Journal of AL-Qadisiyah for computer science and mathematics Vol.9 No.2 Year 2017
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Coefficients Bounds for a General Subclasses of m-Fold Symmetric Bi-Univalent Functions

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Recived : 7\5\2017 Revised : 19\6\2017 Accepted : 9\7\2017

Abstract: In this paper, we introduce and investigate a new general subclasses $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\alpha)$ and $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$ of Σ_m consisting of analytic and m-fold symmetric bi-univalent functions in the open unit disk U. We obtain estimates on the Taylor-Maclaurin coefficients $|a_{m+1}|$ and $|a_{2m+1}|$. Also, we obtain new special cases for our results.

Keywords: Analytic function, Univalent function, Bi-Univalent function, m-Fold symmetric function, m-Fold symmetric bi-univalent function.

Mathematics subject classification : 30C45. 30C50. 30C80.

1. Introduction

Symbolized by $\mathcal{A}$ the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1.1)$$

which are analytic in the open unit disk $U = \{z: z \in \mathbb{C} \; and \; |z| < 1\}$. Also, symbolized by $\mathcal{S}$ the class of all functions in $\mathcal{A}$ which are univalent and normalized by $f(0) = 0 = f'(0) - 1$ in U. The well-investigated subclasses of the univalent function class $\mathcal{S}$ are the class of starlike functions of order α $(0 \le \alpha < 1)$, symbolized by $\mathcal{S}^*(\alpha)$ and the class of convex functions of order α symbolized by $\mathcal{K}(\alpha)$ in U.

The Koebe One-Quarter Theorem [1] shows that the image of U under every function f from $\mathcal{S}$ contains a disk of radius 1/4. Thereby every such univalent function has an inverse f^{-1} which satisfies

$$f^{-1}(f(z)) = z \qquad (z \in U)$$

and

$$f(f^{-1}(w)) = w \quad \left(|w| < r_0(f);\; r_0(f) \ge \frac{1}{4}\right),$$

where

$$f^{-1}(w) = g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots. \qquad (1.2)$$

The function $f \in \mathcal{A}$ is considered bi-univalent in U if both f and f^{-1} are univalent in U. Given by the Taylor-Maclaurin series expansion (1.1), the class of all bi-univalent functions in U can be symbolized by Σ .

For each function $f \in \mathcal{S}$, the function

$$h(z) = \sqrt[m]{f(z^m)} \quad (z \in U, m \in \mathbb{N}) \qquad (1.3)$$

is univalent and the unit disk U can be mapped into a region with m-fold symmetry. It is considered m-fold symmetric (see [2, 3]) if the function has the following normalized form:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \quad (z \in U, m \in \mathbb{N}). \qquad (1.4)$$

The class of m-fold symmetric univalent functions in U, which are normalized by the series expansion (1.4) can be symbolized by $\mathcal{S}_m$ and the functions in the class $\mathcal{S}$ are one-fold symmetric (that is, m=1).

In [4] Srivastava et al. specified that m-fold symmetric bi-univalent function analogues to the concept of m-fold symmetric univalent functions and these gave some important results, such as each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each $m \in \mathbb{N}$, in their study. As for as the normalized form of f given by (1.4) is concerned, they obtained the series expansion for f^{-1} as follows:

$$f^{-1}(w) = g(w) = w - a_{m+1}w^{m+1} + [(m+1)a_{m+1}^2 - a_{2m+1}]w^{2m+1} - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots, \quad (1.5)$$

where $f^{-1} = g$. The class of m-fold symmetric bi-univalent functions in U can be symbolized by Σ_m. For $m = 1$, formula (1.5) coincides with formula (1.2) of the class Σ.

