University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Existence and Uniqueness Common Fixed Point in Fuzzy b-Matric Space

Ban Nazar Ali[a] ,Alia Shani Hassan[b] *

[a]*Departmentof Mathematics, University of Al-Qadisiyah, Diwaniyah , Iraq.Email : banqser@gmail.com*

[b]*Departmentof Mathematics, University of Al-Qadisiyah, Diwaniyah , Iraq.Email: alia.shani@qu.edu.iq*

A R T I C L E I N F O

Article history:
Received: 21 /2/2024
Rrevised form: 22 /3/2024
Accepted : 29 /3/2024
Available online: 30 /3/2024

Keywords:

Fuzzy metric space , Fuzzy b-metric, common Fixed points

A B S T R A C T

Multiple fixed points common to fuzzy spaces b-metric are presented in this work. We provide an appropriate requirement for a sequence on the fuzzy space b-metric to be Cauchy, which is a significant outcome. As a result, we streamline the numerous theorem of point fixed proofs on a fuzzy space for b-metrics with established contraction conditions.

MSC..

https://doi.org/10.29304/jqcsm.2024.16.11448

1. Introduction:

Zadeh introduced the concept of fuzzy reasoning [1]. In contrast to conventional reasoning, which determines whether a component belongs or does not belong to fuzzy logic indicates a set to which the membership of an element in a set is a numerical value within a specific range [2]. Zadeh was motivated to investigate fuzzy set theory to address the issue of uncertainty, which is a fundamental aspect of real-world problems. The concept of the fuzzy space metric , the value for a point fixed can be approached from various perspectives, one of which involves the use of fuzzy logic [3]**.** Heilpern introduced fuzzy mapping as a concept and demonstrated a fixed-point theorem in linear spaces metric of fuzzy contraction mapping. This theorem serves as a hazy extension of the contraction of Banach principle, which has attracted the attention of numerous authors who have since explored different contraction circumstances inside the context of mapping fuzzy. The imprecision resulting from non-exact distances between elements is accounted for within the fuzzy metric spaces, as defined the Seikkala and Kaleva [3,4]. Aside from fuzzy metric spaces, numerous expansions exist between metric space and metric concepts. Czerwik and Bakhtin proposed a framework in which a less strict requirement, rather than the triangle inequality, was observed. This was done to generalize the Banach contraction idea. We called these spaces b-metric . [2], [5]. Fuzzy spaces metric and B-metrics are compared in [6]. In contrast, Sedghi et al., 2012 presented a b-metric space that is fuzzy relaxes triangle inequality. The study targets a certain spatial category. We prove the utility of a lemma in fuzzy b-metric spaces by using a countable extension of the t-norm [7]. The lemma proves that

*Corresponding author Ban Nazar Ali

Email addresses: *banqser@gmail.com*

Communicated by 'sub etitor'

$\{u_n\}$The sequence is Cauchy. This Lemma simplifies the evidence of numerous well-known theorems of fixed points. In this current paper, we streamline the demonstrations of numerous renowned fixed-point theorems.

Definition 1.1 [8]

A binary operation T : [0, 1] × [0, 1] → [0, 1] is a continuous

t-norm if T satisfies the following conditions:

(b1) T is associative and commutative,

(b2) T is continuous,

(b3) T(a, 1) = a for all a ∈ [0, 1],

(b4) T(a, b) ≤ T(c, d) for a, b, c, d ∈ [0, 1] such that a ≤ c and b ≤ d. Typical examples of a continuous t-norm are T_p (a, b) = a · b, T_{min}(a, b) = min{a, b} and T_L (a, b) = max{a + b − 1, 0}.

Definition 1.2 [9] A 3-tuple (X,M,T) is called a fuzzy metric space if X is an arbitrary (nonempty) set, T is a continuous t-norm, and M is a fuzzy set on X^2 × (0,∞) satisfying the following conditions for all u, v, w∈ X and

t ,s > 0:

(fm1) M(u, v, t) > 0,

(fm2) M(u, v, t) =1 if and only if u = v,

(fm3) M(u, v, t) = M(v, u,t),

(fm4) T(M(u, v, t),M(v, w ,s)) ≤ M(u, w, t + s),

(fm5) M(u, v,·) : (0,∞) → [0, 1] is continuous.

Remark 1.3

We use X^2= X × X in this paper.

Definition 1.4 [7]

A 3-tuple (X,M,T) is called a fuzzy b-metric space if X is an arbitrary (nonempty) set, T is a continuous t-norm, and M is a fuzzy set on X^2 × (0,∞) satisfying the following conditions for all u, v, w ∈ X, t, s > 0 and a given real number b ≥ 1:

(bF1) M(u, v, t) > 0,

(bF2) M(u, v, t)=1 if and only if x = y,

(bF3) M(u, v, t) = M(v, u, t),

(bF4) T(M(u, v, $\frac{t}{b}$),M(y, z, , $\frac{s}{b}$)) ≤ M(u, w, t + s), (bF5) M(u, v,·) : (0,∞) → [0, 1] is continuous

Note 1.5:

The class of fuzzy b-metric spaces is effectively larger than that of fuzzy metric spaces, since a fuzzy b-metric is a fuzzy metric when b = 1. The next example shows that a fuzzy b-metric on X need not be a fuzzy metric on X.

Example 1.1 [10]

Assume that M (u, v, t) = Exp ($\frac{|u-v|^p}{t}$) , where p > 1 is a real number. Then M is a fuzzy b-metric with b $=2^{p-1}$. Noted that in the preceding example, for p = 2, it is easy to see that (X,M,T) is not a fuzzy metric space.

Example 1.2 [10]

Let M(u, v, t) = $e^{\frac{-f(u,v)}{t}}$ or M(x, y,t) =Exp($\frac{t}{t+f(u,v)}$), where f is a b-metric on X, and let T(a, c) = a · c for a, c ∈ [0, 1]. Then it is easy to show that M is a fuzzy b-metric.

Definition 1.6 [10]

A function f : R → R is called b-no decreasing if u > b v implies f (u) ≥ f (v) for all u, v ∈ R.

Lemma 1.7 [7]

Let M(u, v,·) be a fuzzy b-metric space. Then M(u, v, t) is b-no decreasing with respect to t for all u, v ∈ X.

