Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

Dynamical Properties of Rough Group Spaces

Eman Hatef Fahem[a] , Sattar Hameed Hamzah[b]

[a]*Department of Mathematics ,College of Education ,Uinversity of Al-Qadisiyah,Iraq .Email:Math.edu.post12@qu.edu.iq*

[b]*Department of Mathematics ,College of Education ,Uinversity of Al-Qadisiyah,Iraq.Email: sattar.hamzah@qu.edu.iq*

ARTICLE INFO

Article history:
Received: 16 /08/2024
Rrevised form: 27 /08/2024
Accepted : 29 /08/2024
Available online: 30 /09/2024

Keywords:

Rough topological group

Rough action

Perodic points

Critical points

ABSTRACT

Our main aim is introduced some concepts in dynamical system in rough theory . We give the definition of periodic points and critical points and investigate their properties in rough actions . Also, we illustrated the relation between them.

MSC..

https://doi.org/ 10.29304/jqcsm.2024.16.31664

1.Introduction:

Pawlak in 1982 introduce a rough sets [1] . The applications of rough sets appeared in many fields[4,5]. In this work, we explained some properties of dynamical concepts in rough theory .We define the periodic points, and the critical points in rough action . We also explained the relation between them by giving some theorems about them.

2. Fundamental Concepts.

Some basic properties about rough sets and related concepts are introduced in this section .

∗Corresponding author: Eman Hatef Fahem

Email addresses: *Math.edu.post12@qu.edu.iq*

Communicated by 'sub etitor'

Definition 2.1:[1]

A Pawlak approximation space is a pair $\mathcal{W} = (\mathcal{U}, \mathcal{R})$ where $\mathcal{U}$ be a non-empty set and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$.For any sub set $\chi \subseteq \mathcal{U}$, $\underline{\mathcal{R}}(\chi)$ called lower approximation and $\overline{\mathcal{R}}(\chi)$ called upper approximation and are defined as follows:

$\underline{\mathcal{R}}(\chi) = \{ x \in \mathcal{U}: x\mathcal{R} \subseteq \chi \}$

$\overline{\mathcal{R}}(\chi) = \{ x \in \mathcal{U} : x\mathcal{R} \cap \chi \neq \emptyset \}$

Where $x\mathcal{R}$ being the equivalence class of x with respect to $\mathcal{R}$.

Remark 2.2:

We used symbol $\underline{\chi}$ instead $\underline{\mathcal{R}}(\chi)$ and symbol $\overline{\chi}$ instead $\overline{\mathcal{R}}(\chi)$.

Definition 2.3:[1]

Aset $\chi = (\underline{\chi}, \overline{\chi})$ called rough in $\mathcal{W}$ if and only if $\overline{\chi} \neq \underline{\chi}$, otherwise,$\chi$is an exact set in $\mathcal{W}$.

Example 2.4

Let $\mathcal{U} = \{1,2,3,4\}$ and $\mathcal{R} = \{(1,1),(2,2),(3,3),(4,4),(1,3),(3,1)\}$ be an equivalence relation on $\mathcal{U}$, then $\mathcal{W} = (\mathcal{U},\mathcal{R})$ being an approximation space. The quotient set of $\mathcal{W}$ is $\mathcal{U}\backslash\mathcal{R} = \{ \{1,3\},\{2\},\{4\} \}$, for $\chi = \{1,4\}$ we have $\underline{\chi} = \{4\}$, and $\overline{\chi} = \{1,3,4\}$ i.e., $\overline{\chi} \neq \underline{\chi}$ in $\mathcal{W}$, then χ is rough set in $\mathcal{W}$

Example 2.5

Let $\mathcal{U} = \{1,2,3\}$ and $\mathcal{R} = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$ be an equivalence relation on $\mathcal{U}$,then $\mathcal{W} = (\mathcal{U},\mathcal{R})$ being an approximation space .The quotient set of $\mathcal{W}$ is $\mathcal{U}\backslash\mathcal{R} = \{\{1,3\},\{2\}\}$ for $\chi = \{1,3\}$ .Then $\underline{\chi} = \{1,3\}$, and $\overline{\chi} = \{1,3\}$ i.e., $\overline{\chi} = \underline{\chi}$ in $\mathcal{W}$, then χ is exact set in $\mathcal{W} = (\mathcal{U},\mathcal{R})$.