Mathematicians such as Lewin [5], Brannan and Clunie [6] as well as Netanyahu [7] studied the functions in Σ and proved the following, the first investigated the bi-univalent function class Σ and showed that $|a_2| < 1.51$, the second showed that $|a_2| < \sqrt{2}$, whereas the third showed that max $|a_2| = 4/3$ if $f(z) \in \Sigma$, but the best known estimate for functions in Σ were obtained by Tan [8] in 1984, that is, $|a_2| \le 1.485$. The coefficient estimate problem involving the bound of $|a_2|$ $(n \in \mathbb{N}\backslash\{1,2\})$ for each $f \in \Sigma$ given by (1.4) is still an open problem. In fact, the aforecited work of Srivastava et al. [9] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years. Recently, many authors investigated bounds for various subclasses of bi-univalent functions (see[10, 11, 12, 13, 14, 9, 15]).

The object of the present paper is to obtain estimates on the Taylor-Maclaurin coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions belonging to the new general subclasses $\mathcal{R}_{\Sigma_m}(\tau, \lambda, \eta; \alpha)$ and $\mathcal{R}_{\Sigma_m}(\tau, \lambda, \eta; \beta)$ of Σ_m. Also, some interesting applications of the results presented here are also discussed.

In order to derive our main result, we have to recall here the following lemma [1].

Lemma 1.1. If $h \in \mathcal{P}$, then $|c_k| \le 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the family of all functions h, analytic in U, for which

$$R\big(h(z)\big) > 0 \quad (z \in U),$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots \quad (z \in U).$$

2. Coefficients bounds for the function class $\mathcal{R}_{\Sigma_m}(\tau, \lambda, \eta; \alpha)$

Definition 2.1. A function $f \in \Sigma_m$ given by (1.4) is said to be in the class $\mathcal{R}_{\Sigma_m}(\tau, \lambda, \eta; \alpha)$ if the following conditions are satisfied:

$$\left|\arg\left(1 + \frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) + \eta z f''(z) - 1\right]\right)\right| < \frac{\alpha\pi}{2} \quad (z \in U) \quad (2.1)$$

and

$$\left|\arg\left(1 + \frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) + \eta w g''(w) - 1\right]\right)\right| < \frac{\alpha\pi}{2} \quad (w \in U), \quad (2.2)$$

where $(\tau \in \mathbb{C}\backslash\{0\};\ \lambda \ge 1;\ 0 \le \eta \le 1;\ 0 < \alpha \le 1)$ and the function g is given by (1.5).

Theorem 2.1. Let the function $f(z)$, given by (1.4), be in the class $\mathcal{R}_{\Sigma_m}(\tau, \lambda, \eta; \alpha)$. Then

$$|a_{m+1}| \le \frac{2\alpha|\tau|}{\sqrt{\left|\alpha\tau(m+1)\big(1 + 2\lambda m + 2\eta m(2m+1)\big) + (1-\alpha)\big(1 + \lambda m + \eta m(m+1)\big)^2\right|}} \quad \ldots (2.3)$$

Journal of AL-Qadisiyah for computer science and mathematics Vol.9 No.2 Year 2017
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

and

$$|a_{2m+1}| \leq \frac{2\alpha^2|\tau|^2(m+1)}{\left(1+\lambda m+\eta m(m+1)\right)^2} + \frac{2\alpha|\tau|}{\left(1+2\lambda m+2\eta m(2m+1)\right)}. \quad (2.4)$$

Proof. Let $f \in \mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\alpha)$. Then

$$1+\frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)+\eta z f''(z)-1\right] = [p(z)]^\alpha \quad (2.5)$$

and

$$1+\frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)+\eta w g''(w)-1\right] = [q(w)]^\alpha, \quad (2.6)$$

where $g = f^{-1}$, $p(z)$, $q(z)$ in $\mathcal{P}$ and have the forms

$$p(z) = 1 + p_m z^m + p_{2m} z^{2m} + p_{3m} z^{3m} + \cdots \quad (2.7)$$

and

$$q(w) = 1 + q_m w^m + q_{2m} w^{2m} + q_{3m} w^{3m} + \cdots. \quad (2.8)$$

Now, equating the coefficients in (2.5) and (2.6), we get

$$\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right) a_{m+1} = \alpha p_m, \quad (2.9)$$

