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On Triple Sequences Space of Fuzzy Numbers Identified by Simple Elliptic Orlicz Function

Aqeel Mohammed Hussein [a]

[a]*Department of Mathematics, Faculty of Education, University of Al-Qadisiyah, Al-Najaf , Iraq.Email: aqeel.hussein@qu.edu.iq*

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ABSTRACT

In this study, we provide the triple sequences space of fuzzy numbers identified by simple elliptic Orlicz function and discuss some properties like the space $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathfrak{i}}_{\mathfrak{j}})$ is complete, $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathfrak{i}}_{\mathfrak{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M} \circ \mathbb{M}_1,\triangle^{\mathfrak{i}}_{\mathfrak{j}})$, $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathfrak{i}}_{\mathfrak{j}}) \cap (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_2,\triangle^{\mathfrak{i}}_{\mathfrak{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1 + \mathbb{M}_2,\triangle^{\mathfrak{i}}_{\mathfrak{j}})$, $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathfrak{l}}_{\mathfrak{j}}) \subset (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathfrak{i}}_{\mathfrak{j}})$, for $0 \preccurlyeq \mathfrak{l} \prec \mathfrak{i}$.

MSC..

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1. Introduction

In this work ,we offer the triple sequence spaces of fuzzy numbers identified by simplified elliptic Orlicz function. We examine the implementations of further groupings of sequence spaces using Orlicz functions by Esi [2], Savas [6], Parashar and Choudhary [5], Altin, Et and Tripathy[1], and Isik, Et and Tripathy [3]. Esi [2], Savas [6], Parashar and Choudhary [5], Altin, Et and Tripathy [1], Tripathy and Sarma [7], Tripathy and Mahanta [8], Mursaleen, Khan, and Qamaruddin [4], Tripathy and Sarma [7], Tripathy and Mahanta [8], and others have also studied a number of algebraic and topological properties. In this paper , we introduce the triple sequences space $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathfrak{i}}_{\mathfrak{j}})$ of fuzzy numbers .

$\Theta : [0,\infty) \to [0,\infty)$ is called an Orlicz function which is convex, continuous, and non-decreasing with $\Theta(0) = 0$, $\Theta(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\Theta(\mathfrak{A}) \to \infty$ as $\mathfrak{A} \to \infty$.

∗Corresponding author *Aqeel Mohammed Hussein*

Email addresses: *aqeel.hussein@qu.edu.iq*

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2. Definitions and Preliminaries

$\mathbb{M}$ is called a simple elliptic Orlicz function if $\mathbb{M}(\mathfrak{A}) = \Theta(\mathfrak{A}) - \mathfrak{A}$ with Θ is an Orlicz function .

$\mathbb{R}(\mathbb{I})$ is denoted the collection of all upper-semi-continuous, regular fuzzy integers . $\mathbb{R}^*(\mathbb{I})$ is denoted the set non-negative fuzzy integers of all $\mathbb{R}(\mathbb{I})$.

$(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}}) = \left\{(\mathfrak{X}_{abc}) \in \mathbb{W}^3_{\mathbb{F}} : \sup_{abc}\left[\mathbb{M}\left(\frac{d_\infty(\triangle^{\mathrm{i}}_{\mathrm{j}}\mathfrak{X}_{abc}, \bar{0})}{\rho}\right)\right] \prec \infty, \text{for some } \rho \succ 0\right\}$, where $\mathbb{W}^3_{\mathbb{F}}$ is a space of all triple sequences .