Definition 2.6: [1]

A rough set $A = (\underline{A}, \overline{A})$ called a rough subset of rough set $B = (\underline{B}, \overline{B})$ iff $\underline{A} \subset \underline{B}$ and $\overline{A} \subset \overline{B}$.

Definition 2.7: [1]

The union of rough sets $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B})$ defined as $A \cup B = (\underline{A} \cup \underline{B}, \overline{A} \cup \overline{B})$ and their intersection defined as $A \cap B = (\underline{A} \cap \underline{B}, \overline{A} \cap \overline{B})$.

Definition 2.8 : [2]

Let $\mathcal{W} = (\mathcal{U}, \mathcal{R})$ be an approximation space and $(\cdot)$ be a binary operation on $\mathcal{U}$. A rough group is a subset $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ of $\mathcal{W}$ satisfies the following conditions :

1. $x \cdot y \in \overline{\Gamma}$, $\forall$ x ,y$\in\Gamma$.

2.$(x \cdot y) \cdot z = x \cdot (y \cdot z)$, $\forall$ x,y,z$\in\overline{\Gamma}$.

3.$\forall$ x$\in\Gamma$, $\exists$ e$\in\overline{\Gamma}$ such that $x \cdot e = e \cdot x = x$.

4.$\forall$ x$\in\Gamma$, $\exists$ y$\in\Gamma$ such that $x \cdot y = y \cdot x = e$.

. The element e called the rough identity and the element y called the rough inverse of x and denoted by x^{-1}. We used the symbol xy instead of $x \cdot y$

Theorem 2.9 : [2]

A necessary and sufficient condition for a subset $\mathcal{H}$ of a rough group Γ to be a rough subgroup is that:

1. $\forall x,y \in \mathcal{H}, xy \in \overline{\mathcal{H}}$.

2. $\forall y \in \mathcal{H}, y^{-1} \in \mathcal{H}$.

Definition 2.10 : [5]

A rough group $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ with a topology τ on $\overline{\Gamma}$ and the topology τ_Γ on Γ induced by τ is a topological rough group if the following conditions are satisfied:

1.The multiplication mapping $\mu: \Gamma \times \Gamma \to \overline{\Gamma}$, $\mu(x,y) = xy$ is continuous.

2. The inversion mapping $\mathcal{N}: \Gamma \to \Gamma$, $\mathcal{N}(x) = x^{-1}$ is continuous. We used the symbol $\mathcal{TRG}$ instead of topological rough group.

Example 2.11 :

Let $\mathcal{U} = \{a, b, c, d\}$ be group with four elements and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ such that $\mathcal{U}/\mathcal{R} = \{\{a, c\}, \{b\}, \{d\}\}$. Let $\Gamma = \{b, c\}$, then $\underline{\Gamma} = \{b\}$, $\overline{\Gamma} = \{a, b, c\}$. A topology τ on $\overline{\Gamma}$ is $\tau_{dis} = \{\overline{\Gamma}, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ then the relative topology is $\tau_\Gamma = \{\Gamma, \phi, \{b\}, \{c\}\}$.Hence Γ is a $\mathcal{TRG}$.

Definition 2.12:

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$ and fix $g \in \Gamma$. Then

(1) The mapping $l_g: \Gamma \to \overline{\Gamma}$ defined by $l_g(x) = gx$, is called rough left translate.

(2)The mapping $R_g: \Gamma \to \overline{\Gamma}$ defined by $R_g(x) = xg$, is called rough right translate.

(3) The mapping $\mathcal{N}: \Gamma \to \Gamma$, $\mathcal{N}(x) = x^{-1}$ is called the inversion mapping.

Theorem 2.13 :

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$. Then

(1) The rough left translate l_g is injective and continuous, $\forall g \in \Gamma$.

(2) The rough right translate R_g is injective and continuous, $\forall g \in \Gamma$.

(3) The inversion $\mathcal{N}$ is homeomorphism.

Proof : clear.

3. Rough Actions

The definition of rough action and some of properties are introduced in this section.

Definition 3.1 :[3]

A left rough action of Γ on χ is a continuous map $\varphi : \overline{\Gamma} \times \overline{\chi} \longrightarrow \overline{\chi}$ satisfies the following conditions:

i. $\varphi(g_1, \varphi(g_2, x)) = \varphi(g_1 g_2, x)$, $\forall\, g_1, g_2 \in \overline{\Gamma}$ and $x \in \overline{\chi}$.

ii. $\varphi(e, x) = x$, $\forall\, x \in \overline{\chi}$.