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right) a_{2m+1} = \alpha p_{2m} + \frac{1}{2}\alpha(\alpha-1)p_m^2, \quad (2.10)$$

$$-\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right) a_{m+1} = \alpha q_m \quad (2.11)$$

and

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)[(m+1)a_{m+1}^2 - a_{2m+1}] = \alpha q_{2m} + \frac{1}{2}\alpha(\alpha-1)q_m^2. \quad (2.12)$$

From (2.9) and (2.11), we find

$$p_m = -q_m \quad (2.13)$$

and

$$2\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right)^2 a_{m+1}^2 = \alpha^2(p_m^2+q_m^2). \quad (2.14)$$

From (2.10), (2.12) and (2.14), we get

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)(m+1)a_{m+1}^2 = \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2+q_m^2)$$
$$= \alpha(p_{2m}+q_{2m}) + \frac{(\alpha-1)}{\alpha}\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right)^2 a_{m+1}^2. \quad (2.15)$$

Therefore, we have

$$a_{m+1}^2 = \frac{\alpha^2\tau^2(p_{2m}+q_{2m})}{\alpha\tau(m+1)\left(1+2\lambda m+2\eta m(2m+1)\right)+(1-\alpha)((1+\lambda m+\eta m(m+1))^2}. \quad \ldots (2.16)$$

Applying Lemma (1.1) for the coefficients p_{2m} and q_{2m}, we immediately have

$$|a_{m+1}| \leq \frac{2\alpha|\tau|}{\sqrt{\left|\alpha\tau(m+1)\left(1+2\lambda m+2\eta m(2m+1)\right)+(1-\alpha)\left(1+\lambda m+\eta m(m+1)\right)^2\right|}} \quad \ldots (2.17)$$

The last inequality gives the desired estimate on $|a_{m+1}|$ given in (2.3).

Next, in order to find the bound on $|a_{2m+1}|$, by subtracting (2.12) from (2.10), we obtain

$$2\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)a_{2m+1} - \left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)(m+1)a_{m+1}^2 = \alpha(p_{2m}-q_{2m}) + \frac{\alpha(\alpha-1)}{2}(p_m^2-q_m^2). \quad (2.18)$$

It follows from (2.13), (2.14) and (2.18) that

$$a_{2m+1} = \frac{\alpha^2\tau^2(m+1)(p_m^2+q_m^2)}{4\left(1+\lambda m+\eta m(m+1)\right)^2} + \frac{\alpha\tau(p_{2m}-q_{2m})}{2\left(1+2\lambda m+2\eta m(2m+1)\right)}. \quad (2.19)$$

Applying Lemma (1.1) once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we readily get

$$|a_{2m+1}| \leq \frac{2\alpha^2|\tau|^2(m+1)}{\left(1+\lambda m+\eta m(m+1)\right)^2} + \frac{2\alpha|\tau|}{\left(1+2\lambda m+2\eta m(2m+1)\right)}.$$

This completes the proof of Theorem (2.1).

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ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

3. Coefficients bounds for the function class $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$

Definition 3.1. A function $f\in\Sigma_m$ given by (1.4) is said to be in the class $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$ if the following conditions are satisfied:

$$Re\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)+\eta z f''(z)-1\right]\right)>\beta \quad (z\in U) \tag{3.1}$$

and

$$Re\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)+\eta w g''(w)-1\right]\right)>\beta \quad (w\in U), \tag{3.2}$$

where ($\tau\in\mathbb{C}\backslash\{0\};\ \lambda\geq 1;\ 0\leq\eta\leq 1;\ 0\leq\beta<1$) and the function g is given by (1.5).

