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On Semi –Complete Bornological Vector Space

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Abstract:

The definition of semi-convergence of nets, semi-cauchy nets in convex bornological vector space and semi-complete bornological vector space and the relationship among these concepts have been studied in this paper. Also, we introduce some theorems of these concepts and get some results. The main results of this study are of considerable interest in many situations.

Keywords: bornological vector space, convergence, semi-bounded

Mathematics subject classification: 46A17.

1.Introduction

The concept of bornological convergence had been studied in [5], [6] and [7].The convergent net in convex bornological vector space and its results studied [1]. In general every bornologically convergent net is topologically convergent and the converse is false. Semi bounded sets were first introduced and investigated in [2].Since that time semi bounded sets have been used to define and study many new bornological proprieties. We define semi-convergent net in convex bornological vector space and introduce definition of bornological semi-cauchy nets in convex bornological vector space in the context of this study and then semi-complete bornological convex vector has been investigated in section four. space some results.

Definition 1.1.[6]:- A bornology on a set X is a family β of subsets of X satisfy the following axioms:

(i) β is a covering of X, i.e. $X = \bigcup_{B\in\beta} B$;

(ii) β is hereditary under inclusion i.e. if $A \in \beta$ and B is a subset of X contained in A, then $B \in \beta$;

(iii) β is stable under finite union.

A pair (X, β) consisting of a set X and a bornology β on X is called a bornological space, and the elements of β are called the bounded subsets of X.

Example 1.2.[6]:- Let R be a field with the absolute value. The collection:

β = $\{A \subseteq R: A$ is bounded subset of R in the usual sense for the absolute value$\}$. Then β is a bornology on R called the Canonical Bornology of R.

Definition 1.3.[6]:- Let E be a vector space over the field K and β be a bornology on E then β is called vector bornology on E .If β is stable under vector addition, homothetic transformations and the formation of circled hulls, in other words, if the sets $A+B, \lambda A$, $\bigcup_{|\alpha\leq 1|} \alpha A$ belongs to β whenever A and B belong to β and $\lambda \in K$. The pair (E, β) is called a bornological vector space.

Definition 1.4.[6]:- A bornological vector space is called a convex bornological vector space if the disked hull of every bounded set is bounded i.e. it is stable under the formation of disked hull.

Definition 1.5.[6]:- A separated bornological vector space (E, β) is one where $\{0\}$ is the only bounded vector subspace of E.

Definition 1.6.[6]:- Let E be a bornological vector space. A subset $A \subseteq E$ is said to be bornologically closed (briefly, b- closed) if the conditions $(x_\gamma)_{\gamma\in\Gamma}$and $x_\gamma \to x$ in E imply that $x \in A$.

Remark 1.7.[6] :- Let E and F be bornological vector spaces and let: $u: E \longrightarrow F$ be a bounded linear map. The inverse image under u of b-closed subset of F is b-closed in E, since $x_\gamma \longrightarrow x$ in E implies $u(x_\gamma) \longrightarrow u(x)\ in\ F$.

Theorem 1.8.[6] :- A bornological vector space E is separated if and only if the vector subspace $\{0\}$ is b- closed in E.

Definition 1.9.[6]:- Let E be a bornological vector space. A sequence $\{x_n\}$ in E is said to be converge bornologically to 0 if there exists a circled bounded subset B of E and a sequence $\{\lambda_n\}$ of scalars tending to 0,such that $x_n \in B$ and $x_n \in \lambda_n B$, for every integer $n \in N$. Bornological convergence is also called Macky-Convergence writes $x_n \overset{M}{\to} 0\ if\ (x_n - x) \overset{M}{\to} 0$, and we write $x_n \overset{M}{\to} x$.

Definition 1.10.[2]:- A subset A of a bornological space X is called a semi-bounded (s-bound) if and only if there is bounded subset B of X such that $B \subseteq A \subseteq \bar{B}$ where $\bar{B}$ is the set of all upper and lower bounds of B which contains B.

Remark 1.11.[2]:- In any bornological space, every bounded set is s-bounded. But the converse is not true in general.