Then (χ, φ) called rough group space. For short a rough Γ- space or $\mathcal{R}\Gamma$-space. We used the symbol gx instead of $\varphi(g, x)$.Right rough action defined in similar way.

Example3.2 :

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$ such that $\overline{\Gamma}$ is group. Then Γ acts on itself by multiplication, $\varphi: \overline{\Gamma} \times \overline{\Gamma} \longrightarrow \overline{\Gamma}, (g, g') \longrightarrow gg'$.

Definition 3.3:

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$ and (χ, φ) be $\mathcal{R}\Gamma$-space. Then for every $g \in \Gamma$.

(i) The map $L_g: \overline{\chi} \longrightarrow \overline{\chi}$; $L_g(x) = \varphi(g\, x)$ is called g-left transformation.

(ii) The map $R_g: \overline{\chi} \longrightarrow \overline{\chi}$; $R_g(x) = \varphi(x, g)$ is called g-right transformation.

Theorem 3.4:

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$ and (χ, φ) be $\mathcal{R}\Gamma$-space. Then L_g & R_g are homeomorphism, for every $g \in \Gamma g$.

Proof :

The map L_g is bijective with inverse $(L_g)^{-1} = L_g{}^{-1}$ and both are continuous by the continuity of the action φ .Therefore L_g is homeomorphism.

Theorem 3.5:

Let (χ, φ) be $\mathcal{R}\Gamma$-space. Then

(i) gA is open in $\overline{\chi}$ for each open set A in $\overline{\chi}$ and $g \in \overline{\Gamma}$

(ii) gA is closed in $\overline{\chi}$ for each closed set A in $\overline{\chi}$ and $g \in \overline{\Gamma}$

Proof:

By using theorem3.4 and the fact $L_g(A) = gA, \forall g \in \Gamma$.

Definition 3.6 : [3]

Let (χ, φ) be $\mathcal{R}\Gamma$-space and $x \in \overline{\chi}$. Then

1.The rough orbit of x is the set $\mathcal{R}\Gamma(x) = \{\varphi(g, x): g \in \overline{\Gamma}\}$.

2.The rough stabilizer of x is the set $\mathcal{R}S(x) = \{g \in \overline{\Gamma}: \varphi(g, x) = x\}$.

3. The rough kernel of the action is the set $\mathcal{R}\ker(\varphi) = \{g \in \overline{\Gamma}: \varphi(g, x) = x, \forall x \in \overline{\chi}\}$.

Theorem 3.7 :

Let (χ, φ) be $\mathcal{R}\Gamma$-space such that $\overline{\Gamma}$ is a group. Then $\mathcal{R}S(x)$ being a subgroup in $\overline{\Gamma}$.

Proof :

1. we have $\varphi(g_1, x) = \varphi(g_2, x) = x, \forall\, g_1, g_2 \in \mathcal{R}S(x)$ and then $\varphi(g_1 g_2, x) = x$. Hence $g_1 g_2 \in \mathcal{R}S(x)$.

2. $e \in \mathcal{R}S(x)$ by definition 3.1,ii .

3. Let $g \in \mathcal{R}S(x)$. Then $\varphi(g^{-1}, x) = \varphi\left(g^{-1}, \varphi(g, x)\right) = \varphi(g^{-1}g, x) = \varphi(e, x) = x$.Thus $g^{-1} \in \mathcal{R}S(x)$. Hence $\mathcal{R}S(x)$ being a subgroup of $\overline{\Gamma}$.

4. Periodic Points and Critical Points

The definition of periodic points and critical points in $\mathcal{R}\Gamma$-spaces and some of properties are introduced in this section.

Definition 4.1

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$. A subset $\mathcal{H} = (\underline{\mathcal{H}}, \overline{\mathcal{H}})$ of $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ called rough syndetic if a compact set $\mathcal{K}$ of Γ exist with $\mu(\mathcal{K}, \mathcal{H}) = \overline{\Gamma}$.