Theorem 3.1. Let the function $f(z)$, given by (1.4), be in the class $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$. Then

$$|a_{m+1}|\leq\sqrt{\frac{4|\tau|(1-\beta)}{(m+1)\big(1+2\lambda m+2\eta m(2m+1)\big)}} \tag{3.3}$$

and

$$|a_{2m+1}|\leq\frac{2|\tau|^2(1-\beta)^2(m+1)}{\big(1+\lambda m+\eta m(m+1)\big)^2}+\frac{2|\tau|(1-\beta)}{\big(1+2\lambda m+2\eta m(2m+1)\big)}. \tag{3.4}$$

Proof. It follows from (3.1) and (3.2) that there exist $p,q\in\mathcal{P}$ such that

$$1+\frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)+\eta z f''(z)-1\right]=\beta+(1-B)p(z) \tag{3.5}$$

and

$$1+\frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)+\eta w g''(w)-1\right]=\beta+(1-B)q(w), \tag{3.6}$$

where $p(z)$ and $q(w)$ have the forms (2.7) and (2.8), respectively. Equating coefficients in (3.5) and (3.6), we get

$$\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right)a_{m+1}=(1-\beta)p_m, \tag{3.7}$$

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)a_{2m+1}=(1-\beta)p_{2m}, \tag{3.8}$$

$$-\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right)a_{m+1}=(1-\beta)q_m \tag{3.9}$$

and

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)\left[(m+1)a_{m+1}^2-a_{2m+1}\right]=(1-\beta)q_{2m}. \tag{3.10}$$

From (3.7) and (3.9), we find

$$p_m=-q_m \tag{3.11}$$

and

$$2\left(\frac{1+\lambda m+\eta m(m+1)}{\tau}\right)^2a_{m+1}^2=(1-\beta)^2(p_m^2+q_m^2). \tag{3.12}$$

Adding (3.8) and (3.10), we have

$$\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)(m+1)a_{m+1}^2=(1-\beta)(p_{2m}+q_{2m}). \tag{3.13}$$

Therefore, we obtain

$$a_{m+1}^2=\frac{\tau(1-\beta)(p_{2m}+q_{2m})}{(m+1)\big(1+2\lambda m+2\eta m(2m+1)\big)}. \tag{3.14}$$

Applying Lemma (1.1) for coefficients p_{2m} and q_{2m}, we obtain

$$|a_{m+1}|\leq\sqrt{\frac{4|\tau|(1-\beta)}{(m+1)\big(1+2\lambda m+2\eta m(2m+1)\big)}}.$$

This gives the bound on $|a_{m+1}|$ as asserted in (3.3).

In order to find the bound on $|a_{2m+1}|$, by subtracting (3.10) from (3.8), we get

$$2\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)a_{2m+1}-\left(\frac{1+2\lambda m+2\eta m(2m+1)}{\tau}\right)(m+1)a_{m+1}^2=(1-\beta)(p_{2m}-q_{2m}) \tag{3.15}$$

or equivalently

$$a_{2m+1}=\frac{(m+1)}{2}a_{m+1}^2+\frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2\big(1+2\lambda m+2\eta m(2m+1)\big)}. \tag{3.16}$$

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ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

Upon substituting the value of a_{m+1}^2 from (3.12), we get

$$a_{2m+1}=\frac{\tau^2(1-\beta)^2(m+1)(p_m^2+q_m^2)}{4\big(1+\lambda m+\eta m(m+1)\big)^2}+\frac{\tau(1-\beta)(p_{2m}-q_{2m})}{2\big(1+2\lambda m+2\eta m(2m+1)\big)}. \quad (3.17)$$

Applying Lemma (1.1) once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we find

$$|a_{2m+1}|\leq\frac{2|\tau|^2(1-\beta)^2(m+1)}{\big(1+\lambda m+\eta m(m+1)\big)^2}+\frac{2|\tau|(1-\beta)}{\big(1+2\lambda m+2\eta m(2m+1)\big)}.$$

This completes the proof of Theorem (3.1).

4. Corollaries and Consequences

For one-fold symmetric bi-univalent functions, the classes $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\alpha)$ and $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$ reduce to the classes $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\alpha)$ and $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\beta)$ and thus, Theorem (2.1) and Theorem (3.1) reduce to Corollary (4.1) and Corollary(4.2), respectively.