Definition 1.8 [7]

Let (X,M,T) be a fuzzy b-metric space. For t > 0, A sequence $\{u_n\}$:

(a) converges to u if M (u_n, u, t) → 1 as n → ∞ for each t > 0. In this case, we write $\lim_{n\to\infty} u_n = x^*$;

(b) is called a Cauchy sequence if for all 0 < ε < 1 and t > 0, there exists u_0 ∈ N such that M (u_n , u_m,t) >1− ε for all n, m ≥ u_0.

Definition1.9 [7]

The fuzzy b-metric space (X,M,T) is said to be complete if every Cauchy sequence is convergent.

Lemma 1.10 [7]

In a fuzzy b-metric space (X,M,T) we have:

(i) If a sequence $\{u_n\}$ in X converges to x^*, then x^*, is unique,

(ii) If a sequence $\{u_n\}$ in X converges to x^*,, then it is a Cauchy sequence.

Remark 1.11

In general ,a fuzzy b-metric is not continuous.

Example 1.3 Assume that X = [0,∞), $M(u, v, t) = e^{-\frac{f(u,v)}{t}}$, $T = T_P$

$$f(u,v) = \begin{cases} 0, & u = v, \\ 2|u - v|, & u, v \in [0, 1), \\ 2\,|x - y| & \text{otherwise.} \end{cases}$$

Then (X,M,T) is a fuzzy b-metric space with b = 4. The b-metric f in this example is taken from [12].

Note that the fuzzy b-metric M is not continuous.

Definition1.12

Given is fuzzy b-metric space (M, T, X). and two maps of oneself (A,B) in X, u in X is referred to as common fixed point of A and B if Au=Bu=u.

we will employ a fuzzy b-metric space with the additional condition $M(u, v, t) \to 1$ as

$n \to \infty$ in the sense of Definition 1.3.

2-Main results

Lemma 2.1 [11]

Let $\{u_n\}$ be a sequence in a fuzzy b-metric space (X,M,T). Suppose that there exists $\alpha \in (0, \frac{1}{b})$ such that

$$M(u_n, u_{n+1}, t) \geq M(u_{n-1}, u_n, , \tfrac{t}{\alpha}) \qquad (1.2)$$

and there exist$u_0, u_1 \in X$ and $v \in (0, 1)$ such that

$$\lim_{n\to\infty} T_{i=n}^{\infty} \left(u_0, u_1, \tfrac{t}{v^i}\right) = 1, t > 0 \qquad (2.2)$$

Then $\{u_n\}$ is a Cauchy sequence.

Corollary 2.2 [11]

Let$\{u_n\}$ be a sequence in a fuzzy b-metric space (X,M,T), and let T be of H-type. If there exists $\alpha \in (0, \frac{1}{b})$ such that

$$M(u_n, u_{n+1}, t) \geq M(u_{n-1}, u_n, \tfrac{t}{\alpha}) \text{ where } t > 0 \text{ and } n \in N \qquad (2.3)$$

, then$\{u_n\}$ is a Cauchy sequence.

Lemma 2.3[7]

If for some $\alpha \in (0, 1)$ and $u, v \in X$,

$$M(u, v, t) \geq M(u, v, \tfrac{t}{\alpha}),\ t > 0, \text{ then } u = v.$$

Theorem (2. 1): Let (X,M,T) be a complete fuzzy b-metric space ,and Let A,B:X→X be two self-mappings on X, Suppose that there exists

$0 < \alpha < \frac{1}{b}$ such that

$M(Au, Bv, t) \geq \min\{ M(u, v, \frac{t}{\alpha}), M(Au, u, \frac{t}{\alpha}), M(Bv, v, \frac{t}{\alpha})\}$, (2.1.1)

For all u ,v ∈X,t>o,

Then A,B have a unique common fixed point in X.

Proof:

Using Picard iteration, we define $\{u_n\}$ as the sequence in fuzzy b-metric space

(M, T, X). $u_{2n+1}=Au_{2n}$, $u_{2n+2}=Bu_{2n+1}$ For all n∈ N ... (2.1.2)

Now , using (2.1.1)and (2.1.2) , we have

$M(u_{2n+1}, u_{2n+2}, t) = M(Au_{2n}, Bu_{2n+1}, t) \geq$

$\min\{ M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(Au_{2n}, u_{2n}, \frac{t}{\alpha}), M(Bu_{2n+1}, u_{2n+1}, \frac{t}{\alpha})\}$,

$= \min\{ M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(u_{2n+1}, u_{2n}, \frac{t}{\alpha}), M(u_{2n+2}, u_{2n+1}, \frac{t}{\alpha})\}$

$\geq \min\{ M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha})\}$ where $n \in N$, t is greater than 0.

Then

Either $M(u_{2n+1}, u_{2n+2}, t) \geq M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha})$
,where t is greater than 0 and n in N , there for It implies through Lemma (2.3) that

$u_{2n+1} = u_{2n+2}$, n in N .

or $M(u_{2n+1}, u_{2n+2}, t) \geq M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}),)$ where t greater than 0

and n *in* N ,then we can determine that It is Cauchy sequence$\{u_n\}$. by using Lemma (2.1).

By definition of completeness of (M , X , T) , we can say

that $\{u_n\}$ is convergent. Therefore, there is u ∈ X so that

where $u_n \to u$ whene n→∞ and $M(u, u_n, t)=1$

whene n→∞ ,t greater than 0 ...(2.1.3)

To show that u is (C.F.P) for A and B .suppose $\lambda_1 \in (\alpha b^2, 1)$ and $\lambda_2 = 1-\lambda_1$ .By using (2.1.1) and(fm4) we have

$M(Au, u, t) \geq \min\left(M\left(Au, Bu_n, \frac{t\lambda_1}{b}\right), M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\right)$

$\geq T\left(\min\left\{M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(Bu_n, u_n, \frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\right)$

$= T\left(\min\left\{M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(u_{n+1}, u_n, \frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\right)$ $M(Au, u, t) \geq T\left(\min\left\{M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(u_{n+1}, u_n, \frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\right)$

$t > 0$ for each n in N. setting limit as $n \to \infty$ for above and making use of (2.1.3) We obtain

$\geq T\left(\min\left\{1, M\left(u, Au, \frac{t\lambda_1}{\alpha b}\right), 1\right\}, 1\right)$

$= T\left(\min\left\{1, M\left(u, Au, \frac{t\lambda_1}{\alpha b}\right)\right\}, 1\right)$

