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Review on graph theory-based image segmentation with its methods

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ABSTRACT

In areas of digital image processing and computer vision, image segmentation is defined as a crucial process that divides an image into many segments for more straightforward and accurate object analysis. Making use of graph-based techniques as an effective tool for segmenting images has drawn more consideration recently. Since graph-based techniques are attractive and increasingly prevalent and can designate image properties, in this article, some of the primary graph-based techniques have been presented. This scheme utilizes graph theory to create a graph depiction of an image in which each pixel is represented as a node and the edges show the degree of similarity between two pixels. When items are represented by vertices and an edge connects them, a graph may be used to depict the relationship between them. To divide a graph into sub-graphs that reflect significant items of interest, this study explores some graph theoretical approaches for image segmentation, including minimum spanning tree, pyramid-based, graph cut-based, and interactive image segmentation and their employing in significant image processing fields such as medical image analysis for infection diagnosis, and remote sensing.

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1. Introduction

Image segmentation aims to achieve regional similarity within each segmented region and minimize regional similarity between segments, a common clustering and optimization problem. Graph theory has emerged as a new area of research in image segmentation technology, using edges to depict relationships between items [1]. A weighted graph in graph-based approaches is marked as an image where the pixels are referred to by nodes, and the edge depicts the association between pixels. By utilizing similarity measure the weight quantifies these pixels association. Relevant image areas are shown in the specification of image segmentation, which is then represented as a graph divided into sub-graphs [2]. Graph-Based Technique: region assessment is taken into account for construction in this procedure. Graph G with its vertices V and edges E is considered in this type of segmentation, where the two vertices are associated with a single edge and have their weight which refers to variation between two adjacent components vi and vj. Where w (vi, vj) refers to a weight between components [3]. In a graph scheme, segmentation (S) divides (V) into regions such that every effective region C ∈ S matches a corresponding section in a graph G_ = (V, E_), whereby E_ ⊆ E. By means, any segmentation has been created by a subset of the edges that exist in (E). To achieve the scaling of segmentation quality several procedures are available, the requirement in this methodology is that the similarity in parts of the section is satisfying and other parts in different sections imply dissimilarity. Fig 1 for example uses the graph technique [4]. In this technique image features for example (image pixels, and pixel intensities) have been arranged in mathematically inclusive structures [5].

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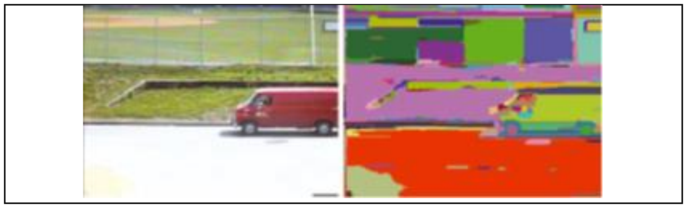

Fig.1 uses a graph in segmentation [2]

The segmentation methodology is classified into four categories: first, graph-cut-based, then interactive process, then minimum spanning tree, and finally pyramid technique [6].

2. Graph-based image segmentation categories

2.1. Graph cuts based

For complicated scene images, automated image segmentation may yield unsatisfactory results. To achieve desired outcomes, interactive segmentation approaches that need previous knowledge with the background and the foreground (object) are utilized. This method (graph-cut) has drawn a lot of interest for interactive image segmentation because it offers a global minimum solution for the Gibbs function that is made up of the segment region's and its boundary's energy [7].

One of the best techniques for segmenting images, Graph Cut, effectively integrates both kinds of data. Graph Cut replaces the input images with grid graphs that contain edge weights that are predetermined. The graph's minimum cut is determined to segment the images utilizing a minimum cut algorithm [8].

Two associated sections A and B are yielded from dividing an existing graph G neglecting the edges of these sections. The whole weight of edge elimination is considered for scaling the degree of related A and B, known as a graph cut, and a symbolic representation of the graph cut is shown in Fig. 2. This graph cut value is minimized by an ideal bi-partition. By appropriately separating the graph made from an image using the graph cut, many homogenous zones could be constructed. The minimal cut algorithm and a normalized cut algorithm are important works in this field. Equation 1 represents the cost c (A, B) of this cut calculation [6]. Fig. 3 is an example of using a graph cut in [9]

$$cut(A,B) = \sum_{\substack{u \in A \\ v \in B}} w(u,v) \tag{1}$$

where A and B correspond to the two disjoint sets of nodes of the resulting bipartition.

