\{s\} \neq \phi$. Then $(\mathcal{N}_{ms}(s), \supseteq)$ is directed set by inclusion. Since $K \cap E - \{s\} \neq \phi$, for all $K \in \mathcal{N}_{ms}(s)$ then there is $\chi_K \in K \cap E - \{s\}$. Define $\chi : \mathcal{N}_{ms}(s) \to E - \{s\}$ by $(K) = \chi_K$ for all $K \in \mathcal{N}_{ms}(s)$, then $(\chi_K)_{K \in \mathcal{N}_{ms}(s)}$ is anet in $E - \{s\}$. To prove it $\chi_K \xrightarrow{ms} s$, let $K \in \mathcal{N}_{ms}(s)$ to finder $x_0 \in \mathring{\circ}$ so that $\chi_x \in K$ for all $x \geq x_0$. suppose $x_0 = K$ then for all $x \geq x_0$,$x = L \in \mathcal{N}_{ms}(s)$, i.e., $L \geq K \Leftrightarrow L \subseteq K$. Then $\chi_x = \chi(x) = \chi(L) = \chi_L \in L \cap E - \{s\} \subseteq L \subseteq K$ then $\chi_L \in K$ for all $x \geq x_0$. Thus $\chi_K \xrightarrow{ms} s$.

Conversely: Suppose that there is a net $(\chi_x)_{x \in X}$ in $E - \{s\}$ so that $\chi_x \xrightarrow{ms} s$. To prove s is *ms*-limit point of E. Let $K \in \mathcal{N}_{ms}(s)$, since $\chi_x \xrightarrow{ms} s$, then there is $x_0 \in X$ such that $\chi_x \in K$ for all $x \geq x_0$. But $\chi_x \in E - \{s\}$ foreach $x \in X$, then $K \cap E - \{s\} \neq \emptyset$ for all $K \in \mathcal{N}_{ms}(s)$. Thus s is *ms*-limit point of E.

(ii) Let E is *ms*-closed set in S and there's anet $(\chi_x)_{x \in X}$ in E when $\chi_x \xrightarrow{ms} s$ and $s \in E^c$. Then $s \in E^{ms}$, since E is *ms*-closed set, then $E = \bar{E}^{ms}$, hence $s \in E$, then $E \cap E^c \neq \emptyset$, this's acontradiction. Thus not net in E *ms*-convergence to apoint in E^c.

Conversely: Let not net in E *ms*-convergent to apoint in E^c. Let $s \in \bar{E}^{ms}$ then there is a net in A so that $\chi_x \xrightarrow{ms} s$. By hypotheses, given each net in E *ms*-convergence to apoint in E. Thus $s \in E$, so $E = \bar{E}^{ms}$ implies that E is *ms*-closed.

(iii) By using (i).

Remark 2.6:

i. Let $f: S \to V$ be afunction from *ms*-space S into *ms*-space V. if $(\chi_x)_{x \in X}$ is anet in S, then $\{f(\chi_x)\}_{x \in X}$ is anet in V.

ii. Let $f: S \to V$ be afunction from an *ms*-space S onto an *ms*-space V and $(y_x)_{x \in X}$ be a net in V. Then there's a net $(\chi_x)_{x \in X}$ in S so that $f(\chi_x) = y_x$ for ever $x \in X$.

Theorem 2.7: If S and V be *ms*-spaces. A function $f: S \to V$ is *ms*-continuous if and only if whenever $(\chi_x)_{x \in X}$ is a net in S so that $\chi_x \xrightarrow{ms} s$, then $f(\chi_x) \xrightarrow{ms} f(s)$.

Proof: Suppose $f: S \to V$ is *ms*-continuous and $(\chi_x)_{x \in X}$ is anet in S so that $\chi_x \xrightarrow{ms} s$. To prove $f(\chi_x) \xrightarrow{ms} f(s)$. Let $L \in \mathcal{N}_{ms}(f(s))$ in V. Then $f^{-1}(L) \in \mathcal{N}_{ms}(s)$, for some $x_0 \in X$, $x \geq x_0$ implies that $\chi_x \in f^{-1}(L)$. Thus, showing that $f(\chi_x) \xrightarrow{ms} f(s)$, since $(\chi_x)_{x \in X}$ is eventual in each *ms*-neighborhod of s, then $(f(\chi_x))_{x \in X}$ is a net in V which is eventually in each *ms*-neighborhood of $f(s)$. Therefore $f(\chi_x) \xrightarrow{ms} f(s)$.

Conversely: To prove $f: S \to V$ is *ms*-continuous. Supposenot. Then there is $L \in \mathcal{N}_{ms}(f(s))$ so that $f(K) \not\subset L$ forever $K \in \mathcal{N}_{ms}(s)$. Thus for ever $K \in \mathcal{N}_{ms}(s)$ we can $\chi_K \in K$ such that $f(\chi_K) \notin L$. But $(\chi_K)_{K \in \mathcal{N}_{ms}(s)}$ is a net in S with $\chi_K \xrightarrow{ms} s$ while $(f(\chi_K))_{K \in \mathcal{N}_{ms}(s)}$ doesn't *ms*-converge to $f(s)$. This is acontradiction, then f is *ms*-continuous function.

Definition 2.8 [13]: Let (S, M_s) be an *ms*-space. S is called *ms*-compact if for ever cover of S by sets of M_S has afinite subcover. A subset K of S is called *ms*-compact if for all cover of K by asubsets of M_S has afinite subcover.

Note that S is an *ms*-compact iff for all *ms*-open cover of S has afinite subcover.