Example 4.2 :

Let $\mathcal{U} = Z$ *and* $\mathcal{R}$ be identity relation, $\Gamma = (Z, +)$ with discrete topology, then $\overline{\Gamma} = \overline{Z} = Z$ and $\mu: \Gamma \times \Gamma \to \overline{\Gamma}$, $\mu(x, y) = x + y$. Then Z_e is rough syndetic set, since $\mathcal{K} = \{-1,3\}$ compact set in $\Gamma = Z$ such that $\mu(Z, \mathcal{K}) = Z + \mathcal{K} = \Gamma$.

Corollary 4.3:

Let $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ be a $\mathcal{TRG}$. Then

1. If $\mathcal{H}$ be a rough syndetic of Γ, then both $A\mathcal{H}$, $\mathcal{H}A$ are rough syndetic subset of Γ.

2. If $\mathcal{H}$ sub group of Γ, then $\mathcal{H}$ be a left rough syndetic set if and only if $\mathcal{H}^{-1}$ be a right rough syndetic .

Proof : By using theorem 2.13.

Corollary 4.4 :

Let $\mathcal{H}$ be a rough syndetic subset of Γ, then $A\mathcal{H}A^{-1}$ rough syndetic subset of Γ for all A.

Proof : By using theorems 2.13 and 4.3 .

Definition 4.5 :

Let (χ, φ) be a $\mathcal{R}\Gamma$-space, a rough subset $\mathcal{P}_x = (\underline{\mathcal{P}_x}, \overline{\mathcal{P}_x})$ of $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ is said to be period of x under Γ if $\overline{\mathcal{P}_x}$ being a maximal set in $\overline{\Gamma}$ such that $\varphi(\overline{\mathcal{P}_x}, x) = x$, $\forall\, x \in \overline{\chi}$.

Theorem 4.6 :

Let (χ, φ) be a $\mathcal{R}\Gamma$-space and $\mathcal{P}_x = (\underline{\mathcal{P}_x}, \overline{\mathcal{P}_x})$ period of x. Then $\overline{\mathcal{P}_x} = \phi_x^{-1}(x)$, where $\phi_x: \overline{\Gamma} \to \overline{\chi}$, $\phi_x(h) = \varphi(h, x)$.

Proof: By definition (4.5) $\overline{\mathcal{P}_x}$ being maximal subset of $\overline{\Gamma}$ so we have $\overline{\mathcal{P}_x} = \{h \in \overline{\Gamma}: \varphi(h, x) = x\}$ and then $\overline{\mathcal{P}_x} = \{h \in \overline{\Gamma}: \phi_x(h) = x\} = \phi_x^{-1}(x)$.

Theorem 4.7:

Let (χ, φ) be a $\mathcal{R}\Gamma$-space and $\mathcal{P}_x$ period of x. Then $g^{-1}\mathcal{P}_x g$ being period of $\varphi(g, x)$, $\forall\, g \in \overline{\Gamma}$.

Proof:

Assume that $\mathcal{H}$ period of $\varphi(g, x)$, so by theorem (4.6) $\mathcal{H} = \phi_{\varphi(g,x)}^{-1}(\varphi(g, x))$. Then $\varphi(\mathcal{H}, \varphi(g, x)) = \varphi(g, x)$

Since $\varphi(\mathcal{H}, \varphi(g, x)) = \{\varphi(h, \varphi(g, x)): h \in \mathcal{H}\}$, Then $\varphi(hg, x) = \varphi(g, x)$ for all $h \in \mathcal{H}$, $\varphi(g^{-1}hg, x) = x$, Since $\mathcal{P}_x$ is period of x, then $g^{-1}hg \in \mathcal{P}_x$.Now , let $k = g^{-1}hg$ then $h = gkg^{-1}$ and $\varphi(hg, x) = \varphi(h, \varphi(g, x)) = \varphi(g, x)$, then $gkg^{-1} \in \mathcal{H}$. Thus $\mathcal{H} = g\mathcal{P}_x g^{-1}$ .Then $g\mathcal{P}_x g^{-1}$ is period of $\varphi(g, x)$

Definition 4.8:

Let (χ, φ) be a $\mathcal{R}\Gamma$-space, a rough subset $\mathcal{P} = (\underline{\mathcal{P}}, \overline{\mathcal{P}})$ of $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$ called period of Γ if $\overline{\mathcal{P}}$ being a maximal set in $\overline{\Gamma}$ such that $\varphi(\overline{\mathcal{P}}, x) = x$, $\forall\, x \in \overline{\chi}$.