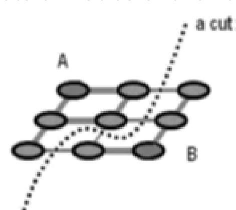

Fig. 2- graph cut [6].

Fig. 3- Image segmentation using graph cut [9].

The normalized cut graph-based image segmentation method is widely used and considers image segmentation as a graph partitioning problem. It addresses overall similarity among regions as well as total dissimilarity between regions, and its Segmentation algorithm is explained in the following steps [10].

Step 1: Graph G = (V, E) is used to begin the task. Affinity matrix (A) and degree matrix (D) are described.
Step 2: Determine which eigenvectors have the least Eigenvalues by solving the equation $(D - A)\ x = \lambda D_x$.
Step 3: Assume that x_2 = eigenvectors whose eigenvalues are the second smallest ($\lambda 2$).
Step 4: To get the binary-valued vector x', set a threshold of x_2, such that for all possible thresholds t, ncut(x') = ncut(x'_2).
Step 5: If ncut is less than threshold T for each of the two new regions, then recurse on the region.

The minimal cut problem is to divide the vertices of an undirected graph with non-negative edge weights into two groups such that the total of the edge weights between the two groups is as little as possible. A graph's edge connectivity is another term that's frequently used to describe a minimal cut [11].

2.2. Interactive image segmentation

The high-quality labels can be developed with less effort when utilizing interactive segmentation algorithms, which enable users to annotate several images rapidly. the complementary skills of humans and robots are brought together in interactive image segmentation procedures. While robots can handle enormous amounts of data in specific tasks, humans are fast at identifying things [12].
In computer vision, there is an issue that is considered the main which is interactive segmentation where extensively researched for a long time. Various kinds of user inputs are used in many interactive segmentation procedures that have tried to segment a target object. These user inputs include bounding boxes, contours, and scribbles Many techniques have been presented, including GrabCut, random walks, geodesics, and approaches using shape prior [13]. Many applications can use this method directly, such as editing an image and analyzing the medical image. So, the goal of this method is to segment the required interested occurrences with minimal user input. Fig. 4 shows an implementation with Interactive segmentation [14]. Interactive segmentation algorithms permit users to control the estimates via interactive input at some repetitions, unlike common semantic and instance segmentation algorithms when a segmentation mask may be produced in a single pass. So, by using this interaction the object can be probably recognized and prediction errors can be corrected [15].

Below are the essential steps that are implemented in the interactive graph-based segmentation process [6]:
First: obtain the user preferences.
Second: rendering to the user's choices and demonstrating an optimal solution—if not, a suboptimal one—is produced.

Fig.4- using Interactive image segmentation [14].

2.3. Minimum spanning tree

Discovery summation with the least value to weights of edge in a spanning tree is considered a problem in graph theory. The connected graph is referred to as a tree. So, the segmentation operation to an image issue search for associated regions that have the least variances has been transformed into the minimum spanning tree delinquent [1]. A subgraph that connects all of the graph's vertices is called a spanning tree in a connected undirected graph, and there should only be one path that connects any two vertices. Numerous spanning trees were identified for the graph. A spanning tree that has an edge weight total that is less than or equal to every other spanning tree's edge weight is known as a minimal spanning tree (MST). Created from an image, a minimal spanning tree of a graph depicts the potential weakest connections. The pixels make up the graph's vertices, and the edges show how similar the vertices are to one another. Different partitions with greater intrinsic affinities might be identified by appropriately deleting the lowest weighted edges [6].
two diverse strategies are considered a minimum spanning tree which are top-down dividing and bottom-up merging for image segmentation. Built on an image graph model that has been formed, the top-down separation method initially employs the minimum spanning tree approach to generate a minimal spanning tree that connects the entire image. This approach is separated rendering a specific principle [1].
Prim's Algorithm is described as a scheme employed for finding the minimum spanning tree in a graph by detecting a subset of edges that reduces the weights sum of the edges, preliminary with a single node, and discovering adjacent nodes with connecting edges. where Kruskal's Algorithm is explained as a method used to determine a connected weighted graph's minimum spanning tree, pointing to recognize the edges that allow traversal of every vertex, the following are algorithms of these two methods [16].

- **Prim's Algorithm**

1- Eliminate all edges which are looped and parallel.
2- Indicate any random node as a root node.
3- examine the yielded edges and the one with the least cost is chosen.
4- Elect an edge with the minimum cost that does not form a cycle.

- **Kruskal's Algorithm**

1- Elimination of all edges which looped and parallel.
2- Order every edge in the increasing order.
3- An addition is made to the edges that have the minimum weightage.
4- Choose an edge with the least cost that does not form a cycle.