Remark 2.9: If the *ms*-space (S, M_S) is *ms*-compact then the space (S, T_{M_S}) is compact for all T_{M_S} induced by M_S. But the converse isn't true in general. As the following example shows.

Example 2.10: Let $S = \Re$, $M_S = \{A : A = [a, a+1)$ such that a is odd integer number$\}$. Then $T_{iM_S} = \{\phi, X, [i, i+2)\}$ is topological space induced by M_S for all $i \in \Lambda$ so that $\Lambda = \{i : i$ is odd integer number$\}$. Then T_{iM_S} is compact for all $i \in \Lambda$ and M_S is not *ms*-compact since $\bigcup_{i\in\Lambda}[i, i+2)$ is *ms*-open cover for $\Re$ but there is not finite subcover.

Theorem 2.11: Let (S, M_S) be an *ms*-space. Then:

i. If S is an *ms*-compact and $S - K \in M_S$, then K is an *ms*-compact in S.

ii. If $f : (S, M_S) \to (V, M_V)$ is an ms_*-continuous and E is an *ms*-compact subset of S, then $f(E)$ is acompact in V.

Proof:

(i) Let (S, M_S) be an *ms*-compact and $S - K \in M_S$. Let $\{U_i \in M_S : i \in I\}$ be an *ms*-opencover of K, then $(S - K) \cap \{U_i \in M_S : i \in I\} = S$. Since S is an *ms*-compact, there exists a finite subset $I_\circ$ of I such that $(S - K) \cap \{U_i \in M_S : i \in I_o\} = S$ then $K \subseteq \bigcup\{U_i \in M_S : i \in I_o\}$ and so K is an *ms*-compact in S.

(ii) Let $\{L_i : i \in I\}$ be acover of $f(E)$. Then $E \subseteq \bigcup\{f^{-1}(L_i) : i \in I\}$, where $f^{-1}(L_i) \in M_S$. Since E is an *ms*-compact, there exists a finite subset $I_\circ$ of I so that $E \subseteq \bigcup\{f^{-1}(L_i) \in M_S : i \in I_\circ\}$. Then $f(E) \subseteq \bigcup\{L_i \in M_S : i \in I_\circ\}$ and so $f(E)$ is a compact in V.

Theorem 2.12 [11]: A spaces S is compact iff every net in S has acluster point in s.

Theorem 2.13: Let S be an *ms*-space, then S is *ms*-compact then ever net in S has an*ms*-cluster point in S.

Proof : Let (S, M_S) be an *ms*-compact space and $(\chi_x)_{x\in X}$ be a net in S, then (S, T_{M_S}) is acompact space for all T_{M_S} in S. Then by Theorem (2.12), the net $(\chi_x)_{x\in X}$ has cluster point s in (S, T_{M_S}) then s is *ms*-clusterpoint of the net $(\chi_x)_{x\in X}$. (i.e. $\chi_x \overset{ms}{\propto} s$) Hence for all net in S has *ms*-clusterpoint in S.

Corollary 2.14: If S be *ms*-space. Then S is *ms*-compact iff every net in S has asub net which *ms*-convergence to a point in S.

Theorem 2.15 [9]: Let (S, M_S) and (V, M_V) be two *ms*-spaces then $M_{S\times V} = \{K \times L : K \in M_S \text{ and } L \in M_V\}$ is an *m*-structure on $S \times V$.

Remark 2.16: Let (S_i, M_{S_i}) be an m-structure $\forall i \in \Lambda$ then its clear to show that the projection function $\Pr_\lambda : \prod_{i\in\Lambda} S_i \to S_\lambda$ is an *ms*-continuous $\forall \lambda \in \Lambda$.

Theorem 2.17: A net $(\chi_x)_{x\in X}$ in a product *ims*-space $\prod S_\lambda$, $\lambda \in \Lambda$ is *ms*-convergence to $s \in \prod S_\lambda$, if and only if $Pr_\lambda(\chi_x) \overset{ms}{\to} Pr_\lambda(s)$ for all $\lambda \in \Lambda$ (where Pr_λ is the λ -th projection function).

Proof: If $\chi_x \overset{ms}{\to} s$ in $\prod \chi_\lambda$, since $Pr_\lambda$are *ms*-continuous function, then by the theorem (2.7) we have $Pr_\lambda(\chi_x) \overset{ms}{\to} Pr_\lambda(s)$.

Conversely: Suppose that $Pr_\lambda(\chi_x) \overset{ms}{\to} Pr_\lambda(s)$ for all $\lambda \in \Lambda$. Let $Pr_{\lambda_1}^{-1}(K_{\lambda_1}) \cap Pr_{\lambda_2}^{-1}(K_{\lambda_2}) \cap \ldots .\cap Pr_{\lambda_n}^{-1}(K_{\lambda_n})$ be a basis *ms*-neighborhood of s in $\prod S_\lambda$. Then for all $i = 1, 2, \ldots., n$, there is x_i so that whenever $x \geq x_i$, $Pr_{\lambda_i} \in K_{\lambda_i}$. Then x_0 greater than for all x_i, $i = 1, 2, \ldots., n$, we have $Pr_{\lambda_i} \in K_{\lambda_i}$ for all $x \geq x_0$. It follows that for all $x \geq x_0$, $\chi_x \in \cap Pr_{\lambda_i}^{-1}(K_{\lambda_i})$, $i = 1, 2, .., n$. So $\chi_x \overset{ms}{\to} s$.