Theorem 4.9 :

Let (χ, φ) be a $\mathcal{R}\Gamma$-space and $\mathcal{P} = (\underline{\mathcal{P}}, \overline{\mathcal{P}})$ a period of $\Gamma = (\underline{\Gamma}, \overline{\Gamma})$, then $\mathcal{P} = \bigcap_{x \in \chi} \mathcal{P}_x$ where $\mathcal{P}_x = (\underline{\mathcal{P}_x}, \overline{\mathcal{P}_x})$ is a period of x.

Proof: clear

Definition 4.10 :

Let (χ,φ) be a $\mathcal{R}\Gamma$-space, a point $x\in\overline{\chi}$ called periodic if $\overline{\mathcal{P}_x}$ being rough syndetic in $\overline{\Gamma}$.

Theorem 4.11 :

Let (χ,φ) be a $\mathcal{R}\Gamma$-space and $\mathcal{P}_x$ a period of x , then $\overline{\mathcal{P}_x}$ being a rough subgroup in $\overline{\Gamma}$.

Proof:

By definition 4.5 we have $\varphi(\overline{\mathcal{P}_x},x)=x$ so $\varphi(p,x)=x$, $\forall\, p\in\overline{\mathcal{P}_x}$,Thus $\varphi(p^{-1},x)=x$, hence $p^{-1}\in\overline{\mathcal{P}_x}$. Now, if $p_1,p_2\in\mathcal{P}_x$, then $\varphi(p_1p_2,x)=\varphi(p_1,\varphi(p_2,x)))=x$. Hence $p_1p_2\in\overline{\mathcal{P}_x}$. Thus $\overline{\mathcal{P}_x}$ is a rough subgroup in $\overline{\Gamma}$

Theorem 4.12 :

Let (χ,φ) be a Hausdorff $\mathcal{R}\Gamma$-space and $x\in\overline{\chi}$ be periodic point, then $\overline{\mathcal{P}_x}$ being closed set in $\overline{\Gamma}$.

Proof:

. Since $\{x\}$ is closed set and from (theorem 4.6) we have $\overline{\mathcal{P}_x}=\phi_x^{-1}(x)$ and then $\overline{\mathcal{P}_x}$ is closed set.

Theorem 4.13 :

Let (χ,φ) be a $\mathcal{R}\Gamma$-space and $x\in\overline{\chi}$ be periodic point, then a compact set $\mathcal{K}$ of $\overline{\Gamma}$ exist with $\varphi(\mathcal{K},x)=\mathcal{R}\Gamma(x)$.

Proof:

From definition (4.10) $\overline{\mathcal{P}_x}$ is rough syndetic set, so a compact set $\mathcal{K}\subseteq\overline{\Gamma}$ exist such that $\varphi(\mathcal{K},\overline{\mathcal{P}_x})=\overline{\Gamma}$, and then $\varphi(h,x)=\varphi(\varphi(\mathcal{K},\overline{\mathcal{P}_x}),x)=\varphi(\mathcal{K},x)$.Hence $\mathcal{R}\Gamma(x)=\varphi(\mathcal{K},x)$.

Theorem 4.14 :

Let (χ,φ) be a $\mathcal{R}\Gamma$-space and $x\in\overline{\chi}$ be periodic , then $\mathcal{R}\Gamma(x)$ being compact set.

Proof:

By theorem (4.13) a compact set $\mathcal{K}$ of $\overline{\Gamma}$ exist with $\mathcal{R}\Gamma(x)=\varphi(\mathcal{K},x)$.Thus $\phi_x(\mathcal{K})=\varphi(\mathcal{K},x)=\mathcal{R}\Gamma(x)$ is compact. Since $\phi_x:\overline{\Gamma}\to\overline{\chi}$ is continuous

Definition 4.15:

Let (χ,φ) be a $\mathcal{R}\Gamma$-space, apoint $x\in\overline{\chi}$ called a critical if $\varphi(g,x)=x \quad \forall\, g\in\overline{\Gamma}$ .The set of all critical points denoted by $\mathcal{CR}(\chi)$.

Example 4.16 :

Let (χ,φ) be a $\mathcal{R}\Gamma$-space where $\Gamma=(\mathbb{R},.)$ with discrete topology, and identity relation, $\chi=\mathbb{R}$. and $\varphi:\overline{\Gamma}\times\overline{\chi}\to\overline{\chi}$ by $\varphi(g,h)=h$, then (χ,φ) is $\mathcal{R}\Gamma$-space has critical point $x=\{1\}$.