3. Main Results

Theorem 3.1 :

$(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}})$ is complete space by the metric

$$\mathcal{G}(\mathfrak{X},\mathcal{Y}) = \sum_{\mathfrak{n}=1}^{\mathrm{ij}}\sum_{\mathfrak{m}=1}^{\mathrm{ij}}\sum_{\mathfrak{z}=1}^{\mathrm{ij}} d_\infty(\mathfrak{X}_{\mathfrak{nmz}}, \mathcal{Y}_{\mathfrak{nmz}}) + \inf\left\{\rho \succ 0 : \sup_{abc}\left[\mathbb{M}\left(\frac{d_\infty(\triangle^{\mathrm{i}}_{\mathrm{j}}\mathfrak{X}_{abc}, \triangle^{\mathrm{i}}_{\mathrm{j}}\mathcal{Y}_{abc})}{\rho}\right)\right] \preccurlyeq 1\right\}, \forall\, \mathfrak{X}, \mathcal{Y} \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}}).$$

Proof :

Assume $(\mathfrak{X}^{(\ell k j)})$ be a Cauchy triple sequence in $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}})$ with the function that $\mathfrak{X}^{(\ell k j)} = (\mathfrak{X}^{(\ell k j)}_{d b a})^\infty_{d,c,a=1}$. Let $\varepsilon \succ 0$. choose $r \succ 0 \ni \mathbb{M}\left(\frac{r x_0}{2}\right) \succcurlyeq 1$. then $\forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0$, $\exists$ positive integer $\mathfrak{n}_0 = \mathfrak{n}_0(\varepsilon)$, $\mathfrak{n}_0 \succ 0 \ni$ $\mathcal{G}(\mathfrak{X}^{(\ell k j)}, \mathfrak{X}^{(w v u)}) \prec \frac{\epsilon}{r x_0}$.

From the definition of $\mathcal{G}$, we have ,

$$\sum_{\mathfrak{n}=1}^{\mathrm{ij}}\sum_{\mathfrak{m}=1}^{\mathrm{ij}}\sum_{\mathfrak{z}=1}^{\mathrm{ij}} d_\infty\left(\mathfrak{X}^{(\ell k j)}_{\mathfrak{nmz}}, \mathcal{Y}^{(w v u)}_{\mathfrak{nmz}}\right) + \inf\left[\rho \succ 0 : \sup_{abc}\left[\mathbb{M}\left(\frac{d_\infty(\triangle^{\mathrm{i}}_{\mathrm{j}}\mathfrak{X}^{(\ell k j)}_{abc}, \triangle^{\mathrm{i}}_{\mathrm{j}}\mathcal{Y}^{(w v u)}_{abc})}{\rho}\right)\right] \preccurlyeq 1\right] \prec \varepsilon, \forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0 \quad (3\text{-}1).$$

which implies that,

$$\sum_{\mathfrak{n}=1}^{\mathrm{ij}}\sum_{\mathfrak{m}=1}^{\mathrm{ij}}\sum_{\mathfrak{z}=1}^{\mathrm{ij}} d_\infty\left(\mathfrak{X}^{(\ell k j)}_{\mathfrak{nmz}}, \mathcal{Y}^{(w v u)}_{\mathfrak{nmz}}\right) \prec \varepsilon, \forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0$$

$$\Longrightarrow d_\infty\left(\mathfrak{X}^{(\ell k j)}_{\mathfrak{nmz}}, \mathcal{Y}^{(w v u)}_{\mathfrak{nmz}}\right) \prec \varepsilon, \forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0, \mathfrak{n}, \mathfrak{m}, \mathfrak{z} = 1,2,3, \dots, \mathrm{ij}.$$

Therefore $\left(\mathfrak{X}^{(\ell k j)}_{\mathfrak{nmz}}\right)$ is a Cauchy triple sequence in $\mathbb{R}^n(\mathbb{I})$, so is convergent in $\mathbb{R}^n(\mathbb{I})$ by the completeness property of $\mathbb{R}^n(\mathbb{I})$, $\forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{z} = 1,2,3, \dots, \mathrm{ij}$

Let

$$\lim_{\ell, k, j \to \infty} \mathfrak{X}^{(\ell k j)}_{\mathfrak{nmz}} = \mathfrak{X}_{\mathfrak{nmz}}, \forall\, \mathfrak{n}, \mathfrak{m}, \mathfrak{z} = 1,2,3, \dots, \mathrm{ij}. \quad \dots\dots\dots (3\text{-}2)$$