Corollary 2.18: If $(\chi_x)_{x\in X}$ be a net in a product *ims*-space $\prod S_\lambda$ having $s \in \prod S_\lambda$ as *ms*-cluster point, then for each $\lambda \in \Lambda$, $(Pr_\lambda(\chi_x))_{x\in X}$ has $Pr_\lambda(s)$ for *ms*-cluster point.

Now, we give the definition of *ms*-proper functions and some results which are related to this concept.

Definition 2.19: Afunction $f : (S, M_S) \to (V, M_V)$ is called:

i. *ms*-closed if foreach *ms*-close set H of S, $f(H)$ is *ms*-closed in V. [5]

ii. ms_*-closed if there is non-indiscrete topology T_{M_S} such that for each *ms*-closed set H of S, $f(H)$ is closed in V. [3]

Definition 2.20: Let f be a function of an *ms*-space S into an *ms*-space V then f is said to be an:

i. *ms*-proper function if f is an *ms*-continuous function and the function $f \times i_Z : S \times Z \to V \times Z$ is an *ms*-closed for ever space Z. [12]

ii. ms_*-proper function if f is an ms_*continuous function and the function $f \times i_Z : S \times Z \to V \times Z$ is an ms_*-closed for ever space Z. [3]

Recall that a sub set E_f of $f(S)$ is called exceptional set of f which defined by:
$E_f = \{t \in f(S)$: *there is a net* $(\chi_x)_{x\in X}$ *in* S *with* $\chi_x \to \infty$ and $f(\chi_x) \to t\}$, where f is a function from an *ms*-space S into an *ms*-space V. We shall introduce a new characterization, which is very useful for *ms*-proper function by using a special set namely, *ms*-exceptional (for brief msE_f) set of f.

Definition 2.21: Let f be afunction from an *ms*-space S into an *ms*-space V, the *ms*-exceptional set of f (for brief msE_f) set is a subset of $f(S)$ which defined by $msE_f = \{t \in f(S)$: ther's anet $(\chi_x)_{x\in X}$ in S with $\chi_x \overset{ms}{\to} \infty$ and $f(\chi_x) \overset{ms}{\to} t\}$

Now, we shall use msE_f to characterize *ms*-proper and ms_*-proper functions.

Theorem 2.22: If $f: S \to V$ be an *ms*-continuous function ,where S is an *ms*-compact , S and V be an *ms*-Hausdorff *im*-spaces. Then thefollowing statements are equivalent:

i. f is *ms* -proper function .
ii. If $(\chi_x)_{x\in X}$ is a net in S and $t \in V$ is *ms* - cluster point of $f\{(\chi_x)\}$, then ther's *ms* - cluster point $s \in S$ of $(\chi_x)_{x\in X}$ so that $f(s) = t$.

Proof: (i → ii) Since f be an *ms*-proper function. Then $f$is an *ms*- closed function and $f^{-1}\{t\}$ is an *ms*-compact, $\forall t \in V$. If $(\chi_x)_{x\in X}$ be a net in S and $t \in V$ be an *ms*-cluster point of a net $f(\chi_x)_{x\in X}$ in V. Claim $f^{-1}\{s\} \neq \emptyset$, if $f^{-1}\{t\} = \emptyset$, then $t \notin f(S) \Rightarrow t \in (f(S))^c$ since S is an *ms*-closed set in S and f is an *ms*-proper (*ms* -closed), then $f(S)$ is an *ms*-closed set in V. That $(f(S))^c$ is an *ms*-open set in V. Ther efore $f(\chi_x)_{x\in X}$ is frequently in $(f(S))^c$. But $f(\chi_x) \in f(S)$, for all $x \in X$. Then $f(S) \cap (f(S))^c \neq \emptyset$, an d thi s is a contradiction. Thus $f^{-1}\{t\} \neq \emptyset$,is't frequently.
Now, suppose that the statements (ii) isn't true, that me ans for $s \in f^{-1}\{t\}$ there's *ms*-open set K_s in S conta ins s so th at $(\chi_x)_{x\in X}$ is't fr equ ently in K_s .
Notice that $f^{-1}\{t\} = \bigcup_{s\in f^{-1}\{t\}}\{s\}$.
Therefore the family $\{K_s : s \in f^{-1}\{t\}\}$ is *ms*-open cover of $f^{-1}\{t\}$, but $f^{-1}\{t\}$ is *ms*-compact set. There're $s_1, s_2, \dots, s_n$ so that $f^{-1}\{t\} \subseteq \bigcup_{i=1}^n K_{s_I}$, then $f^{-1}\{t\} \cap (\bigcup_{i=1}^n K_{s_i})^c = \emptyset$. Then $f^{-1}\{t\} \cap (\bigcap_{i=1}^n K_{s_i}^c) = \emptyset$. But $(\chi_x)_{x\in X}$ is not frequently in $K_{s_i}$for each $i = 1, 2, \dots, n$. Thus is not frequently in $\bigcup_{i=1}^n K_{s_I}$, but $\bigcup_{i=1}^n K_{s_i}$is *ms*-open set in S, so $(\bigcap_{i=1}^n K_{s_i}^c)$ is *ms*-closedset in S . Th us by ass umption $f(\bigcap_{i=1}^n K_{s_i}^c)$ is *ms*-clo sed set in Y.
Claim $t \notin f(\bigcap_{i=1}^n K_{s_i}^c)$, if $t \in f(\bigcap_{i=1}^n K_{s_i}^c)$ then there's $s \in \bigcap_{i=1}^n K_{s_i}^c$ so that $f(s) = t$, thus $s \notin \bigcup_{i=1}^n K_{s_i}$ but $s \in f^{-1}\{t\}$, therefore $f^{-1}\{t\}$ isn't subset of $\bigcup_{i=1}^n K_{s_i}$, this is a contradiction. Then there is *ms*-op en set E in S so that $t \in E$ and $\cap f(\bigcap_{i=1}^n K_{s_i}^c)$. That is $f^{-1}(E) \cap f^{-1}(f(\bigcap_{i=1}^n K_{s_i}^c)) = \emptyset$, also $f^{-1}(E) \cap (\bigcap_{i=1}^n K_{s_i}^c) = \emptyset$. So $f^{-1}(E) \subseteq \bigcup_{i=1}^n K_{s_i}$. But $f\{(\chi_x)\}$ is fre quently in , then $(\chi_x)_{x\in X}$ is freque ntly in $f^{-1}(E)$ an d th en it is freq uently in $\bigcup_{i=1}^n K_{s_i}$. This is a contradiction, there 's *ms*-clu ster poi nt $s \in S$ so that $f(s) = t$.
(ii → i) To prove tha t $f \times I_Z: S \times Z \to V \times Z$ is *ms*-closed for any space Z . Le t F be *ms*-closed subset of $S \times Z$ and $(f \times I_Z)(F) = G$. To prove G is *ms*-clos ed sub set of $V \times Z$. Le t $(t, z) \in \bar{G}^{ms}$, then there's a net $\{(y_x, z_x)\}_{x\in X}$ in G so that $\{(y_x, z_x)^{ms}_{\propto}(t, z)\}$. Thus there's a net $\{(\chi_x, z_x)\}_{x\in X}$ in F so that $(f \times I_Z)(\{(\chi_x, z_x)\}) = \{(y_x, z_x)\}$ for ever $x \in X$. Sin ce $\{(y_x, z_x)^{ms}_{\propto}(t, z)\}$ by corollary (2.18) ,then $y_x \overset{ms}{\propto} y$ and$z_x \overset{ms}{\propto} z$ and $f(s) = t$. Since $\{(\chi_x, z_x)\}_{x\in X}$ in F and F is *ms*-closed. So $F = \bar{F}^{ms}$, then $(t, z) = (f \times I_Z)(s, z) \in G$. Then $G = \bar{G}^{ms}$hence G is *ms*-closed subset of $V \times Z$. Then $f \times I_Z$is *ms*-closed function , thus $f \times I_Z$ is *ms*-proper function.