$= T\left(M\left(u, Au, \frac{t\lambda_1}{\alpha b}\right), 1\right)$

$= M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right)$, $t>0$, then since $v_1 = \frac{b\alpha}{\lambda_1}$

$0 < v_1 < 1$ additionally through Lemma (2.3)leads to the conclusion that whene

t is greater than 0 then u equal Au

(ii) $M(u, Bu, t) \geq T\left(M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), M\left(Au_n, Bu, \frac{t\lambda_2}{b}\right)\right)$

$\geq T\left(M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right),\right.$

$\left.\min\left\{M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right), M\left(Au_n, u_n, \frac{t\lambda_2}{\alpha b}\right), M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right)\right\}\right)$

Through utilizing $n \to \infty$ and using(2.1.3) wet obtain

$M(u, Bu, t) \geq T\left(1, \min\left\{1, 1, M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right)\right\}\right)$

$= M\left(Bu, u, \frac{t}{v_2}\right)$, $t>0$, where $v_2 = \frac{b\alpha}{\lambda_2} \in (0,1)$

and through lemma(2.3)It Consequently follows that $Bu = u$

Finally Assume that there exist two (C.F.P) of A and B

We refer to them as u and r using $u \neq r$. From((2.1.1)

$M(u, r, t) = M(Au, Br, t) \geq$

$\left(\min\left\{M\left(u, r, \frac{t}{\alpha}\right), M\left(Au, u, \frac{t}{\alpha}\right), M\left(Br, r, \frac{t}{\alpha}\right)\right\}\right)$

$= (\min\{ M(u, r, \frac{t}{\alpha}), M\left(u, u, \frac{t}{\alpha}\right), M(r, r, \frac{t}{\alpha})\})$

$= M\left(u, r, \frac{t}{\alpha}\right)$

When t is greater than 0,
and by lemma(2.3) Therefore, it follows that u=r.

If B=A ,we have (Th.2.5[11])

Corollary 2.1: Let (X,M,T) be a complete fuzzy b-metric space ,and Let $A: X \to X$ self − mapping on X Suppose that there exists

$0 < \alpha < \frac{1}{b}$ such that

$M(Au, Av, t) \geq \min\{ M(u, v, \frac{t}{\alpha}), M(Au, u, \frac{t}{\alpha}), M(Av, v, \frac{t}{\alpha})\},$ ... (2.1.1)

For all u, v ∈X,t>o,Then A have a unique fixed point in X.

Theorem (2. 2): Let (X, M ,Tmin) be a complete fuzzy b-metric space, and Let A,B:X⟶X be two self-mappings on X, Suppose that there exists

$0 < \alpha < \frac{1}{b^2}$ such that the following condition holds:

$M(Au, Bv, t) \geq \min\{ M(u, v, \frac{t}{\alpha}), M(Au, u, \frac{t}{\alpha}), M(Bv, v, \frac{t}{\alpha}), M(Au, v, \frac{2t}{\alpha}), M(u, Bv, \frac{t}{\alpha})\},$
... (2.2.1)

For all u,v ∈X,t>o,

then A,B have a unique common fixed point in X.

Proof: For every sequence on a complete fuzzy (M,X,T)b-metric space, we define $\{u_n\}$.

with Picard iteration $u_{2n+1}=Au_{2n}$, $u_{2n+2}=Bu_{2n+1}$ for all $n \in N$ ... (2.2.2)

Now , utilizing (2.2.1) additionally (2.2.2) , we possess

$M(u_{2n+1}, u_{2n+2}, t) = M(Au_{2n}, Bu_{2n+1}, t)$

$\geq \min\{ M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(Au_{2n}, u_{2n}, \frac{t}{\alpha}), M(Bu_{2n+1}, u_{2n+1}, \frac{t}{\alpha}), M(Au_{2n}, u_{2n+1}, \frac{2t}{\alpha}), M(u_{2n}, Bu_{2n+1}, \frac{t}{\alpha})\}$

$= \min\{ M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(u_{2n+1}, u_{2n}, \frac{t}{\alpha}),$

$M(u_{2n+2}, u_{2n+1}, \frac{t}{\alpha}), M(u_{2n+1}, u_{2n+1}, \frac{2t}{\alpha}), M(u_{2n}, u_{2n+2}, \frac{t}{\alpha})\}$

$\geq \min \{M(u_{2n}, u_{2n+1}, \frac{t}{\alpha}), M(u_{2n+2}, u_{2n+1}, \frac{t}{\alpha}), 1, \min\{M(u_{2n}, u_{2n+1}, \frac{t}{2b\alpha}), M(u_{2n+1}, u_{2n+2}, \frac{t}{2b\alpha})\}\}$

$\geq \min\{M(u_{2n}, u_{2n+1}, \frac{t}{2b\alpha}), M(u_{2n+1}, u_{2n+2}, \frac{t}{2b\alpha})\}$, If $t>0$ and $n \in N$

Thus

Either $M(u_{2n+1}, u_{2n+2}, t) \geq M(u_{2n+1}, u_{2n+2}, \frac{t}{2\alpha b})$
,where $t>0$,$n \in N$ therefor through lemma(2.3) it implese

$u_{2n+1} = u_{2n+2}$,n in N .

thu, or $M(u_{2n+1}, u_{2n+2}, t) \geq M(u_{2n}, u_{2n+1}, \frac{t}{2\alpha b})$,

where $t > 0$, $n \in N$,then through lemma(2.1) we find that a sequence

$\{u_n\}$ is Cauchy By definition of completeness of(X , M ,T) ,we can say

that $\{u_n\}$ is convergent. Thus there is $u \in X$ so that

$u_n \to u$ whene $n \to \infty$ and $M(u, u_n, t) \to 1$

whene $n \to \infty$,tis greater than 0 ...(2.2.3)

To show that u is (C.F.P) for A and B .suppose $\lambda_1 \in (\alpha b^2, 1)$and $\lambda_2 = 1 - \lambda_1$. By using (2.1.1) and bF4 we find

(i) $M(Au, u, t) \geq \min\{M\left(Au, Bu_n, \frac{t\lambda_1}{b}\right), M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\}$

$\geq \min \{\min\left\{M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(Bu_n, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u_n, \frac{2t\lambda_1}{\alpha b}\right), M\left(u, Bu_n, \frac{t\lambda_1}{\alpha b}\right)\right\}, M(u_{n+1}, u, \frac{t\lambda_2}{b})\}$

$= \min \{\min\left\{M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(u_{n+1}, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u_n, \frac{2t\lambda_1}{\alpha b}\right), M\left(u, u_{n+1}, \frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\}$

$$\geq \min \{\min\left\{\begin{matrix} M\left(u, u_n, \frac{t\lambda_1}{\alpha b}\right), M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), M\left(u_{n+1}, u_n, \frac{t\lambda_1}{\alpha b}\right), \\ \min\left\{M\left(Au, u, \frac{t\lambda_1}{\alpha b^2}\right), M\left(u, u_n, \frac{t\lambda_1}{\alpha b^2}\right)\right\}, M\left(u, u_{n+1}, \frac{t\lambda_1}{\alpha b}\right) \end{matrix}\right\}$$

$,M(u_{n+1}, u, \frac{t\lambda_2}{b})$ $t > 0$ for each n in N.