Definition 1.12.[2]:- A map f from bornological space X into a bornological space Y is said to be semi bounded map if the image under f of every bounded subset of X is semi bounded in Y.

Definition 1.13.[2]:- Let E and F be two bornological vector spaces. A semi bounded linear map of E into F is that map which is both linear and semi bounded at the same time.

2.Bornological semi-convergence net

In this section, we define semi-convergence of net in every convex bornological vector space, and the main propositions and theorems about this concept.

Definition 2.1:- Let $(x_\gamma)_{\gamma\in\Gamma}$ be a net in a convex bornological vector space E. We say that (x_γ) bornologically semi-converges to 0 $\left((x_\gamma)\overset{bs}{\rightarrow}0\right)$, if there is an absolutely convex set and semi-bounded S of E and a net (λ_γ) in K converging to 0, such that $x_\gamma \in \lambda_\gamma S$, for every $\gamma \in \Gamma$. Then A net (x_γ) bornologically semi-converges to a point $x \in E$ and $\left(x_\gamma \overset{bs}{\rightarrow} x\right)$ when $\left((x_\gamma - x)\overset{bs}{\rightarrow}0\right)$.

Remark 2.2:- Let E be a convex bornological vector space. A net $(x_\gamma)_{\gamma\in\Gamma}$ in E is bornologically semi- convergent to a point $x \in E$, if there is a decreasing net (λ_γ) of positive real numbers tending to zero such that the net $(\frac{x_\gamma - x}{\lambda_\gamma})$ is a semi-bounded.

Theorem 2.3:- Every bornologically semi-convergent net is semi-bounded.

Proof:- Let E be a convex bornological vector space and a net (x_γ) bornologically semi-converges to a point $x \in E$. i.e. $(x_\gamma - x)\overset{bs}{\rightarrow}0$, there is an absolutely convex subset and semi-bounded S of E and a net (λ_γ) of scalars tend to 0, such that $x_\gamma \in S$ and $(x_\gamma - x) \in \lambda_\gamma S$ for every $\gamma \in \Gamma$.

Since (λ_γ) is a net of scalars tend to 0, and E is a convex bornological vector space then $\lambda_\gamma\, S$ is a semi-bounded subset of E ,

i.e. $\left\{(x_\gamma - x)_{\gamma\in\Gamma}\right\} \subseteq \left\{(x_\gamma - x)_{\gamma\in\Gamma} \subseteq \lambda_\gamma S\right\}$ (every subset of semi-bounded is semi-bounded [3])

implies $(x_\gamma - x)$ is a semi-bounded subset of E, then (x_γ) is semi-bounded.

Theorem 2.4:- Let E and F be convex bornological vector space. Then the image of a bornologically semi-convergent net under a semi-bounded linear map of E into F is a bornologically semi-convergent net.

Proof:- Let (x_γ) be a net semi- converges bornologically to a point x in E, and let $u: E \longrightarrow F$ be a semi-bounded linear map. Since $(x_\gamma - x)\overset{bs}{\rightarrow}0$ in E, then there is an absolutely convex set and semi-bounded S of E and a net (λ_γ) of scalars tending to 0, such that $x_\gamma \in S$ and $x_\gamma - x \in \lambda_\gamma\, S$ for every $\gamma \in \Gamma$.

Then $u(x_\gamma - x) \in u(\lambda_\gamma S)$ for every $\gamma \in \Gamma$. Since u is a linear map and (λ_γ) is a net of scalars, then $u(x_\gamma) - u(x) \in \lambda_\gamma u(S)$, since u is a semi-bounded map, we have $u(S)$ is semi-bounded and disk (if absolutely convex set then it is disked [6]) when S is semi-bounded and disked by definition 2.1 we have $u(x_\gamma) - u(x) \overset{bs}{\rightarrow} 0$ then $u(x_\gamma)$ bornologically semi-converges to a point $u(x)$ in F.