Theorem 2.23: Let f is function from ms_*-space S into *ms*-space V. Then f is *ms*-proper if and only if $msE_f = \phi$.

Proof : Let $msE_f = \emptyset$. To see that $f \times I_Z: S \times Z \to V \times Z$ is *ms*-closed function for all space Z. suppose F is a closed subset of $S \times Z$ and let $(f \times I_Z)(F) = G$. To prove that G is *ms*-closed subset of $V \times Z$ let $(t_0, z_0) \in \bar{G}^{ms}$, then there's anet $\{(t_x, z_x)\}$ in G so that $(t_x, z_x) \overset{ms}{\to} (t_0, z_0)$. Therfore there's anet $\{(\chi_x, z_x)\}_{x\in X}$ in so that $(f \times I_Z)(\{(\chi_x, z_x)\}) = \{(t_x, z_x)\}$, for ever $x \in X$, by theorem (2.17), thus $f(\chi_x) \overset{ms}{\to} t_0$ and $I_Z(z_x) \overset{ms}{\to} z_0$. Since $msE_f = \phi$, then $\chi_x \overset{ms}{\to} s_0$ for some $s_0 \in S$, thus by theorem (2.17) $(\chi_x, z_x) \overset{ms}{\to} (s_0, z_0)$. Since F is closed (*ms*-closed) then $(s_0, z_0) \in F$. By ms_*-continuous of $f \times I_Z$, we has $(f \times I_Z)(\{(\chi_x, z_x)\}) = (f(\chi_x), I_Z(z_x)) \overset{ms}{\to} ((f \times I_Z(s_0, z_0)) = (f(s_0), I_Z(Z_x))$, also we have $f(s_0) = t$, which implies to $(t_0, z_0) \in G$ which means that G is *ms*-closed set.

Conversely: Let f is *ms*-proper function. To show that $msE_f = \phi$. If not, then there is a point $t_o \in msE_f$, there's anet $(\chi_x)_{x\in X}$ in S with $\chi_s \overset{ms}{\to} \infty$ so that $f(\chi_x) \overset{ms}{\to} t_o$. By remark (2.6.ii), there's apoint $s_o \in S$ so that $\chi_x \overset{ms}{\to} s_o$ and $f(s_o) = t_o$. Thus we have the net $(\chi_x)_{x\in X}$ is *ms*-convergent, this is a contradiction. Therefore $msE_f = \phi$.

Examples 2.24:

i. If f is function from *ms*-space (R, M_R) such that $M_R = T_U$ into itself defined by $f(x) = e^x$, for all $x \in X$. It's clear f is ms_*-continuous function and for each net $(\chi_n)_{n\in N}$ in R with $\chi_n \overset{ms}{\to} \infty$, then $f(\chi_n) = e^{\chi_n} \overset{ms}{\to} \infty$ therefore $msE_f = \phi$, hence f is ms_* -proper function.

ii. Let Z and R are m-structure such that M_Z, M_R denoted indiscrete and discrete m-structure respectively and $f: Z \to R$ is afunction defined by $f(x) = \frac{1}{x}$ for all $x \in Z$. Clear that f isn't ms_*-continuous and the net $(n)_{n\in N}$ in Z has ms-limit, since $f(n) = \frac{1}{n} \xrightarrow{ms} 0$, $0 \in R$ thus $0 \in msE_f$. Also, it's clearthat if $(\chi_n)_{n\in N}$ is a net in Z with $\chi_n \xrightarrow{ms} \infty$, then $f(\chi_n) = \frac{1}{\chi_n} \xrightarrow{ms} 0 \in f(Z)$, therefore $msE_f \neq \phi$ and f is not ms_*-proper.