Setting a limit as $n \to \infty$ for above and applying (2.2.3), we obtain

$M(Au, u, t) \geq \min\{\min\{1, M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), 1, \min\left\{M\left(Au, u, \frac{t\lambda_1}{\alpha b^2}\right), 1\right\}, 1\}, 1\}$

$\geq \min\{1, M\left(Au, u, \frac{t\lambda_1}{\alpha b}\right), 1, \min\left\{M\left(Au, u, \frac{t\lambda_1}{\alpha b^2}\right), 1\right\}, 1\}$

$= M\left(Au, u, \frac{t\lambda_1}{\alpha b^2}\right)$, $t>0$, where $v_1=\frac{b\alpha}{\lambda_1}$, $0<v_1<1$.

Then, since $v_1=\frac{b\alpha}{\lambda_1}$,

$0 < v_1 < 1$ additionally through Lemma (2.3)leads to the conclusion that whene

t is greater than 0 then u equal Au

(ii) $M(u, Bu, t) \geq \min\left(M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), M\left(Au_n, Bu, \frac{t\lambda_2}{b}\right)\right)$

$\geq \min\left\{M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), \min\left\{M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right), M\left(Au_n, u_n, \frac{t\lambda_2}{\alpha b}\right), M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right), M\left(Au_n, u, \frac{2t\lambda_2}{\alpha b}\right), M\left(u_n, Bu, \frac{t\lambda_2}{\alpha b}\right)\right\}\right\}$

$= \min\left\{M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), \min\left\{M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right), M\left(u_{n+1}, u_n, \frac{t\lambda_2}{\alpha b}\right), M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right), M\left(u_{n+1}, u, \frac{2t\lambda_2}{\alpha b}\right), M\left(u_n, Bu, \frac{t\lambda_2}{\alpha b}\right)\right\}\right\}$

$$\geq \min\left\{M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), \min\left\{\begin{array}{l} M\left(u_n, u, \frac{t\lambda_2}{b\alpha}\right), M\left(u_{n+1}, u_n, \frac{\lambda_2 t}{b\alpha}\right), M\left(Bu, u, \frac{\lambda_2 t}{b\alpha}\right), \\ M\left(u_{n+1}, u, \frac{2t\lambda_2}{b\alpha}\right), \min\left\{M\left(u_n, u, \frac{t\lambda_2}{\alpha b^2}\right), M\left(u, Bu, \frac{t\lambda_2}{\alpha b^2}\right)\right\} \end{array}\right\}\right\}$$

For every , $t > 0$. n in N.

setting limit as $n\to\infty$ for above and applying (2.2.3), we obtain

$M(u, Bu, t) \geq \min\left\{1, \min\left\{1, 1, M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right), 1, \min\left\{1, M\left(u, Bu, \frac{t\lambda_2}{\alpha b^2}\right)\right\}\right\}\right\}$

$\geq \min\left\{1, 1, M\left(Bu, u, \frac{t\lambda_2}{\alpha b}\right), 1, \min\left\{1, M\left(u, Bu, \frac{t\lambda_2}{\alpha b^2}\right)\right\}\right\}$

$= M\left(u, Bu, \frac{t\lambda_2}{\alpha b^2}\right)$, $t>0$, since $v_2=\frac{\alpha b^2}{\lambda_2}$, $0<v_2<1$.

additionally through Lemma (2.3) leads to the conclusion that when t is greater than 0

then $u = Bu$

Finally suppose that there exist two (C.F.P) of A and B

We call then u and r with $u \neq r$, From((2.2.1)

$M(Au, Br, t) \geq \min\left\{M\left(u, r, \frac{t}{\alpha}\right), M\left(Au, u, \frac{t}{\alpha}\right), M\left(Br, r, \frac{t}{\alpha}\right), M\left(Au, r, \frac{2t}{\alpha}\right), M\left(u, Br, \frac{t}{\alpha}\right)\right\}$

$= \min\left\{M\left(u, r, \frac{t}{\alpha}\right), M\left(u, u, \frac{t}{\alpha}\right), M\left(r, r, \frac{t}{\alpha}\right), M\left(u, r, \frac{2t}{\alpha}\right), M\left(u, r, \frac{t}{\alpha}\right)\right\}$

$\geq \min\{ M(u, r, \frac{t}{\alpha}),\ M\left(u, u, \frac{t}{\alpha}\right), M(r, r, \frac{t}{\alpha}), \min\left\{ M\left(u, u, \frac{t}{\alpha b}\right), M\left(u, r, \frac{t}{\alpha b}\right)\right\}, M(r, u, \frac{t}{\alpha})\}$

$= \min\{ M(u, r, \frac{t}{\alpha}),\ 1,\ 1,\ \min\left\{1,\ M\left(u, r, \frac{t}{\alpha}\right)\right\},\ M(r, u, \frac{t}{\alpha})\}$

$= M\left(u, r, \frac{t}{\alpha b}\right) = M\left(Au, Br, \frac{t}{\alpha b}\right), t>0,$
additionally, lemma (2.3) indicates that r=u.

If B=A ,we have (Th.2.6[11])

Corollary 2.2: Let (X, M ,Tmin) be a complete fuzzy b-metric space, and Let $A: X \to X$ be self-mapping on X, Suppose that there exists

$0 < \alpha < \frac{1}{b^2}$ such that the following condition holds:

$M(Au,Av,t) \geq \min\{M(u, v, \frac{t}{\alpha}), M(Au, u, \frac{t}{\alpha}), M(Av, v, \frac{t}{\alpha}), M(Au, v, \frac{2t}{\alpha}), M(u, Av, \frac{t}{\alpha})\},$
... (3.1.2)

For all u,v ∈X,t>o,

Then A has a unique fixed point.