Theorem 2.5:- Suppose that $(x_\gamma)_{\gamma\in\Gamma}$ and $(y_\gamma)_{\gamma\in\Gamma}$ are bornologically semi-convergent nets in a convex bornological vector space E, (λ_γ) is a convergent net in K such that $x_\gamma \overset{bs}{\rightarrow} x$, $y_\gamma \overset{bs}{\rightarrow} y$ and $\lambda_\gamma \longrightarrow \lambda$ then

(i) $x_\gamma + y_\gamma \overset{bs}{\rightarrow} x + y$;

(ii) $cx_\gamma \overset{bs}{\rightarrow} cx$, for any number c ∈ K;

(iii) $\lambda_\gamma x_\gamma \overset{bs}{\rightarrow} \lambda x$.

Proof:-

(i) Since $x_\gamma \overset{bs}{\rightarrow} x$, $y_\gamma \overset{bs}{\rightarrow} y$ in E i.e. $x_\gamma - x \overset{bs}{\rightarrow} 0$, $y_\gamma - y \overset{bs}{\rightarrow} 0$ in E, then there exist an absolutely convex and semi-bounded sets S_1, S_2 of E and nets (α_γ), (β_γ) of scalars tending to 0, such that :

$(x_\gamma - x) \in \alpha_\gamma S_1$ and $(y_\gamma - y) \in \beta_\gamma S_2$ for every $\gamma \in \Gamma$. Then $x_\gamma - x + y_\gamma - y \in \alpha_\gamma S_1 + \beta_\gamma S_2 = (\alpha_\gamma + \beta_\gamma)(S_1 + S_2) - \alpha_\gamma S_2 - \beta_\gamma S_1$.

Since $\alpha_\gamma \longrightarrow 0$ and $\beta_\gamma \longrightarrow 0$.

Then $\left((x_\gamma + y_\gamma) - (x + y)\right) \in \alpha_\gamma S_1 + \beta_\gamma S_2 \subseteq (\alpha_\gamma + \beta_\gamma)(S_1 + S_2)$

Now, if $\alpha_\gamma \longrightarrow 0$ and $\beta_\gamma \longrightarrow 0$ in K then $\alpha_\gamma + \beta_\gamma \longrightarrow 0$ in K and S_1+S_2 is semi-bounded and absolutely convex set when S_1 and S_2 are semi-bounded and absolutely convex sets and since E is a bornological vector space

implies $\left((x_\gamma + y_\gamma) - (x + y)\right) \overset{bs}{\rightarrow} 0$ then $(x_\gamma + y_\gamma) \overset{bs}{\rightarrow} (x + y)$.

(ii) If $x_\gamma \overset{bs}{\rightarrow} x$ then $(x_\gamma - x) \overset{bs}{\rightarrow} 0$ and so there is an absolutely convex set and semi-bounded S of E and a net (α_γ) of scalars tends to 0, such that $x_\gamma \in S$ and $(x_\gamma - x) \in \alpha_\gamma S$ for every $\gamma \in \Gamma$ $c(x_\gamma - x) \in c\alpha_\gamma S$ then $cx_\gamma - cx \in \alpha_\gamma(cS)$. Since $c \in K$ and E is a convex bornological vector space then cS is an absolutely convex set and semi-bounded of E when S is an absolutely convex set and semi-bounded of E by(definition 2.1) $cx_\gamma - cx \overset{bs}{\rightarrow} 0$, then $cx_\gamma \overset{bs}{\rightarrow} cx$.

(iii) If (x_γ) bornologically semi-converges to x in E, then there is an absolutely convex and semi-bounded set S of E and a net (α_γ) of scalars tends to 0, such that

