Some Properties of Generalized Feebly Closed Maps and Feebly Closed Maps

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Abstract:

The main purpose of this paper is study and investigate some properties of generalized feebly closed maps and feebly closed maps ,we give several results about that by using some concepts of topological as feebly closed sets,feebly open sets,generalized feebly closed sets and feebly normal space.

***Key words**:Generalized feebly closed maps,feebly closed maps,feebly open sets,feebly closed sets and feebly normal space.*

1.Introduction

In fact the study of generalized closed maps and feebly closed maps has found considerable interest among general topologists. One reason in these objects are natural generalizations of closed sets . More importantly generalized feebly closed maps and feebly closed maps suggest some new results and concepts of topology.

Levine[1] initated the investigation of so called generalized closed sets by definition , a subset A of a topological space X is called generalized closed , briefly g-closed if $clA\subset U$ whenever $A\subset U$ and U is open . Dalal[2]proved ,let $f:X\rightarrow Y$ be a mapping if the image of every closed subset of X is open and g-closed in Y,then f is feebly closed where $f:X\rightarrow Y$ is feebly closed mapping if the image of each closed set in X is feebly closed in Y.

Recently Dalal[3] proved,let (X,T) be a topological space and $A\subset X$, if A is gf-closed , then (fclA-A) is gf-open ,where intersection of all feebly closed sets containing set A is the feebly – closure of A and is denoted by fclA and a subset A of X is called generalized feebly closed (briefly gf-closed) if

$fclA\subset U$, whenever $A\subset U$ and U is open . Throughout this paper (X,T) and (Y,V) represent non empty topological spaces on which no separation axioms are assumed unless stated explicitly .If $f:(X,T)\rightarrow(Y,V)$ be a mapping, f^{-1} (H) is the inverse image of a subset H of (X,T), H^o,clH and f/H denote the interior, closure and restriction of mapping f to subset H respectively.The purpose of this paper is study and investigate some properties of generalized feebly closed maps and feebly closed maps ,we give several results about that by using some concepts of topological .

2- Preliminaries

A subset H of a topological space (X,T) is said to be semi open [4] if there exists an open set U of X such that $U\subset H\subset ClU$ and is said semi closed if there exists closed set U such that $U^{\circ}\subset H\subset U$. The complement of semi open set said to be semi closed . The semi closure [5]and [6] of a subset H of X denoted by scl_x (H) briefly scl(H) , is defined to be the intersection of all semi closed sets containing H . scl(H) is a semi closed set [5] ,[6] . The semi interior [6] of H denoted by sint (H) is defined to be union of all semi open sets contained in H . In a topological space (X,T) ,H is called feebly open [7] if $H\subset$scl int H and accustomed H is said to be feebly closed if sint $clH\subset H$. In [8] proved every open set is feebly open set and every closed set is feebly closed set and [9] proved the complement of feebly open set is feebly closed set . Intersection of all feebly closed sets containing H is feebly–closure [10] of H and is denoted by fclH and a function $f: (X,T)\rightarrow(Y,V)$ is called feebly closed if the image of each closed set in X is feebly closed in Y . A function $f: (X,T)\rightarrow(Y,V)$is called feebly open [2] if the image of each open set in X is feebly open in Y and proved every closed function is feebly closed function . A subset H of a topological space (X,T) is called generalized feebly closed (briefly gf-closed) [3] if $fclH\subset U$ whenever $H\subset U$ and U is open set in X and proved every feebly closed set is gf-closed set . A function $f : (X,T)\rightarrow (Y,V)$ is called gf-closed[11] if the image of every closed subset of X is gf-closed in Y. In [12] a topological space (X,T) is called $T½$- space if and only if for each $x\in X$, singleton {x} is open or closed . In [8] a topological space (X,T) is called feebly regular (briefly f- regular) if for all $x\in X$ and for all open set A containing x there exists feebly open set H such that $x\in H\subset fclH\subset A$ and a topological space (X,T) is called feebly normal (briefly f-normal) if for all disjoint closed sets

H_1, H_2 in X , there exist feebly open sets U_1, U_2 in X such that $H_1 \subset U_1, H_2 \subset U_2$ and $U_1 \cap U_2 = \phi$. Let $f : (X,T) \rightarrow (Y,V)$ be a function and $A \subset X$, then A is called inverse set [13] if $A = f^{-1}(f(A))$.To a chieve our purposes,we mention the following results.