Theorem (2. 3): Let (X,M,T) be a complete fuzzy b-metric space , and Let A,B:X→ X.

be two self-mappings on X, Suppose that there exists $0 < \alpha < \frac{1}{b}$ such that

$$M(Au, Bv, t) \geq \min\left\{\frac{M(v, Bv, \frac{t}{\alpha})[1+ M\left(u, Au, \frac{t}{\alpha}\right)+ M\left(v, Au, \frac{t}{\alpha}\right)]}{M(u, v, \frac{t}{\alpha})}, M(u, v, \frac{t}{\alpha})\right\} \quad \text{... (2.3.1)}$$

For all u,v ∈ X.

Then A,B have a unique common fixed point in X.

Proof: We define $\{u_n\}$ be sequences in complete (M,X,T) fuzzy b_ metric

with Picard iteration $u_{2n+1}=Au_{2n}$, $u_{2n+2}=Bu_{2n+1}$ For all n∈ N ... (2.3.2)

Now , using (2.3.1)and (2.3.2) , We've got

$M(u_{2n+1}, u_{2n+2}, t) = M(Au_{2n}, Bu_{2n+1}, t) \geq \min$
$$\left\{\frac{M\left(u_{2n+1} Bu_{2n+1}, \frac{t}{\alpha}\right)\left[1+ M\left(u_{2n}, Au_{2n}, \frac{t}{\alpha}\right)+ M\left(u_{2n+1}, Au_{2n}, \frac{t}{\alpha}\right)\right]}{M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)}, M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)\right\}$$

$$= \min\left\{\frac{M\left(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha}\right)\left[1+ M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)+ M\left(u_{2n+1}, u_{2n+1}, \frac{t}{\alpha}\right)\right]}{M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})}, M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})\right\}$$

$$= \min\left\{\frac{M\left(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha}\right)\left[1+ M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)+ 1\right]}{M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)}, M\left(u_{2n}, u_{2n+1}, \frac{t}{\alpha}\right)\right\}$$

$$\geq \min\left\{\frac{M\left(u_{2n+1}\, u_{2n+2}, ,\frac{t}{\alpha}\right)\left[\,2+\; M\;\left(u_{2n}\, ,u_{2n+1}\, ,\frac{t}{\alpha}\right)\right]}{M\;\left(u_{2n}\, ,\; u_{2n+1}\, ,\frac{t}{\alpha}\right)}\; , M\left(u_{2n}\, ,u_{2n+1}\, ,\frac{t}{\alpha}\;\right)\right\}$$

$$\geq \min\left\{\frac{M\;\left(u_{2n+1},\; u_{2n+2},\; ,\frac{t}{\alpha}\right)\left[\,2+\; M\left(\;u_{2n},u_{2n+1}\, ,\frac{t}{\alpha}\right)\right]}{2+\; M(u_{2n}\, ,u_{2n+1}\, ,\frac{t}{\alpha})}\; , M(u_{2n}\, ,u_{2n+1}\, ,\frac{t}{\alpha}\;)\right\}$$

$$\geq\; \min\{\; M\;\;(\, u_{2n}\;\; , u_{2n+1}\, ,\frac{t}{\alpha})\;\; ,\;\; M\;\;(u_{2n+1}\;\; , u_{2n+2}\;\; ,\frac{t}{\alpha})\}$$

Thus

Either $M\,(\,u_{2n+1}\, , u_{2n+2}\, ,t\,)\ \geq\ M\ (\,u_{2n+1}\ , u_{2n+2}\, ,\ \frac{t}{\alpha})$ $t>0$, $n\in N$ Consequently, lemma (2.3) suggestive of $u_{2n+1} = u_{2n+2}$,n in N .

Thus, $M\,(u_{2n+1}\, , u_{2n+2}\, ,t\,) \geq\ M\ (\,u_{2n}\, , u_{2n+1}\, ,\ \frac{t}{\alpha})$, When n is in N and t is greater than zero,

then through lemma (2.1) , the sequence $\{u_n\}$ becomes clear to us isCauchy .

By definition based on the completeness of (M, T, X), we can state that $\{u_n\}$ convergent. There for a where u for X exists $u_n \to u$

as $n\to\infty$ and $M(u\, , u_n\, ,t)\to 1$ as $n\to\infty$, t is greater than zero ...(2.3.3)

To show that u is (C.F.P) for A and B .suppose $\lambda_1\in(\alpha b\, ,1)$ and $\lambda_2=1-\lambda_1$ .By using (2.1.1) and bF4 we have

(i) $$M\;\,(\,Au\;\; , u\, ,t\,)\;\geq T\;\,(\,M\;\,(\,Au\, , Bu_n\, ,\;\frac{t\lambda_1}{b}\;),\;\, M\,(u_{n+1}\;\; , u\, ,\;\;\frac{t\lambda_2}{b}))$$

$$\geq T(\min\left\{\frac{M\left(u_n\, ,Bu_{n+1}\, ,\frac{t\lambda_1}{\alpha b}\right)\left[1+M\left(u\;\, ,Au\, ,, \frac{t\lambda_1}{\alpha b}\right)+ M\left(u_n\, ,Au, \frac{t\lambda_1}{\alpha b}\right)\right]}{M\left(u\, , u_n\, , \frac{t\lambda_1}{\alpha b}\right)}, M\left(u\, ,u_n\, ,\;\frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}\, , u,\;\frac{t\lambda_2}{b}\right))$$

$$=T(\min\;\;\left\{\frac{M\left(u_n\, , u_{n+1}\, , \frac{t\lambda_1}{\alpha b}\right)\left[1+M\left(u\;\; ,Au\, , \frac{t\lambda_1}{\alpha b}\right)+ M\left(u_n\, ,Au,\frac{t\lambda_1}{\alpha b}\right)\right]}{M\left(u\, , u_n\, ,\frac{t\lambda_1}{\alpha b}\right)}, M\left(u\, , u_n\, ,\;\frac{t\lambda_1}{\alpha b}\right)\right\}, M\left(u_{n+1}\, , u,\;\frac{t\lambda_2}{b}\right))$$