$x_\gamma \in S$ and $(x_\gamma - x) \in \alpha_\gamma S$ for every $\gamma \in \Gamma$. $\lambda_\gamma x_\gamma - \lambda x = (x_\gamma - x)(\lambda_\gamma - \lambda) + x(\lambda_\gamma - \lambda) + \lambda(x_\gamma - x)$. Now $\lambda_\gamma \longrightarrow \lambda$, then $\lambda_\gamma - \lambda \longrightarrow 0$ and $x_\gamma \overset{bs}{\rightarrow} x$. If $(x_\gamma - x) \overset{bs}{\rightarrow} 0$ then $\lambda_\gamma x_\gamma - \lambda x = \lambda_\gamma x_\gamma - \lambda x = (x_\gamma - x)(\lambda_\gamma - \lambda) \in \alpha_\gamma \left((\lambda_\gamma - \lambda)S\right)$ i.e. $\lambda_\gamma x_\gamma - \lambda x \in \alpha_\gamma \left((\lambda_\gamma - \lambda)S\right)$. Since E is a convex bornological vector space, then $(\lambda_\gamma - \lambda)S$ is an absolutely convex and semi-bounded set when S is an absolutely convex and semi-bounded set of E by (definition 2.1) $\lambda_\gamma x_\gamma - \lambda x \overset{bs}{\rightarrow} 0$ then $\lambda_\gamma x_\gamma \overset{bs}{\rightarrow} \lambda x$.

Theorem 2.6:- A convex bornological vector space E is separated if and only if every bornologically semi-convergent net in E has a unique limit.

Proof:- Necessity**:** Let E be a separated bornological vector space. If a net (x_γ) in E bornologically semi-converges to x and y. Then the net $z_\gamma = x_\gamma - x_\gamma = 0$ semi-converges to $z = x - y$.

Thus it suffices to show that the limit z of the net $(z_\gamma = 0)$ must be the element 0 .

Let (λ_γ) be a net of real numbers tends to 0 and let S be a semi-bounded subset of E such that $z - z_\gamma = z \in \lambda_\gamma S$ for every $\gamma \geq 1$.

If $z \neq 0$, then the line spanned by z (i.e. the subspace (K_z) is contained in S,that contradicts the hypothesis that E is separated.

Sufficiency**:** Assume the uniqueness of limits, and suppose that there is an element $z \neq 0$ such that the line spanned by z is semi-bounded. Then we can find a semi-bounded set $S \subset E$ such that $z \in \left(\frac{1}{\gamma}\right) S$ for every $\gamma \geq 1$ and hence the net $(z_\gamma = z)$ semi-converges to 0.

But clearly this net also semi-converges to z , whence, by uniqueness of limits, $z = 0$ we have reached a contradiction.

Theorem 2.7:- Let E be a convex bornological vector space and let (x_γ) be a net in E then the following are equivalent:

(i) The net (x_γ) bornologically semi-converges to 0;
(ii) there is an absolutely convex set and semi-bounded $S \subset E$ and a decreasing net (α_γ) of positive real numbers, tends to 0, such that $x_\gamma \in \alpha_\gamma S$ for every $\gamma \in \Gamma$;
(iii) there is an absolutely convex and semi-bounded set $S \subset E$ such that, given any $\varepsilon > 0$, we can find an integer $\Gamma(\varepsilon)$ for which $x_\gamma \in \varepsilon S$ whenever $\gamma \geq \Gamma(\varepsilon)$.
(iv) there is a semi-bounded disk $S \subset E$ such that (x_γ) belongs to the semi-normed . E_S and semi-converges to 0 in E_S.

Proof:- (i) ⟹(ii):

For any integer $p \in \Gamma$ there is $\Gamma_p \in \Gamma$ such that if $\gamma \geq \Gamma_{\text{p}}$ then $\lambda\gamma \leq \frac{1}{\text{p}}$; hence $\lambda_\gamma S \subset \left(\frac{1}{p}\right) S$, since S is disk. We may assume that the net Γ_p is strictly increasing, and, for $\Gamma_p \leq K \leq \Gamma_{p+1}$,

Let $\alpha_K = \frac{1}{p}$. Then the net $\{\alpha_K\}$ satisfies the conditions of assertion (ii).

Clearly (ii) ⟹ (iii).

To show that (iii) ⟹ (i), let , for every $\gamma \in \Gamma, \varepsilon = inf\ \{\varepsilon > 0;\ x_\gamma \in \varepsilon S\}$, and $\lambda_\gamma = \varepsilon_\gamma + \frac{1}{\gamma}$,

then the net (λ_γ) converges to 0 and $x_\gamma \in \lambda_\gamma S$ for every $\gamma \in \Gamma$.