Also,

$$\sup_{abc}\left[\mathbb{M}\left(\frac{d_\infty(\triangle^{\mathrm{i}}_{\mathrm{j}}\mathfrak{X}^{(\ell k j)}_{abc}, \triangle^{\mathrm{i}}_{\mathrm{j}}\mathcal{Y}^{(w v u)}_{abc})}{\rho}\right)\right] \preccurlyeq 1, \forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0 \dots (3\text{-}3)$$

$$\Longrightarrow \left[\mathbb{M}\left(\frac{d_\infty(\triangle^{\mathrm{i}}_{\mathrm{j}}\mathfrak{X}^{(\ell k j)}_{abc}, \triangle^{\mathrm{i}}_{\mathrm{j}}\mathcal{Y}^{(w v u)}_{abc})}{\rho}\right)\right] \preccurlyeq 1 \preccurlyeq \mathbb{M}\left(\frac{r x_0}{2}\right), \forall\, \ell, k, j, w, v, u \succcurlyeq \mathfrak{n}_0 .$$

By the continuity of $\mathbb{M}$, we arrive that,

$$\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)},\ \triangle_j^i\ \mathcal{Y}_{abc}^{(wvu)}\right) \leqslant \frac{r x_0}{2} .\ \mathcal{G}(\mathfrak{X}^{(\ell k j)}, \mathfrak{X}^{(wvu)}),\ \forall\ \ell, k, j, w, v, u \geqslant n_0.$$

$$\Longrightarrow \mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)},\ \triangle_j^i\ \mathcal{Y}_{abc}^{(wvu)}\right) \leqslant \frac{r x_0}{2} . \frac{\varepsilon}{r x_0} = \frac{\varepsilon}{2},\ \forall\ \ell, k, j, w, v, u \geqslant n_0.$$

$$\Longrightarrow \mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)},\ \triangle_j^i\ \mathcal{Y}_{abc}^{(wvu)}\right) \leqslant \frac{\varepsilon}{2}, \forall\ \ell, k, j, w, v, u \geqslant n_0.$$

Therefore $\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}\right)$ is a Cauchy triple sequence in $\mathbb{R}^n(\mathbb{I})$, so is convergent in $\mathbb{R}^n(\mathbb{I})$ by the completeness property of $\mathbb{R}^n(\mathbb{I})$. Let $\lim_{\ell k j} \triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)} = \mathcal{Y}_{abc}$ in $\mathbb{R}^n(\mathbb{I})$, $\forall\ a, b, c \in \mathbb{N}$.

We must prove ,

$$\lim_{\ell k j} \mathfrak{X}^{(\ell k j)} = \mathfrak{X} \text{ and } \mathfrak{X} \in (\ell_\infty)_{\mathbb{F}}^3(\mathbb{M}, \triangle_j^i) .$$

$$\triangle_j^i\ \mathfrak{X}_{lkf} = \sum_{r=0}^{i} (-1) \binom{i}{r} \mathfrak{X}_{(l+rj)(k+rj)(f+rj)} \text{……. } (**)$$

and

$$\sum_{n=1}^{ij} \sum_{m=1}^{ij} \sum_{z=1}^{ij} \mathbb{d}_\infty\left(\mathfrak{X}_{nmz}^{(\ell k j)}, \mathcal{Y}_{nmz}^{(wvu)}\right) + \inf\left[\rho > 0 : \sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}, \triangle_j^i\ \mathcal{Y}_{abc}^{(wvu)}\right)}{\rho}\right)\right] \leqslant 1\right] < \varepsilon, \forall\ \ell, k, j, w, v, u \geqslant n_0 \text{ ...(3-1).}$$

$\forall\ a, b, c = 1$, from (**) and (3-1), we get ,

$$\lim_{\ell, k, j \to \infty} \mathfrak{X}_{ij+1}^{(\ell k j)} = \mathfrak{X}_{ij+1}, \text{ for } i \geqslant 1, j \geqslant 1.$$