Now, we give some results on *ms*-proper functions, new proofs by using our exceptional set msE_f which makes the proofs much simpler.

Remark 2.25: Clear that if S is *ms*-compact, then an *ms*-continuous function $f: S \to V$ is *ms*-proper, when S and V are *ms*-Hausdorff spaces. This follow from thefact that all net in *ms*-compactness space has *ms*-convergent subnet, therefore $msE_f = \emptyset$.

3. *ms*-ConvergenceOf Filters:

In that section, I'm introduce anew type of converge namely, *ms*-converge of filter. Also, we given examples and theorem about this concept.

Now, we review some basic definitions, theorems and remarks about a filter.

Definition 3.1 [14]: Let ξ be a nonempty collection of a nonempty subset of a non-empty set S. We say that ξ is afilter on S if :

i. If $F_1, F_2 \in \xi$, then $F_1 \cap F_2 \in \xi$.

ii. If $F_1 \in \xi$ and $F_1 \subseteq F_2$ then $F_2 \in \xi$.

Definition 3.2 [10]: A sub collection ξ_0 of a filter ξ on a non-empty set S is called afilter base if and only if each element of ξ contains some element of ξ_0 .i.e. each $F \in \xi$ there is $F_0 \in \xi_0$ such that $F_0 \subseteq F$.

Remark 3.3 [10]: If ξ_0 is a filter base for a filter ξ on a non-empty set S. Then $\xi = \{F \subseteq S : F_0 \subseteq F$, for some $F_0 \in \xi_0\}$ is called filter generated by ξ_0.

Definition 3.4 [10]: A filter ξ on a space S is called be converge to apoint $s \in S$ (written $\xi \to s$) if and only if $\mathcal{N}(s) \subseteq \xi$. The point is $s \in S$ called a limit point of ξ. Also, we said $s \in S$ is acluster point of ξ and it is denoted by $(\xi \propto s)$ iff $F \cap K \neq \emptyset$, for all $F \in \xi$ and $K \in \mathcal{N}(x)$.

Remark 3.5 [10]: Let be f afunction from aspace S into aspace V, then:

i. If ξ is a filter on S. Then $f(\xi)$ is a filter on V having for a base the sets $f(F)$, $F \in \xi$.

ii. If ξ_0 is afilter base on S. Then $f(\xi_0)$ is a filter base on V.

Definition 3.6: A filter ξ on an *ms*-space S is called be *ms*-converge to apoint $s \in S$ (written $\xi \xrightarrow{ms} s$) if and only if $\mathcal{N}_{ms}(s) \subseteq \xi$. Also, a filter ξ on an *ms*-space s has $s \in S$ as *ms*-cluster point (written $\xi \overset{ms}{\propto} s$) if and only if $F \in \xi$ meets each $K \in \mathcal{N}_{ms}(s)$.

Theorem 3.7: A filter ξ on an *ms*-space S has $s \in S$ as *ms*-cluster point iff $s \in \overline{E}^{ms}$, forever $E \in \xi$.

Proof: $\xi \overset{ms}{\propto} s \Leftrightarrow s \in \cap \overline{E}^{ms} \Leftrightarrow$ for all $K \in \mathcal{N}_{ms}(s)$ and for all $E \in \xi$, $K \cap E \neq \emptyset \Leftrightarrow s \in \overline{E}^{ms}$, for all $E \in \xi \Leftrightarrow$ $s \in \overline{E}^{ms}$.

Remark 3.8: If ξ be a filter on an *ms*-space S and $s \in S$, then its clear to show that If $\xi \xrightarrow{ms} s$ ($\xi \overset{ms}{\propto} s$) then $\xi \to s$ ($\xi \propto s$) andif $\xi \xrightarrow{ms} s$ then $\xi \overset{ms}{\propto} s$.
The converse in this remark isn't true in general and see that from nextexample:

Examples 3.9:

i. If $S = \{1,2,3\}$, $M_S = \{\emptyset, S, \{2\}, \{3\}, \{2,3\}\}$ and $T_{M_S} = \{\emptyset, S, \{2,3\}\}$, let $\xi = \{S, \{2,3\}\}$ and $\mathcal{N}(3) = \{S, \{2,3\}\}$. Since $\mathcal{N}(3) \subseteq \xi$, then $\xi \to 3$. But $\mathcal{N}_{ms}(3) = \{S, \{3\}, \{2,3\}, \{1,3\}\}$ then $\mathcal{N}_{ms}(3) \not\subset \xi$. Thus ξ does not *ms*-convergence to 3.

ii. let $S = \{1,2,3\}$ such that M_S, T_{M_S} are discrete topology then there is $T_{M_S} = P(S)$. Let $\xi = \{S, \{1,3\}\}$be a filter on S, $\mathcal{N}(2) = S$ then $\xi \propto 2$. Since$\mathcal{N}_{ms}(2) = \{S, \{2\}, \{1,2\}, \{2,3\}\}$, then$\{1,3\} \in \xi$, $\{2\} \in N_{ms}(2)$, then$\{1,3\} \cap \{2\} = \emptyset$. Thus 2 does not *ms*-cluster point at ξ.

iii. Let $S = R$, $M_S = T_U$ and $\xi = \{E \subseteq R : [-1,1] \subseteq E\}$ be a filter on R, then $\xi \overset{ms}{\propto} 0$ but ξ does not *ms*-convergence to 0, since $(-1,1) \in \mathcal{N}_{ms}(0)$, but $(-1,1) \notin \xi$.