By taking the limite $n \to \infty$ as well as by(2.3.3) we find

$$M\;\;(\,Au\;\;, u\, ,t)\geq T\left(\;\min\left\{\frac{1\;\;\;\left[\;\;1+M\left(u\;\;,A\,u\, ,,\;\frac{t\lambda_1}{\alpha b}\right)+M\;\;\left(u_n,\;\,A\,u\;\;,\frac{t\lambda_1}{\alpha b}\;\right)\right]}{1}, 1\right\},1\right)$$

$$= \min\;\left\{\left[\;\;1+ M\left(u\;\;,A\,u\, ,,\;\frac{t\lambda_1}{\alpha b}\right)+ M\left(u_n\, ,A\,u,\frac{t\lambda_1}{\alpha b}\;\right)\right], 1\;\right\}$$

$$M\;\;(\,Au\;\;, u\, ,t\,)\;\;=\;\;1$$

as well by (fm2)it Follows that $Au = u$

(ii) $M(u, Bu, t) \geq T(M(u, u_{n+1}, \frac{t\lambda_1}{b}), M(Au_n, Bu, \frac{t\lambda_2}{b}))$

$$\geq T(M(u, u_{n+1}, \frac{t\lambda_1}{b}), \min\left\{\frac{M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right)\left[1+M\left(u_n, Au_n, \frac{t\lambda_2}{\alpha b}\right)+M\left(u, Au_n, \frac{t\lambda_2}{\alpha b}\right)\right]}{M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right)}, M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right)\right\})$$

$$=T(M\left(u, u_{n+1}, \frac{t\lambda_1}{b}\right), \min\left\{\frac{M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right)\left[1+M\left(u_n, u_{n+1}, \frac{t\lambda_2}{\alpha b}\right)+M\left(u, u_{n+1}, \frac{t\lambda_2}{\alpha b}\right)\right]}{M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right)}, M\left(u_n, u, \frac{t\lambda_2}{\alpha b}\right)\right\})$$

By taking the limiten $\to \infty$ as well as *from*(2.3.3) we have

$$\geq T(1, (\min\left\{\frac{M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right)[1+1+1]}{1}, 1\right\})$$

$$\geq T(1, \min\left\{3M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right), 1\right\}) = \min\left\{3M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right), 1\right\}$$

If $\min\left\{3M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right), 1\right\}=1$, then

by(fm2) , it follows that $u = Bu$.

If $\min\left\{3M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right), 1\right\}=3M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right)>M\left(u, Bu, \frac{t\lambda_2}{\alpha b}\right)$,

Then ,from lemma(2.3) ,we get

$u = Bu$.

Finally suppose that there exist two (C.F.P) of A and B

We refer to them as u and r using $u \neq r$. From (2. 1 . 1).

$$M(u, r, t)= M(Au, Br, t) \geq \min\left\{\frac{M\left(r, Br, \frac{t}{\alpha}\right)\left[1+M\left(u, Au, \frac{t}{\alpha}\right)+M\left(r, Au, \frac{t}{\alpha}\right)\right]}{M\left(u, r, \frac{t}{\alpha}\right)}, M\left(u, r, \frac{t}{\alpha}\right)\right\}$$

$$= \min\left\{\frac{M\left(r, r, \frac{t}{\alpha}\right)\left[1+M\left(u, u, \frac{t}{\alpha}\right)+M\left(r, u, \frac{t}{\alpha}\right)\right]}{M\left(u, r, \frac{t}{\alpha}\right)}, M\left(u, r, \frac{t}{\alpha}\right)\right\}$$

$$\geq \min\left\{\frac{1\left[1+1+M\left(r, u, \frac{t}{\alpha}\right)\right]}{M\left(u, r, \frac{t}{\alpha}\right)}, M\left(u, r, \frac{t}{\alpha}\right)\right\}$$

$$= \min\left\{\frac{1\left[2+M\left(u, r, \frac{t}{\alpha}\right)\right]}{M\left(u, r, \frac{t}{\alpha}\right)}, M\left(u, r, \frac{t}{\alpha}\right)\right\}$$

$$\geq \min\left\{\frac{1\left[2+M\left(u,r,\frac{t}{\alpha}\right)\right]}{M\left(u,r,\frac{t}{\alpha}\right)}, M\left(u,r,\frac{t}{\alpha}\right)\right\}$$

$$= M\left(u,r,\frac{t}{\alpha}\right)$$

for t is greater than zero , then apply lemma (2.3) Consequently, $u = r$ follows.
When B equals A, we get

Corollary 2.3: Let us assume that (M , X, T_{min}) is a fuzzy b-metric space b-metric additionally examine A:X→X. Presuming 0 < α< 1/b exists so that

$$M(Au,Av,t) \geq \min\left\{\frac{M\left(v, Av, \frac{t}{\alpha}\right)\left[1+M\left(u,Au,\frac{t}{\alpha}\right)+M\left(v,Au,\frac{t}{\alpha}\right)\right]}{M\left(u,v,\frac{t}{\alpha}\right)}, M\left(u,v,\frac{t}{\alpha}\right)\right\} \quad \text{... (2.3.1)}$$

Then A has a unique fixed point inX each u and v in X, t is greater than o.

To prove the following theorem, we use the following definition:

Let us a set Φ which consists of all continuous functions

∅ : [0, 1]→ [0, 1]such that ∅(0)= 0, ∅(1)= 1, and

∅(α)> α, for all α ∈ (0, 1)[13].

Theorem (2. 4): Let (X, M ,Tmin) be a complete fuzzy b-metric space, and Let A, B: X → X be two self-mappings on X, Suppose that there exists $0 < \alpha < \frac{1}{b^2}$

such that the following condition holds:

$$M(Au,Bv,t) \geq \min\{\emptyset(M\left(u,v,\frac{t}{\alpha}\right)), \emptyset(M\left(u,Au,\frac{t}{\alpha}\right)), \emptyset(M\left(u,Bv,\frac{2t}{\alpha}\right)), \emptyset(M\left(v,Bv,\frac{t}{\alpha}\right))\}, \quad \text{... (3.1.4)}$$

For all u,v ∈X,t>o,

then A,B have a unique common fixed point in X.