We obtain , using this method of induction,

$$\lim_{\ell, k, j \to \infty} \mathfrak{X}_{abc}^{(\ell k j)} = \mathfrak{X}_{ab} , \forall\ a, b, c \in \mathbb{N}.$$

Moreover, $\lim_{\ell, k, j \to \infty} \triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)} = \triangle_j^i\ \mathfrak{X}_{abc}$, $\forall\ a, b, c \in \mathbb{N}$. Now, taking $w, v, u \to \infty$, keeping fixing $(\ell k j)$ and by continuity of $\mathbb{M}$, we have the following from of (3-3),

$$\sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}, \triangle_j^i \mathfrak{X}_{abc}\right)}{\rho}\right)\right] \leqslant 1, \text{ for some } p > 0 .$$

Now ,taking the infimum of such p's, we obtain

$$\inf\left\{\rho > 0 : \sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}, \triangle_j^i \mathfrak{X}_{abc}\right)}{\rho}\right)\right] \leqslant 1\right\} < \varepsilon, \forall\ \ell, k, j \geqslant n_0 (\text{by } (3\text{-}1))$$

Consequently on taking limit as $w, v, u \to \infty$, we arrive that

$$\sum_{n=1}^{ij} \sum_{m=1}^{ij} \sum_{z=1}^{ij} \mathbb{d}_\infty\left(\mathfrak{X}_{nmz}^{(\ell k j)}, \mathfrak{X}_{nmz}\right) + \inf\left[\rho > 0 : \sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}, \triangle_j^i \mathfrak{X}_{abc}\right)}{\rho}\right)\right] \leqslant 1\right] < \varepsilon + \varepsilon = 2\varepsilon, \forall\ \ell, k, j \geqslant n_0.$$

Which tends to,

$$\mathcal{G}\left(\mathfrak{X}^{(\ell k j)}, \mathfrak{X}\right) < 2\varepsilon, \forall\ \ell, k, j \geqslant n_0.$$

i.e. $\lim_{\ell k j} \mathfrak{X}^{(\ell k j)} = \mathfrak{X}$. Next , we show that $\mathfrak{X} \in (\ell_\infty)_{\mathbb{F}}^3(\mathbb{M}, \triangle_j^i)$. We aware that,

$$\mathcal{G}\left(\triangle_j^i\ \mathfrak{X}_{abc}, \bar{0}\right) \leqslant \mathcal{G}\left(\triangle_j^i\ \mathfrak{X}_{abc},\ \triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}\right) + \mathcal{G}\left(\triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}, \bar{\theta}\right).$$

Since $\mathbb{M}$ is non-decreasing , so is continuous, we have

$$\sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}, \bar{0}\right)}{\rho}\right)\right] \leqslant \sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{abc}, \triangle_j^i\ \mathfrak{X}_{abc}^{(\ell k j)}\right)}{\rho}\right)\right] + \sup_{abc}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle_j^i\ \mathfrak{X}_{ab}^{(\ell k c)}, \bar{0}\right)}{\rho}\right)\right] < \infty.$$

Therefore $\mathfrak{X} \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}})$

Thus,

$(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}})$ is a complete .

Theorem 3.2 :

Let $\mathbb{M}, \mathbb{M}_1$ and $\mathbb{M}_2$ be a maximal Orlicz functions which satisfy the Δ_2-condition .