Definition 3.10: A filter base $\xi_0$on an *ms*-space S is called be *ms*-convergence to $s \in S$ (written $\xi_0 \xrightarrow{ms} s$) if and only if the filter generated by ξ_0 *ms*-convergent to s. Also, we say that a filter base ξ_0 has $s \in S$ as *ms*-cluster point (written $\xi_0 \overset{ms}{\propto} s$) if and only if each $F_0 \in \xi_0$ meets each $K \in \mathcal{N}_{ms}(s)$.

Definition 3.11: Let ξ_0 be a filter base on an *ms*-space S, $s \in S$.Then:

i. A point s is called be *ms*-accumulattion point of ξ_0 if $s \in \cap \overline{F_o}^{ms}$, for every$F_0 \in \xi_0$.

ii. A point s is called be *ms*-adherent point of ξ_0 if $s \in \overline{F_o}^{ms}$, for every $F_0 \in \xi_0$.

Remark 3.12: Every *ms*-adherent point is *ms*-accumulation point.

Theorem 3.13: A filter base ξ_0 on an *ms*-space S is *ms*-convergence to a point $s \in S$ iff foreach $K \in \mathcal{N}_{ms}(s)$, there's $F_0 \in \xi_0$ so that is such that $F_0 \subseteq K$.

Proof: Given $\xi_0 \overset{ms}{\longrightarrow} s$, then a filter ξ generated by ξ_0 and $\xi \overset{ms}{\longrightarrow} s$. Then $\mathcal{N}_{ms}(s) \subseteq \xi$, hence for each $K \in \mathcal{N}_{ms}(s)$, $K \in \xi$ thus there is $F_0 \in \xi_0$ such that $F_0 \subseteq U$.

Conversely: To prove that $\xi_0 \overset{ms}{\longrightarrow} s$ i.e., ξ be a filter on S generated by ξ_0 with $\xi \overset{ms}{\longrightarrow} s$. Let $K \in \mathcal{N}_{ms}(s)$ then by hypotheses, there is $F_0 \in \xi_0$ such that $F_0 \subseteq K$, since ξ is a filter on S, then $K \in \xi$. Hence $K \in \xi$ and $\mathcal{N}_{ms}(s) \subseteq \xi$,therefore $\xi_0 \overset{ms}{\longrightarrow} s$.

Theorem 3.14: A filter ξ on an *ms*-space S has $s \in S$ as an *ms*-cluster point if and only if there's a filter ξ' finer than ξ which *ms*-convergence to s.

Proof: Suppose that $\xi \overset{ms}{\propto} s$, then by definition (3.10) each $F \in \xi$ meets each $K \in N_{ms}(s)$. Then $\xi_0' = \{ K \cap F : K \in \mathcal{N}_{ms}(s), F \in \xi \}$ is a filter base for some filter ξ' which is finer than ξ and *ms*-convergence to s.

Conversely: Give $\xi \subseteq \xi'$ and $\xi' \overset{ms}{\longrightarrow} x$, then $\xi' \overset{ms}{\longrightarrow} s$ and $\mathcal{N}_{ms}(s) \subseteq \xi'$. Hence each $F \in \xi$ and each $K \in \mathcal{N}_{ms}(s)$ belong to ξ'. Since ξ' is a filter, then $K \cap F \neq \emptyset$.

Theorem 3.15: Let S be an *ms*-space, $E \subseteq S$, $s \in S$. Then $s \in \overline{E}^{ms}$ iff there's afilter ξ on S so that $E \in \xi$ and $\xi \overset{ms}{\longrightarrow} s$.

Proof: If $s \in \overline{E}^{ms}$, then $E \cap K \neq \emptyset$ for all $K \in \mathcal{N}_{ms}(s)$. Then $\xi_0 = \{ E \cap K : K \in \mathcal{N}_{ms}(s)\}$ it's afilter base for somefilter ξ. The result filter contain E and $\xi \overset{ms}{\longrightarrow} s$.

Conversely: Let $E \in \xi$ and $\xi \overset{ms}{\longrightarrow} s$, then $\mathcal{N}_{ms}(s) \subseteq \xi$. Since ξ is afilter and $E \cap K \neq \emptyset$ for all $K \in \mathcal{N}_{ms}(s)$. Thus $s \in \overline{E}^{ms}$.

Corollary 3.16: Let S be an *ms*-space, $E \subseteq S$, $s \in S$. Then $s \in \overline{E}^{ms}$ iff there's afilter base ξ_0 on S so that $E \in \xi_0$ and $\xi \overset{ms}{\longrightarrow} s$.

Theorem 3.17: Let $f: S \to V$ is afunction and ξ is afilter on, $s \in S$. Then f is *ms*-continuous if and only if whenever $\xi \overset{ms}{\longrightarrow} s$ in S, then $f(\xi) \overset{ms}{\longrightarrow} f(s)$ in V.