Thus the assertion (i, ii, iii) are equivalent. Suppose that the bornology of E is convex. Clearly (iv) implies (i) with $\lambda_\gamma = P_s(x_\gamma)$ and P_S the gauge of S, while (ii) implies that $x_\gamma \in E_S$ and $P_S(x_\gamma) \leq \alpha_\gamma \to 0$.

Remark 2.8:- It is clear that (x_γ) for every $\gamma \in \Gamma$ bornologically semi-converges to 0 if and only if every subnet of (x_γ) bornologically semi-converges to 0.

Theorem 2.9:- A net $\{x_\gamma\}$ in a product convex bornological vector space $\prod_{i\in I} E_i$ bornologically semi-converges to y if and only if the net $\{x_\gamma^i\}_{i\in I}$ bornologically semi-converges to y_i in convex bornological vector space E_i.

Proof:- Let $x_\gamma \overset{bs}{\rightarrow} y = (y_1, y_2, \dots, y_n, \dots)$ in $\prod_{i\in I} E_i$, since p_i is semi-bounded linear map, then by Theorem 3.4 $p_i(x_\gamma) \overset{bs}{\rightarrow} p_i(y)$, for each $i \in I$, then $x_\gamma^i \overset{bs}{\rightarrow} y_i$ in E_i. Suppose that $x_\gamma^i \overset{bs}{\rightarrow} y_i$ for each $i \in I$, then there is an absolutely convex and semi-bounded set S_i of E_i , $i \in I$ and a net (λ_γ^i) of scalars tending to 0, such that $x_\gamma^i \in S_i$ and for each $i \in I x_\gamma^i - y_i \in \lambda_\gamma^i - S_i, \gamma \in \Gamma$ since a net $\lambda_\gamma^i \to 0, i \in I$, for each $\gamma \in \Gamma$ then there is a diagonal net semi-converging to 0 such that when $i = \gamma$ then $\lambda_\gamma^i \to 0$.

Since $x_\gamma^i - y_i \in \lambda_\gamma^n B_i$ whenever $i = n$ and $= 1,2, \dots, n, \dots$.

If $\gamma \neq i$, since a disk S_i and $|\lambda_\gamma^n| \to 0$ and if $a \in S_i$ then $\lambda_\gamma^n a \in S_i$ then $x_\gamma^i - y_i \in \lambda_\gamma^n S_i$ then $x_\gamma^i - y_i \in \lambda_\gamma^n S_i$. Therefore $x_\gamma = (x_\gamma^1, x_\gamma^2, \dots) - y \in \lambda_\gamma^n \prod_{i\in I} S_i$ $i.e\ x_\gamma \overset{sb}{\rightarrow} y$.

3.Bornological Semi-Cauchy Net

In this section, we introduce the definition of bornological semi- Cauchy nets for bornological vector space and important results of these concepts.

Definition 3.1:- Let E be a convex bornological vector space. $(x_\gamma)_{\gamma\in\Gamma}$ is called a bornological semi-Cauchy net in E if there is an absolutely convex and semi-bounded set $S \subseteq E$ and a null net of postive scalars $(\mu_{\gamma,\gamma'})_{\gamma,\gamma'\in\Gamma\times\Gamma}$ such that $(x_\gamma - x_{\gamma'}) \in \mu_{\gamma,\gamma'} S$.

Proposition 3.2:- Every bornological semi-Cauchy net is semi-bounded.

Proof:- Suppose $(x_\gamma)_{\gamma\in\Gamma}$ is a semi-Cauchy net in a bornological vector space E by definition 3.1, there is an absolutely convex set and semi-bounded $S \subseteq E$ and a null net of postive scalars $(\mu_{\gamma,\gamma'})_{\gamma,\gamma'\in\Gamma\times\Gamma}$ such that $(x_\gamma - x_{\gamma'}) \in \mu_\gamma S$.

Since $x_\gamma \in S$ for all $\gamma \in \Gamma$. Since S is semi-bounded subset of E, then (x_γ) is semi-bounded.