2.4. Pyramid-based methods

Pyramids are hierarchical structures used in segmentation, defining image components with decreasing precision. The hierarchical structure is built, and image segmentation can be accomplished by selecting all vertices at a level or a subset of vertices as roots. Fig. 5 illustrates regular pyramids, a graph hierarchy, as image arrays, while irregular ones require modifying data structures and decimation mechanisms to dynamically alter their image design [17]. A set of graphs with many levels of resolution are graphically represented by a pyramid built from this base graph. At level (L), the vertices and edges are decreased to the vertices and edges of level (L+1) using a reduction function. The level responsible for yielded segmentation has been confirmed to be the pyramid working level. They fall into two groups based on the strategy utilized to form pyramids [6].
Multi-scale contextual information is contained in the spatial pyramid and is crucial for tasks involving dense prediction. A multi-scale procedure is also vital for creating a hierarchical depiction and making the model scale-invariant for graph-structured data. The feature pyramid is considered a useful scheme for taking context with several scales. In dense prediction processes such as object detection and semantic segmentation, it is generally used. Also comprised is a hierarchical depiction that is also exposed to be beneficial for embedding graph-structured data [18].

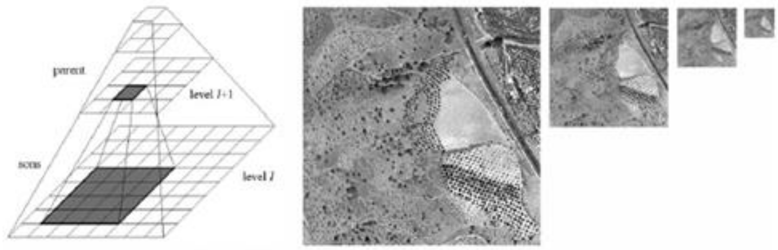

Fig.5- Regular pyramids [17].

The comparison to graph technique methods is summarized in Table 1.

Table 1- Explain the summary in comparison to the Graph technique

Technique	Description	Strategies
Graph cuts based	• A and B are yielded from dividing an existing graph G neglecting the edges of these sections • weight of edge elimination is considered for scaling the degree of related A and B	• minimal cut algorithm • normalized cut algorithm
Interactive	• need for user interaction • Several types of interaction, for instance, some points, line segments, or strokes to mark the object	• GrabCut method • random walks • geodesics • approaches using shape prior
Minimum spanning tree	• A connected graph is referred to as a tree • Discovery summation with the least value to weights of edge	• Kruskal's algorithm • Prim's algorithm
Pyramid-based methods	• hierarchical structures • regular pyramids • irregular pyramids	• A multi-scale technique

3. Related works

Some of the related works are briefly considered and Table 2 presents the summary of these related works.
[19] This research explores the development of an automated anatomy segmentation (AAS) system in clinical radiology, utilizing an iterative graph-cut-ASM technique for object delineation and a hierarchical 3D scale-based multi-object method for object recognition.

[20] An efficient method for segmenting images quickly for high resolution remote sensing data is shown using graph theory and the fractal net evolution approach (FNEA). The approach effectively obtains an initial object layer in time by modeling an image as a weighted undirected graph.

[21] While using enhanced contrast and spatial resolution this research combines PET and CT images to enhance tumor segmentation. The random walk and graph cut approaches deal with the segmentation problem, while the max-flow/min-cut strategy handles the co-segmentation problem, which is an energy reduction challenge. Tumor segmentation variances between PET and CT images are penalized by a network consisting of two sub-graphs and a specific connection.

Inside [22] a hierarchical graph-based image segmentation procedure was introduced for extracting roads from high-resolution data is obtainable. The technique includes creating a graph depiction of the image, extracting characteristics to improve the contrast between road and non-road pixels, merging and splitting segments based on color and shape data, and post-processing to eliminate abnormalities. The method's higher efficiency in urban regions is demonstrated by experiments conducted on three difficult datasets.

[23] Color image segmentation scheme has been introduced using GrabCut simple linear iterative clustering (SLIC) integrated with Bayes classification. This work has been extended to include the Gaussian mixture model (GMM) for SLIC features and is used to define the energy function. The simplified graph cut model makes use of Bayes classification, while the min-cut technique adds more power.

[24] in this proposal, a procedure to segment kidney US images using a graph cuts-based method was introduced, where texture feature maps from Gabor filters with intensity information are used, improving computational efficiency by creating a graph of image pixels near the kidney boundary.