Then :

$\mathbb{i})(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M} \circ \mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}})$

$\mathbb{ii})(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}}) \cap (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1 + \mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

Proof :

$\mathbb{i}$) Assume $(\mathfrak{X}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}})$. Consider $\mathfrak{y} \succ 0$ and $\varepsilon \succ 0 \ni \varepsilon = \mathbb{M}(\mathfrak{y})$. Then,

$$\mathbb{M}_1\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\ \mathbb{L}\right)}{\rho}\right) \prec \mathfrak{y}\text{ , for some } \rho \succ 0.$$

Let

$$\mathcal{Y}_{abc} = \left[\mathbb{M}_1\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\ \mathbb{L}\right)}{\rho}\right)\right]\text{, for some } \rho \succ 0.$$

Since $\mathbb{M}$ is non-decreasing and continuous, we obtain,

$$\mathbb{M}(\mathcal{Y}_{abc}) = \mathbb{M}\left[\mathbb{M}_1\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\ \mathbb{L}\right)}{\rho}\right)\right] \prec \mathbb{M}(\mathfrak{y}) = \varepsilon\text{ , for some } \rho \succ 0.$$

This tends that, $(\mathfrak{X}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M} \circ \mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

Consequently $(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M} \circ \mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

$\mathbb{ii}$) Let $(\mathfrak{X}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}}) \cap (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

Then,

$$\left[\mathbb{M}_1\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\ \mathbb{L}\right)}{\rho}\right)\right] \prec \varepsilon\text{ , for some } \rho \succ 0 \text{ and } \left[\mathbb{M}_2\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\ \mathbb{L}\right)}{\rho}\right)\right] \prec \varepsilon\text{ , for some } \rho \succ 0.$$

$(\mathbb{M}_1 + \mathbb{M}_2)\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\mathbb{L}\right)}{\rho}\right) = \mathbb{M}_1\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\mathbb{L}\right)}{\rho}\right) + \mathbb{M}_2\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}}_{\mathrm{j}}\, \mathfrak{x}_{abc},\mathbb{L}\right)}{\rho}\right) \prec \varepsilon + \varepsilon = 2\varepsilon$, for some $\rho \succ 0$.Therefore $(\mathfrak{X}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1 + \mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

Thus,

$(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1,\triangle^{\mathrm{i}}_{\mathrm{j}}) \cap (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}}) \subseteq (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M}_1 + \mathbb{M}_2,\triangle^{\mathrm{i}}_{\mathrm{j}})$.

Proposition 3.3 :

$(\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{I}}_{\mathrm{j}}) \subset (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}}_{\mathrm{j}})$, for $0 \preccurlyeq \mathrm{I} \prec \mathrm{i}$.

Proof :

Let $(\mathfrak{X}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\mathbb{M},\triangle^{\mathrm{i}-1}_{\mathrm{j}})$. Then we have,

$$\sup_{abc \geqslant 1}\left[\mathbb{M}\left(\frac{\mathbb{d}_\infty\left(\triangle^{\mathrm{i}-1}_{\mathrm{j}}\, \mathfrak{x}_{abc},\bar{0}\right)}{\rho}\right)\right] \prec \infty.$$

Now we have ,

$$\sup_{abc\geqslant 1}\left[\mathbb{M}\left(\frac{d_\infty\left(\triangle_j^i\, x_{abc},\bar{0}\right)}{\rho}\right)\right]=\sup_{abc\geqslant 1}\left[\mathbb{M}\left(\frac{d_\infty\left(\triangle_j^{i-1}\, x_{abc}-\triangle_j^{i-1}\, x_{abc+1},\bar{0}\right)}{2\rho}\right)\right]\leqslant$$

$$\sup_{abc\geqslant 1}\frac{1}{2}\left[\mathbb{M}\left(\frac{d_\infty\left(\triangle_j^{i-1}\, x_{abc},\bar{0}\right)}{\rho}\right)\right]+\sup_{abc\geqslant 1}\frac{1}{2}\left[\mathbb{M}\left(\frac{d_\infty\left(\triangle_j^{i-1}\, x_{abc+1},\bar{0}\right)}{\rho}\right)\right]\prec\infty.$$

In the same way , we have

$$(\ell_\infty)_{\mathbb{F}}^3\left(\mathbb{M},\triangle_j^{l}\right)\subset(\ell_\infty)_{\mathbb{F}}^3\left(\mathbb{M},\triangle_j^{i}\right),\text{ for } 0\leqslant l\prec i\,.$$

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