Corollary 4.1. Let $f(z)$ given by (1.1) be in the class $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\alpha)$. Then

$$|a_{m+1}|\leq\frac{2\alpha|\tau|}{\sqrt{|2\alpha\tau(1+2\lambda+6\eta)+(1-\alpha)(1+\lambda+2\eta)^2|}} \quad (4.1)$$

and

$$|a_{2m+1}|\leq\frac{4\alpha^2|\tau|^2}{(1+\lambda+2\eta)^2}+\frac{2\alpha|\tau|}{(1+2\lambda+6\eta)}. \quad (4.2)$$

Corollary 4.2. Let $f(z)$ given by (1.1) be in the class $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\beta)$. Then

$$|a_{m+1}|\leq\sqrt{\frac{2|\tau|(1-\beta)}{(1+2\lambda+6\eta)}} \quad (4.3)$$

and

$$|a_{2m+1}|\leq\frac{4|\tau|^2(1-\beta)^2}{(1+\lambda+2\eta)^2}+\frac{2|\tau|(1-\beta)}{(1+2\lambda+6\eta)}. \quad (4.4)$$

The classes $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\alpha)$ and $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\beta)$ are defined in the following way:

Definition 4.1. A function $f(z)\in\Sigma$ given by (1.1) is said to be in the class $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\alpha)$ if the following conditions are satisfied:

$$\left|\arg\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)+\eta zf''(z)-1\right]\right)\right|<\frac{\alpha\pi}{2}\quad (z\in U) \quad (4.5)$$

and

$$\left|arg\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)+\eta wg''(w)-1\right]\right)\right|<\frac{\alpha\pi}{2}\quad (w\in U), \quad (4.6)$$

where ($\tau\in\mathbb{C}\backslash\{0\};\ \lambda\geq1;\ 0\leq\eta\leq1;\ 0<\alpha\leq1$) and the function g is given by (1.2).

Definition 4.2. A function $f(z)\in\Sigma$ given by (1.1) is said to be in the class $\mathcal{R}_{\Sigma}(\tau,\lambda,\eta;\beta)$ if the following conditions are satisfied:

$$Re\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{f(z)}{z}+\lambda f'(z)+\eta zf''(z)-1\right]\right)>\beta\quad (z\in U) \quad (4.7)$$

and

$$Re\left(1+\frac{1}{\tau}\left[(1-\lambda)\frac{g(w)}{w}+\lambda g'(w)+\eta wg''(w)-1\right]\right)>\beta\quad (w\in U), \quad (4.8)$$

where ($\tau\in\mathbb{C}\backslash\{0\};\ \lambda\geq1;\ 0\leq\eta\leq1;\ 0\leq\beta<1$) and the function g is given by (1.2).

Remark 4.1. For m-fold symmetric bi-univalent functions, if we put $\lambda=1$ in Theorem (2.1) and Theorem (3.1), we obtain the results which were given by Srivastava et al.[16]. In addition , if we put $\eta=0$ in Theorem (2.1) and Theorem (3.1), we obtain the results which were given by Srivastave et al. [17]

Remark 4.2. For one-fold symmetric bi-univalent functions, if we put $\tau=1$ and $\lambda=1$ in Theorem (2.1) and Theorem (3.1), we obtain the results which were given by Frasin [12]. In addition , if we put $\tau=1$ and $\eta=0$ in Theorem (2.1) and Theorem (3.1), we obtain the results which were given by Frasin and Aouf [13].

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Journal of AL-Qadisiyah for computer science and mathematics Vol.9 No.2 Year 2017
ISSN (Print): 2074 – 0204 ISSN (Online): 2521 – 3504

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المستخلص :

في هذا البحث، قدمنا وناقشنا اصناف جزئية عامة جديدة $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\alpha)$ و $\mathcal{R}_{\Sigma_m}(\tau,\lambda,\eta;\beta)$ من Σ_m والمتكونة من الدوال التحليلية الثنائية التكافؤ المتناظرة المطوية من النمط m في قرص الوحدة المفتوح U . حصلنا على تخمينات حول معاملات مكلورين - تايلور $|a_{m+1}|$ و $|a_{2m+1}|$. حصلنا ايضاً على حالات خاصة جديدة لنتائجنا.