Proof: Supposethat f is *ms*-continuous function and $\xi \overset{ms}{\longrightarrow} s$. To prove $f(\xi) \overset{ms}{\longrightarrow} f(s)$ in V. Let $s \in \mathcal{N}_{ms}(f(s))$, since f be *ms*-continuous, then there's $K \in \mathcal{N}_{ms}(s)$ so that $f(K) \subseteq L$ Since $\xi \overset{ms}{\longrightarrow} s$, then $K \in \xi$. But $L \in f(\xi)$, thus $f(\xi) \overset{ms}{\longrightarrow} f(s)$.

Conversely: Suppose that the condition is holds, to prove that f is *ms*-continuous. Let $\xi = \{K : K \in \mathcal{N}_{ms}(s)\}$ is a filter on S and $\xi \overset{ms}{\longrightarrow} s$. By hypotheses $f(\xi) \overset{ms}{\longrightarrow} f(s)$, for each $L \in \mathcal{N}_{ms}(f(s))$, we have $L \in f(\xi)$. There is $K \in \mathcal{N}_{ms}(s)$ so that $f(K) \subseteq L$. That f is *ms*-continuousfunction.

Theorem 3.18: Let S be an *ms*-space, $E \subseteq S$. Apoint $s \in S$ is *ms*-limitpoint of E iff E-$\{s\}$ belong to somefilter ξ which *ms*-convergence to s.

Proof: Suppose that s is *ms*-limit point,then $K \cap E$-$\{s\} \neq \emptyset$ forevery $K \in \mathcal{N}_{ms}(s)$. $\xi_0 = \{ K \cap E\text{-}\{s\} : K \in \mathcal{N}_{ms}(s)\}$ be afilter base forsome filter ξ. The result filter contain e-$\{s\}$ with $\xi \overset{ms}{\longrightarrow} s$.

Conversely: If E-$\{s\} \in \xi$ with $\xi \overset{ms}{\longrightarrow} s$, then E-$\{s\} \in \xi$. $\mathcal{N}_{ms}(s) \subseteq \xi$. Sincee ξ is afilter. Then $K \cap E$-$\{s\} \neq \emptyset$ for all $K \in \mathcal{N}_{ms}(s)$. Hence s is *ms*-limitpoint of aset E.

Definition 3.19 [15]: Let $(\chi_x)_{x\in X}$ is anet in aspace S, ξ is afilter generate by afilterbase ξ_0 consist of the sets $B_{x_0} = \{\chi_x : x \geq x_0, x_0 \in X\}$ is called a filter generated by $(\chi_x)_{x\in X}$. i.e., $\xi_0 = \{ B_{x_0} \subseteq S$ χ_x is eventually in B_{x_0} $\}$ is a filter base, ξ is a filter on S and it is called a filter associated with the net $(\chi_x)_{x\in X}$.

Theorem 3.20: A net $(\chi_x)_{x\in X}$ in an *ms*-space S *ms*-convergence to $s \in S$ iff a filter ξ generated by $(\chi_x)_{x\in X}$ *ms*-convergent to s.

Proof: A net $(\chi_x)_{x\in X}$ *ms*-convergent to $s \in S$ iff each $K \in N_{ms}(s)$ contains atail of $(\chi_x)_{x\in X}$, sincethe tailsof $(\chi_x)_{x\in X}$ are abase for afilter generate by $(\chi_x)_{x\in X}$, the result follows.

Definition 3.21 [15]: Let ξ_0 be a filter base on a space S. For all $F_1, F_2 \in \xi_0$, we put $F_1 \geq F_2$ iff $F_1 \subseteq F_2$, then $(\xi_0, \geq)$ is a directed set. For all $F \in \xi_0$, define $\chi : \xi_0 \to \cup F, F \in \xi_0$ such that for all $F \in \xi_0$ take (fixed) $\chi_F \in F$ so that $\chi(F) = \chi_F$. Thus $(\chi_F)_{F\in\xi_0}$ is a net in S and it is called a net associated with a filter base ξ_0.

Theorem 3.22: Let $(\chi_F)_{F\in\xi_0}$ be a net associated with a filter base ξ_0 on an *ms*-space S and $s \in S$. If $\xi_0 \overset{ms}{\longrightarrow} s$, then $\chi_F \overset{ms}{\longrightarrow} s$.

Proof: Let $\xi_0 \overset{ms}{\longrightarrow} s$ and $K \in \mathcal{N}_{ms}(s)$. Thus there is $F_0 \in \xi_0$ such that $F_0 \subseteq K$, then $\chi_{F_0} \in K$, so $\chi_F \in K$ for all $F \geq F_0$. Therefore $\chi_F \overset{ms}{\longrightarrow} s$.

The converse of this theorem isn't true in general. See that from nextexample:

Example 3.23: If $S = \{1,2,3\}$ and $M_S = \{\emptyset, S, \{1\}\}$ be m-structure on S. Put $\xi_0 = \{1,3\}$ and $\xi = \{\{1,3\}, S\}$ $\mathcal{N}_{ms}(1) = \{S, \{1\}, \{1,2\}, \{1,3\}\}$. Define $\chi : \xi_0 \to \{1,3\}$ by $\chi(\{1,3\}) = 1$, then χ is a net in S. Thus $\chi \overset{ms}{\longrightarrow} 1$ but ξ_0 does not *ms*-convergence to 1, since $\{1\} \in \mathcal{N}_{ms}(s)$ but $\{1\} \notin \xi$.

Definition 3.24 [15]: Let ξ_0 be afilter base on aspace. Put $X = \{ (s,F) : s \in F, F \in \xi_0 \}$, $(X, \geq)$ is adirected set by relating, $(s_1, F_1) \geq (s_2, F_2)$ if and only if $F_1 \subseteq F_2$, so define a function $\chi: X \to S$, by $\chi(x) = \chi_x \in S$, where $x = (s,F)$. Then $(\chi_x)_{x\in X}$ is called the canonical net (net based) of ξ_0.