Theorem4.17 :

Let (χ,φ) be a $\mathcal{R}\Gamma$ -space and $x\in\overline{\chi}$ be a critical point then x being periodic point with period equal to $\overline{\Gamma}$.

Proof:

From definition 4.15 we have $\mathcal{R}\Gamma(x)=\{x\}$ and $\overline{\Gamma}$ maximal syndetic set such that $\varphi(g,x)=x$, Therefore $\overline{\Gamma}$ being period of x, and then x being periodic.

Theorem 4.18:

Let (χ,φ) be a Hausdorff $\mathcal{R}\Gamma$ -space and $x\in\overline{\chi}$ be not critical point, then there exist two open nbhds U and V with $x\in U$, $\varphi(h,x)\in V$, $V=\varphi(h,U)$ and $U\cap V=\emptyset$.

Proof: Since x is not critical, then $\varphi(h,x)\neq x$ for some $h\in\overline{\Gamma}$ so there exist two open sets W_1,W_2 with $x\in W_1$, $\varphi(h,x)\in W_2$ and $W_1\cap W_2=\emptyset$.Thus $\phi_{xh}(W_1)$ open contains $\varphi(h,x)$. Now, let $V=\phi_{xh}(W_1)\cap W_2$ and $U=\varphi(h^{-1},V)$. Then $V\subset W_2$ and $U\subset W_1$ so that U and V are disjoint and $x\in U$ and $\varphi(h,x)\in V$.

Theorem 4.19 :

Let (χ,φ) be a Hausdorff $\mathcal{R}\Gamma$ -space. Then A point x being a critical point iff each nbhd of x contains an orbit.

Proof:$\Longrightarrow$

From definition 4.15 we have $\mathcal{R}\Gamma(x)=\{x\}$ and if U be a nbhd of x then U contains an orbit.

$\Longleftarrow$

If x is not critical, then $\varphi(h,x)\neq x$ for some $h\in\overline{\Gamma}$ .Thus by theorem (4.18) there exist disjoint open nbhds U of x and V of $\varphi(h,x)$ such that $V=\varphi(h,U)$. Hence for each $y\in U$, we have $\varphi(h,y)\notin U$ since $U\cap V=\emptyset$, So, there is a nbhd U of x don't contain orbit.

Theorem 4.20:

Let (χ,φ) be a $\mathcal{R}\Gamma$-space, then $\mathcal{CR}(\chi)$ closed.

Proof:

Assume that $\mathcal{CR}(\chi)$ is not closed, so there exist a net $\{x_\beta\}_{\beta\in\Omega}$ in $\mathcal{CR}(\chi)$ with $x_\beta\to x$ and x is not critical . Then by theorem (4.18), there exist open sets U containing x and V containing $\varphi(h_\beta,x)$ with $\varphi(h_\beta,U)=V$ and $U\cap V=\emptyset$

for some $h_\beta \in \overline{\Gamma}$ .Since $x_\beta \to x$, then $\{x_\beta\}_{\beta\in\Omega}$ is eventually in U .Thus for all sufficiently large β ,we have $x_\beta \in U$ and then $\varphi(h_\beta, x_\beta) \in V$ and $\varphi(h_\beta, x_\beta) \notin U$. But $\varphi(h_\beta, x_\beta) = x_\beta \in U$ since x_β are critical , and this contradiction. Hence $\mathcal{CR}(\chi)$ is closed

REFERENCES :

[1] Pawlak Z., Rough sets, Int.J.Comput.Inform.Sci.,1982,11(5),341–356.

[2] Biswas R., Nanda S., Rough groups and rough subgroups, Bull.Pol. Acad. Sci. Math. ,1994,42,251–254.

[3] Nof Alharbi ,Alla Altassan ,On Topological Rough Groups, arx:1909.02500v1 [math.GR] 5Sep 2019.

[4] Nof Alharbi*, Alaa Altassan , Hassen Aydi, and Cenap Özel, Rough quotient in topological rough sets Received January 10, 2019; accepted November 4, 2019. https://doi.org/10.1515/math-2019-0138.

[5] Bagirmaz N.,IcenI .,Ozcan A.F. ,Topological roughgroups ,Topological.Algebra Appl. ,2016,4,31–38,DOI:10.1515/taa-2016-0004.