Theorem 3.3:- A bornological semi-convergent net in a convex bornological vector space is a semi-Cauchy net **.**

Proof:- Let (x_γ) in E bornologically semi-converges to a point $x \in E$ by definition 3.1, there is an absolutely convex and semi-bounded set $S \subseteq E$ and a net (μ_γ) of scalars tends to 0 such that $(x_\gamma - x) \in \mu_\gamma S$. for every $\gamma \in \Gamma$ hence if γ' is a positive integer such that $\gamma \geq \gamma'$. $(x_{\gamma'} - x) \in \mu_{\gamma'} S$. For every integer γ' with $\gamma \geq \gamma'$, then $x_{\gamma'} - x \to 0$ implies $x_\gamma - x_{\gamma'} = x_\gamma - x - (x_{\gamma'} - x) \in \mu_{\gamma\gamma'} S$ for all $\gamma, \gamma' \in \Gamma$ with $\gamma \geq \gamma'$, then (x_γ) is a bornological semi-Cauchy net

Theorem 3.4:- Every semi-Cauchy net in a convex bornological vector space E which has a bornologically semi-convergent subnet , it is bornologically semi-convergent.

Proof:- Let (x_γ) be a semi-Cauchy net in a bornological vector space E and let $(x_{k_{\gamma'}})$ be a subnet of (x_γ) semi-converges to $x \in E$, then there is an absolutely convex set and semi-bounded $S_1 \subseteq E$ and a net $(\mu_{\gamma'})$ of scalars tends to 0 such that $x_{k_{\gamma'}} - x \in \mu_{k_{\gamma'}} S_1$ for every integer $k_{\gamma'} \in \Gamma$. Since (x_γ) is a semi-Cauchy net . By definition 3.1, there is an absolutely convex set and semi-bounded $S_2 \subseteq E$ and a null net of positive scalars (μ_γ) such that:-

$$x_\gamma - x_{k_{\gamma'}} \in \mu_{k_{\gamma'}} S_2$$

for every $k_{\gamma',\gamma} \in \Gamma$ with $\gamma \geq k_{\gamma'}$ let $S = S_1 \cup S_2$ then $x_\gamma - x - (x_{k_{\gamma'}} - x) \in \mu_{k_{\gamma'}}S$ for every $k_{\gamma''}, \gamma \in \Gamma$ with $\gamma \geq k_{\gamma'}$.

Since $x_{k_{\gamma'}} - x \to 0$ then $x_\gamma - x \in \mu_{k_{\gamma'}}S$ for every $k_{\gamma''}, \gamma \in \Gamma$ with $\gamma \geq k_{\gamma'}$, then $x_\gamma - x \in \mu_\gamma S$ for every $\gamma \in \Gamma$.

i.e. (x_γ) bornologically semi-converges to a point $x \in E$.

4. Semi-complete bornological vector space

In this section, we introduce the definition of semi-complete bornological vector spaces and investigate its properties such as product, quotient and direct sum.

Definition 4.1:- A separated convex bornological vector space is called a semi-complete bornological vector space if every bornological semi-Cauchy net in E is semi-converges in E .

Theorem 4.2:- Let E be a separated convex bornological vector space and let F be a bornological subspace of E , then:-

(i) If F is semi-complete , then F is b-closed in E ;
(ii) If E is semi-complete and F is b-closed, then F is semi-complete .

Proof: (i) Let (x_γ) be a net in F which bornologically semi-converges to $x \in E$; by proposition 4.6 (x_γ) is a semi-Cauchy net in F. Since F is semi-complete then (x_γ) bornologically semi-converges to y in F but, E is a separated bornological vector space, then $x = y$ implies $x_\gamma \xrightarrow{bs} x$ in F, then F is b-closed in E .

(ii) Let (x_γ) be a semi-Cauchy net in ; Since E is semi-complete , then (x_γ) bornologically semi-converge to x in E. Since F is b-closed, then (x_γ) bornologically semi-converges to x in F, then F is semi-complete .

Theorem 4.3:- Let E be a semi-complete bornological vector space and F be b-closed subspace of E, then the quotient E/F is semi-complete .