Theorem 3.25: A filter base ξ_0 on an *ms*-space S is *ms*-convergence to $s \in S$ iff the canonical net of ξ_0 *ms*-convergence to s.

Proof: Let $\xi_0 \overset{ms}{\longrightarrow} s$ and $K \in \mathcal{N}_{ms}(s)$, then there's $F_0 \in \xi_0$ so that $F_0 \subseteq K$. Since $F_0 \neq \emptyset$, there's $s_0 \in F_0$. Pick $x_0 = (s_0, F_0)$, then $\chi_x \in K$ for all $x \geq x_0$. Therefore $\chi_x \overset{ms}{\longrightarrow} s$.

Conversely: Let $\chi_x \overset{ms}{\longrightarrow} s$ and $K \in \mathcal{N}_{ms}(s)$, there's $x_0 \in X$ so that $\chi_x \in K$ for ever $x \geq x_0$. Thus there's $F_0 \in \xi_0$ and $s_0 \in F_0$ such that $x_0 = (s_0, F_0)$. To prove $F_0 \subseteq K$, let $s_0 \in F_0$. Then $x = (s, F_0) \geq (s_0, F_0) = x_0$, thus $\chi_x \in K$. Hence $F_0 \subseteq K$, therefore $\xi_0 \overset{ms}{\longrightarrow} s$.

Corollary 3.26: A filter base ξ_0 on an *ms*-space S has $s \in S$ as an *ms*-cluster point if and only if the canonical net on ξ_0 has s as an *ms*-cluster point.

Theorem 3.27: An *ms*-space S is $ms - T_2$ *ms*-space if and only if every *ms*-convergefilter in S have aunique *ms*-limitpoint.

Proof: If S be a $ms - T_2$ *ms*-space and ξ be a filter on S so that $\xi \overset{ms}{\longrightarrow} s$ and $\xi \overset{ms}{\longrightarrow} t$ with $s \neq t$. Since be an $ms - T_2$ *ms*-space, then there's $K \in \mathcal{N}_{ms}(s)$ and $L \in \mathcal{N}_{ms}(t)$ so that $K \cap L = \emptyset$. Since $\xi \overset{ms}{\longrightarrow} s$, then $\mathcal{N}_{ms}(s) \subseteq \xi$ and $\xi \overset{ms}{\longrightarrow} t$ then $\mathcal{N}_{ms}(t) \subseteq \xi$. Since be a filter, then $K \cap L \neq \emptyset$. This is a contradiction, hence the result follows.

Conversely: To provethat S is an $ms - T_2$ *ms*-space. Supposenot, then there're $s, t \in S$ with $s \neq t$ so that forever $K \in \mathcal{N}_{ms}(s)$ and forever $L \in \mathcal{N}_{ms}(t), K \cap L \neq \emptyset$. Then $\xi_0 = \{K \cap L \ : K \in \mathcal{N}_{ms}(s) \text{ and } L \in \mathcal{N}_{ms}(t)\}$ is a filter base forsome filter ξ. The result filter *ms*-convergence at s and t. This is a contradiction, thus S is $ms - T_2$ *ms*-space.

Theorem 3.28: An *ms*-space S be *ms*-compactness space iff all filter base ξ_0 with *ms*-adherent point $s \in S$ *ms*-convergence to s.

Proof: Suppose that S be an *ms*-compact and $s \in S$ be an *ms*-adherent point of ξ_0. Then $s \in \overline{E}^{ms}$ for all $E \in \xi_0$, then by corollary (3.16) we have $\xi_0 \overset{ms}{\longrightarrow} s$.

Conversely: Suppose that $\xi_0 \overset{ms}{\longrightarrow} s$, by theorem (3.22) every net associated with a filter base ξ_0 *ms*-convergence to s.Thus by corollary (2.14), every net has a subnet which *ms*-convergence to s. Thus S is *ms*-compact space.

Theorem 3.29: A filter ξ on a product *ms*-space $\prod S_\lambda$, $\lambda \in \Lambda$ is *ms*-convergence to $s \in \prod S_\lambda$ if and only if $Pr_\lambda\,(\xi) \overset{ms}{\longrightarrow} Pr_\lambda(s)$ in S_λ, for ever $\lambda \in \Lambda$.

Proof: If $\xi \overset{ms}{\longrightarrow} s$ in $\prod S_\lambda$, $\lambda \in \Lambda$. Since Pr_λ are *ms*-continuous, by theorem (3.17), $Pr_\lambda\,(\chi_x) \overset{ms}{\longrightarrow} Pr_\lambda(s)$ in S_λ for each $\lambda \in \Lambda$.

Conversely: By using theorem (2.17).

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حول تقارب البنية الاصغرية للشبكات والمُرشِحات

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المستخلص:

في هذا البحث سأقدم وأدرس نوع جديد من التقارب للبنية الاصغرية اطلقت عليه اسم (تقارب البنية الاصغرية للشبكات والمرشحات) باستخدام المجموعات المفتوحة الاصغرية، كما تمكنت من تحقيق بعض الخصائص لهذه النوع . كذلك استخدمت دالتين معرفتين على اساس البنية الاصغرية باشكال مختلفة احدهما تحقق انتقال صفة التراص الاصغري والاخرى تحقق انتقال فيما بين صفتي التراص الاصغري والتراص من جهة واحدة تبعاً ومتطلبات البحث .