Proof: According to our definition, $\{u_n\}$ is sequences in complete (M,X,T) fuzzy b_ metric

with Picard iteration $u_{2n+1}=Au_{2n}$, $u_{2n+2}=Bu_{2n+1}$ $for\ all\ n \in N$ ... (2.4.2)

Now , using (2.4.1)and (2.4.2) , we find

$M(u_{2n+1}, u_{2n+2}, t) = M(Au_{2n}, Bu_{2n+1}, t) \geq (\min\{\emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \emptyset(M(u_{2n}, Au_{2n}, \frac{t}{\alpha})), \emptyset(M(u_{2n}, Bu_{2n+1}, \frac{2t}{\alpha})), \emptyset(M(u_{2n+1}, Bu_{2n+1}, \frac{t}{\alpha}))\}$

$= (\min\{\emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \emptyset(M(u_{2n}, u_{2n+2}, \frac{2t}{\alpha})), \emptyset(M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha})) \}$

$\geq \min\{\emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \min\{ \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha b})), \emptyset(M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha b})) \}$
$, \emptyset(M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha})) \}$

$\geq \min\{\emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha})), \min\{ \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha b})), \emptyset(M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha b})) \}, \emptyset(M(u_{2n+1}, u_{2n+2}$
$, \frac{t}{\alpha})) \}$

$M(u_{2n+1}, u_{2n+2}, t) \geq \min\{ \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{\alpha b})), \emptyset(M(u_{2n+1}, u_{2n+2}, \frac{t}{\alpha b})) \}$

Then

Either $M(u_{2n+1}, u_{2n+2}, t) \geq \emptyset (M(u_{2n+1}, u_{2n+2}, \frac{t}{2\alpha b}))$, $n \in N, t > 0$

Since $\emptyset(a) > a$, We've got

$M (u_{2n+1}, u_{2n+2}, t))) \geq M(u_{2n+1}, u_{2n+2}, \frac{t}{2\alpha b})$

Then by lemma(2.3) it implies that, $u_{2n+1=} \; u_{2n+2}$, $n \in N$.

or $M(u_{2n+1}, u_{2n+2}, t) \geq \emptyset(M(u_{2n}, u_{2n+1}, \frac{t}{2\alpha b})), n \in N, t > 0, then$

Since $\emptyset(a) > a$, we have

$M(u_{2n+1}, u_{2n+2}, t) \geq M(u_{2n}, u_{2n+1}, \frac{t}{2\alpha b})$, $n \in N, t > 0$, therefor,

by lemma(2.1) *we have that* $\{u_n\}$ is *Cauchy sequence*

By definition of completeness of(X , M ,T) ,we can say

that $\{u_n\}$ *is convergent. Hence there exists* $u \in X$ *such that*

$\lim_{n\to\infty} u_n = u$ and $\lim_{n\to\infty} M(u, u_n, t) = 1, t > 0$ ...(2.4.3)

To show that u is (C.F.P) for A and B .suppose $\lambda_1 \in (\alpha b^2, 1)$ and

$\lambda_2 = 1 - \lambda_1$. *By using* (2.4.1) and bF4 we have

(i) $M (Au, u, t) \geq \min\left\{ M \left(Au, Bu_n, \frac{t\lambda_1}{b}\right), M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\right\}$

$\geq \min\left\{\{ \min\{\emptyset(M(u, u_n, \frac{t\lambda_1}{\alpha b})), \emptyset(M(u, Au, \frac{t\lambda_1}{\alpha b})), \emptyset(M(u, Bu_n, \frac{2t\lambda_1}{\alpha b})), \emptyset(M(u_n, Bu_n, \frac{t\lambda_1}{\alpha b})) \}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\}\right\}$

$= \min\left\{\{ \min\{\emptyset(M(u, u_n, \frac{t\lambda_1}{\alpha b})), \emptyset (M(u, Au, \frac{t\lambda_1}{\alpha b})), \emptyset(M(u, u_{n+1}, \frac{2t\lambda_1}{\alpha b})), \emptyset(M(u_n, u_{n+1}, \frac{t\lambda_1}{\alpha b})) \}, M\left(u_{n+1}, u, \frac{t\lambda_2}{b}\right)\}\right\}$

For all n∈N,t> 0.

Taking limit as $n\to\infty$ *for above and using*(2.4.3) *we get*

$$M(Au,u,t)\geq \min\left\{\min\{\emptyset(1),\emptyset(M(u,Au,\frac{t\lambda_1}{\alpha b})),\emptyset(1),\emptyset(1)\},1\right\}$$

$$=\min\left\{\min\left\{1,\ \emptyset\left(M\left(u,Au,\frac{t\lambda_1}{\alpha b}\right)\right),1,1\right\},\ 1\right\}=M(u,Au,\frac{t\lambda_1}{\alpha b})$$

$,t>0$, since $v_1=\frac{b\alpha}{\lambda_1}\in(0,1)$.

additionally, *lemma*(2.3) *it* Follows that $Au=u$

(ii) $M(u,Bu,t)\ \geq\ \min\{M(u,u_{n+1},\frac{t\lambda_1}{b}),M(Au_n,Bu,\frac{t\lambda_2}{b})\}$

$$\geq \min\ \{M(u,u_{n+1},\frac{t\lambda_1}{b}),\ \min\left\{\emptyset\left(M\left(u_n,u,\frac{t\lambda_2}{\alpha b}\right)\right),\ \emptyset\left(M\left(u_n,Au_n,\frac{t\lambda_2}{\alpha b}\right)\right),\emptyset\left(M\left(u_n,Bu,\frac{t\lambda_2}{\alpha b}\right)\right),\emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)\right\}\}$$

$$=\min\ \{M\left(u,u_{n+1},\frac{t\lambda_1}{b}\right),\ \min\left\{\emptyset\left(M\left(u_n,u,\frac{t\lambda_2}{\alpha b}\right)\right),\ \emptyset\left(M\left(u_n,u_{n+1},\frac{t\lambda_2}{\alpha b}\right)\right),\emptyset\left(M\left(u_n,Bu,\frac{2t\lambda_2}{\alpha b}\right)\right),\emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)\right\}\}$$

$$\geq\ \min\ \{M\left(u,u_{n+1},\frac{t\lambda_1}{b}\right),\ \min\left\{\begin{matrix}\emptyset\left(M\left(u_n,u,\frac{t\lambda_2}{\alpha b}\right)\right),\ \emptyset\left(M\left(u_n,u_{n+1},\frac{t\lambda_2}{\alpha b}\right)\right),\\ \min\left\{\emptyset\left(M\left(u_n,u,\frac{t\lambda_2}{\alpha b^2}\right),\emptyset\left(M(u,Bu,\frac{t\lambda_2}{\alpha b^2}\right)\right\},\\ \emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)\end{matrix}\right\}\}$$