Proof: Let $\boldsymbol{\beta_0}$ be a base for the bornology of E. If $\theta: E \to E/F$ is the canonical map, then $\theta(\boldsymbol{\beta_0})$ is a base for the bornology of E/F, Since F is b-closed subspace of E, then by theorem in[6] the quotient E/F is separated and θ is semi bounded linear map (definition 1.12). Thus $\theta(x_\gamma)$ is bornologically semi-convergent net in E/F, for every bornologically semi-convergent (x_γ) in E. Then bornologically semi-converge Cauchy net in E/F. Whence E/F is semi-complete .

Theorem 4.4:- Every product of any family of semi-complete bornological spaces is semi-complete .

Proof:- Let $E_i, i \in \Gamma \neq \Phi$ be a family of semi-complete separated bornological vector space and let $E = \prod_{i\in\Gamma} E_i$ be the product of E_i. If (x_γ) is a semi-Cauchy net in E. Then the canonical projection $p_i: E \to E_i$ of a semi-Cauchy net is semi-Cauchy (x_γ^i) in E_i for every $i \in \Gamma$. Since every E_i is semi-complete and it is clear that separated bornological vector space for every $i \in \Gamma$, then (x_γ^i) bornologically semi-converges to a unique point x_i in E_i for every $i \in$ I , and let $x = (x_i) \in E, i \in \Gamma$, then the net (x_γ) bornologically semi-converges to $x \in E$ Then E is a semi-complete bornological vector space.

Theorem 4.5:- Let $\left(E_i, u_{ji}\right)_{i\in\Gamma}$ an inductive system of semi-complete bornological vector space, i.e. E_i is semi-complete for every $i \in \Gamma$ and let $E = \underset{i\in\Gamma}{\longrightarrow} E_i$. Then E is semi-complete if and only if E is separated.

Proof:- Since every semi-complete space is separated, only the sufficiency needs proving. Assume, then, E to be separated and let u_i be the canonical embedding of E_i in to E. Let $\left(x_\gamma^i\right)_{\gamma\in\Gamma}$ be a semi-Cauchy net in E_i, whenever $i \in \Gamma$. Since E_i is a semi-complete bornological vector space then $\left(x_\gamma^i\right)$ bornologically semi-converges to a point $x \in E_i$; then $u_i\left(x_\gamma^i\right)$ bornologically semi-converges semi-Cauchy net in E whenever $i \in \Gamma$. Since E is separated then $u_i\left(x_\gamma^i\right)$ has a unique limit then E is semi-complete .

Corollary 4.6:- Let $\left(E_i, u_{ji}\right)_{i\in\Gamma}$ an inductive system of semi-complete bornological space and $E = \underset{i\in\Gamma}{\longrightarrow} E_i$. Hence if the maps u_{ji} are injective, then E is semi-complete .

Proof:- the proof is complete by using theorems (4.5 and 4.6).

Corollary 4.7 :- Every bornological direct sum of any family of semi-complete bornological spaces is semi-complete .

Proof:- Let (E_i) be a family of semi-semi-complete bornological space and let $E = \underset{i\in\Gamma}{\oplus} E_i$ be their bornological direct sum. For every $J \in$ I, $E_J = \underset{i\in\Gamma}{\oplus} E_i$. The space E_J is bornologically isomorphic to the product $\prod_{i\in\Gamma} E_i$. Whence is semi-complete theorem 4.4. If $J \subset J'$, denote by $u_{J'J}$ the canonical embedding of E_J into $E_{J'}$. Then E is the bornological inductive limit of the spaces E_J and the assertion follows from Corollary 4.6 .

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حول فضاء المتجهات البرنولوجي شبه الكامل

فاطمة كامل مجيد
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المستخلص :

تعريف شبه التقارب للشبكات، شبه كوشي للشبكات في فضاء المتجهات البرنولوجي المحدب. و فضاء المتجهات البرنولوجي شبه الكامل والعلاقه بين هذه المفاهيم تمت دراستها في هذا البحث. كذلك قدمنا بعض النظريات لهذه المفاهيم و حصلنا على بعض النتائج. النتائج الرئيسية لهذه الدراسة ذات اهتمام معتد به في العديد من المجالات .