Within [25] a pyramid graph cut was produced for grayscale medical image segmentation. In a pyramid-shaped graph structure, gradient sources of information have been utilized with intensity by using a single source node and multiple sink nodes. Depending on their influence at each cutting position, gradient information, and intensity are used in the min-cut.

the authors in [26] introduce research work by employing graph theory via color spatial grouping with the addition of the compromise region. So that the process of comparing the chromatic spatial aggregation with the integration of the compromised area with other traditional theoretical and graphic models to analyze dissimilar quality measures is considered for magnetic resonance imaging (MRI) inputs and X-ray images that are beneficial in medical imaging for improved analysis through diagnosis.

Additionally, in [1] The research offers an image segmentation procedure for UAV images in coastal areas based on Minimum Span Trees (MST). To control the segmentation scale, it is beneficial of a scale control parameter and an edge weight-based optimum criteria (merging predicate) based on statistical learning theory (SLT).

[27] an unsupervised graph coloring technique is utilized to segment an image of a brain tumor, where the brightness difference between a few pixels represents an edge and every pixel is thought of as a node in the network.

[28] This work introduces a lightweight Graph Neural Network (GNN) that optimizes part semantic segmentation without extra post-processing and implicit cluster classification across various graphs, replacing traditional clustering methods.

[29] This work discusses image segmentation using an undirected weighted graph. It focuses on communities, where each section has a corresponding image section. The increasing area method is used to search communities, and parting quality is assessed by the average edge weight. The number of nearest neighbors connected via edges is determined by the correlation radius.

[30] in this suggested work a minimum spanning tree is used to segment brain tumor images. This approach uses a minimum spanning tree without tuning parameters to achieve interactive segmentation. The stages contain preprocessing and graph creation and achieve interactive segmentation using a minimum spanning tree.

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Table 2- A summary of related works

Reference	Author, year	Techniques	Using Description
[19]	Chen et al, (2011)	iterative graph-cut-ASM technique	anatomy segmentation
[20]	Yi Yang et al., (2015)	graph theory and fractal net evolution approach	High-resolution remote sensing imagery
[21]	W. Ju et al., (2015)	graph cut, Random walk	PET-CT images showing co-segmentation of a lung tumor
[1]	Wang et al., (2016)	Minimum spanning tree	UAV image
[22]	Rasha Alshehhia, Prashanth Reddy Marpu, (2017)	a hierarchical graph	Road Networks Extracted from High-Resolution Satellite Images
[23]	Ren, D et al, (2017)	GrabCut	Color image segmentation
[24]	Zheng Q et al., (2018)	Graph cut	Medical images
[25]	Thanongchai et al., (2020)	pyramid graph cut	grayscale medical image
[27]	Zheng Lin and et al., (2020)	Graph Coloring Approach	Medical image
[26]	Sekar et al., (2021)	Graph theory	Medical image
[28]	Belim, S. V., & Belim, S. Y., (2022)	Graph	Image segmentation using an undirected weighted graph representation
[30]	Mayala et al., (2022)	Spanning tree	Medical image
[29]	Aflalo and et al., (2023)	Graph	a lightweight Graph Neural Network (GNN) that optimizes for the same objective function

4. Discussion and Conclusion

As a discussion of the role of employing graph technique in image segmentation and because graph-based techniques may create graph-like depictions of images, there has been a lot of concentration on their application to image segmentation. In the image, each pixel is represented as a node, and the edges show how similar the corresponding pixels are to one another. However, these techniques encounter difficulties because of increased computational complexity, such as polynomial time when searching for a graph's vertex or edge, and exponential complexity when attempting to match scene models with graph object models [6]. By taking into account both local and global information in an image, the graph approach can detect complex characteristics and object shapes, leading to a more secure segmentation process. A major field of image processing study is medical imaging, where graph-based methods show promise for organ, tissue, and cell segmentation and analysis in images for infection diagnosis. The accuracy of segmentation has also improved in remote sensing applications using graphs as shown in related works.

The conclusion from that the graph-based methods used in image segmentation, including minimum spanning tree, pyramid-based, graph cut-based, and interactive image segmentation are reviewed in this article along with their strategies and applications. Graph-based techniques are becoming more common and can describe image properties as seen in related works. As a result, using graph-based techniques is thought to be an effective tool for segmenting images and has received more attention recently. As previously noted, in image processing, the procedure of segmenting images has a vital role because it makes the description of the image more beneficial to the analysis process of interest objects easier and more accurate to implement.

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