Taking limit as $n\to\infty$ *for above and using*(2.4.3) *we get*

$$\geq\min\ \{1,\ \min\left\{\begin{matrix}\emptyset(1),\ \emptyset(1),\ \min\left\{\emptyset(1),\emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b^2}\right)\right)\right\},\\ \emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)\end{matrix}\right\}\}$$

$$=\min\ \{1,\ \min\left\{\begin{matrix}1,\ 1,\min\left\{1,\emptyset(M(u,Bu,\frac{t\lambda_2}{\alpha b^2})\right\},\\ \emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)\end{matrix}\right\}\}$$

$$\geq\emptyset\left(M\left(u,Bu,\frac{t\lambda_2}{\alpha b}\right)\right)$$

$= M\left(u, Bu, \frac{t\lambda_2}{\alpha b^2}\right), t > 0$, then, since $v_2 = \frac{\alpha b^2}{\lambda_2} \in (0,1)$.

additionally *lemma*(2.3) *it* Follows that $u = Bu$

Finally suppose that there exist two (C.F.P) of A and B

We call then u *and* r *with* $u \neq r$, From((2.4.1)

$$M(u, r, t) = M(Au, Br, t) \geq \min\{\emptyset(M(u, r, \tfrac{t}{\alpha})), \emptyset(M(u, Au, \tfrac{t}{\alpha})), \emptyset(M(u, Br, \tfrac{2t}{\alpha})), \emptyset(M(r, Br, \tfrac{t}{\alpha}))\}$$

$$= \min\{\emptyset(M(u, r, \tfrac{t}{\alpha})), \emptyset(M(u, u, \tfrac{t}{\alpha})), \emptyset(M(u, r, \tfrac{2t}{\alpha})), \emptyset(M(r, r, \tfrac{t}{\alpha}))\}$$

$$\geq \min\{\emptyset(M(u, r, \tfrac{t}{\alpha})), \emptyset\left(M\left(u, u, \tfrac{t}{\alpha}\right)\right), \min\left\{\emptyset\left(M\left(u, u, \tfrac{t}{\alpha b}\right)\right), \emptyset\left(M\left(u, r, \tfrac{t}{\alpha b}\right)\right)\right\}, \emptyset(M(r, r, \tfrac{t}{\alpha}))\}$$

$$= \min\{\emptyset(M(u, r, \tfrac{t}{\alpha})), \emptyset(1), \min\left\{\emptyset(1), \emptyset\left(M\left(u, r, \tfrac{t}{\alpha b}\right)\right)\right\}, \emptyset(1)\}$$

$$= \min\{\emptyset(M(u, r, \tfrac{t}{\alpha})), 1, \min\left\{1, \emptyset\left(M\left(u, r, \tfrac{t}{\alpha b}\right)\right)\right\}, 1\}$$

$$\geq \min\{\emptyset\left(M\left(u, r, \tfrac{t}{\alpha b}\right)\right), 1\}$$

$$\geq \emptyset\left(M\left(u, r, \tfrac{t}{\alpha b}\right)\right) \geq M\left(u, r, \tfrac{t}{\alpha b}\right) = M\left(Au, Br, \tfrac{t}{\alpha b}\right), t > 0,$$

and by lemma(2.3) *it* Follows that $u = r$.

If B=A, we have

Corollary 2.4: A*ssume* that (M, X, T_{min}) is a fuzzy b-metric space additionally ., and Assume that A from X to X. assume that

$0 < \alpha < \frac{1}{b^2}$ and let us $\emptyset: [0,1] \to [0,1]$ be a continuous functions such that

$\emptyset(0) = 0, \emptyset(1) = 1$, and $\emptyset(a) > a$, for all $a \in (0,1)$

such that the following condition holds:

$$M(Au, Av, t) \geq \min\{\emptyset(M(u, v, \tfrac{t}{\alpha})), \emptyset(M(u, Au, \tfrac{t}{\alpha})), \emptyset(M(u, Av, \tfrac{2t}{\alpha})), \emptyset(M(v, Av, \tfrac{t}{\alpha}))\}, \quad (2.4.1)$$

There is only one fixed point in X for every u, v □ *X,* t is greater than zero *. as a result,* *.*

References

[1] L.A. Zadeh, "Fuzzy sets", Information and control 8(3) (1965) 338-353.
[2] I. Bakhtin, "The contraction mapping principle in quasimetric spaces", Functional analysis 30 (1989) 26-37.
[3] S. Heilpern, " Fuzzy mappings and fixed point theorem", Journal of Mathematical Analysis and Applications 83(2) (1981) 566-569.
[4] O. Kaleva, S. Seikkala, "On fuzzy metric spaces", Fuzzy sets and systems 12(3) (1984) 215-229.
[5] S. Czerwik, "Contraction mappings in $ b $-metric spaces", Acta mathematica et informatica universitatis ostraviensis 1(1) (1993) 5-11.
[6] Z. Hassanzadeh, "S. Sedghi, Relation between b-metric and fuzzy metric spaces", Mathematica Moravica 22(1) (2018) 55-63.

[7] S. Sedghi, N. Shobe, J. Park, “Common fixed point theorem in b-fuzzy metric space”, Nonlinear Functional Analysis and Applications 17(3) (2012) 349-359.

[8] T. Norms, “Trends in Logic”, Studia Logica Library 8 (2000).

[9] A. George, P. Veeramani, “On some results in fuzzy metric spaces”, Fuzzy sets and systems 64(3) (1994) 395-399.

[10] T. Došenovic, A. Javaheri, S. Sedghi, N. Shobe, “Coupled fixed point theorem in b-fuzzy metric spaces”, Novi Sad J. Math 47(1) (2017) 77-88.

[11] D. Rakić, A. Mukheimer, T. Došenović, Z.D. Mitrović, S. Radenović, “On some new fixed point results in fuzzy b-metric spaces”, Journal of Inequalities and Applications 2020 (2020) 1-14.

[12] Venkata, G., Babu, R., Tolera Masissa, D, “ Fixed points in b-metric spaces via simulation function” , Novi Sad J. Math. 47(2), 133–147 (2017) 3

[13] V. Gupta, N. Mani, and A. Saini, “ Fixed point theorems and its applications in fuzzy metric spaces”, Proceedings of the Conference AEMDS-2013, 2013.