Proposition 2.1[3: Theorem 3.2]

Let (X,T) be a topological space and $H \subset X$, then H is feebly closed if and only if $fclH = H$.

Proposition 2.2[3: Corollary 3.4]

Let (X,T) be a topological space , A and $B \subset X$ if $A \subset B$ then $fclA \subset fclB$.

Proposition 2.3[3: Theorem 3.1]

Let (X,T) be a topological space , A and B be feebly closed subsets of X , then $A \cap B$ is feebly closed set in X .

Proposition 2.4[11: Proposition 3.5]

If A is an open and gf-closed set of a topological space (X,T) ,then A is feebly closed.

Lemma 2.5[11]

(i) Every closed map is gf-closed map .(Remark3.1) .

(ii) Every feebly closed map is gf-closed map. (Proposition 3.3) .

3- Basic properties of feebly closed maps

Theorem 3.1

If $f: (X,T) \rightarrow (Y,V)$ is feebly closed and A is a closed set of X , then $f/A: (A,T_A) \rightarrow (Y,V)$ is feebly closed.

Proof:

Let H is a closed in A , to prove $f_A(H)$ is feebly closed in Y, since A is closed in X , then H is closed in X, since f is feebly closed , then $f(H)$ is feebly closed in Y, but $f_A(H) = f(H)$, then $f_A(H)$ is feebly closed in Y. Thus f/A is feebly closed.

Theorem 3.2

If $f : X \rightarrow Y$ is feebly closed and $A = f^{-1}(B)$, where B is closed set in Y , then $f/A: A \rightarrow Y$ is feebly closed.

Proof :

Let H is closed in A , then there exists closed set G in X such that $H = G \cap A$, since $f_A(H) = f(H)$, then $f_A(H) = f(G \cap A) = f(G \cap f^{-1}(B)) = f(G) \cap B$, since f is feebly closed and G is closed in X , then $f(G)$ is feebly closed in Y and B is closed , then B is feebly closed then by Proposition 2.3 , $f(G) \cap B$ is feebly closed in Y thus $f_A(H)$ is feebly closed in Y, hence f/A is feebly closed .

If B is not closed set in Y , then Theorem 3.2, is not necessarily true , the *following example illustrates that .*

Example 3.3

Let $T=\{\phi,X,\{b\},\{c\},\{b,c\}\}$ be a topology on $X = \{a,b,c\}$ and $V=\{\phi,Y,\{2\},\{3\},\{2,3\}\}$ be a topology on $Y =\{1,2,3\}$.

Let $f:X\to Y$ defined by : $f(a)=1$, $f(b)=2$ and $f(c)=3$.

Let $B=\{2,3\}$, then $A=f^{-1}(B)=\{b,c\}$ .it is clear $\{c\}$ is closed set in T_A with respect to T , but $f_A\{c\} =\{3\}$ which is not feebly closed in Y , thus $f/A:A\to Y$ is not feebly closed.

Theorem 3.4

If $f:X\to Y$ is a continuous , feebly open and feebly closed surjection from a regular space X to a space Y , then Y is feebly regular .

Proof :

Let U be an open set containing a point y inY . Let x be a point of X such that $y=f(x)$. Since X is regular then there is an open set H in X such that $x\in H\subset clH\subset f^{-1}(U)$, where $f^{-1}(U)$ is open since f is continuous , therefore , $y\in f(H)\subset f(clH)\subset U$, where $f(H)$ is feebly open since f is feebly open , since clH is closed set and f is feebly closed then $f(clH)$ is feebly closed in Y , then by Proposition 2.1, $fcl(f(clH)) = f(clH)$. Since $H\subset clH$, then $f(H)\subset f(clH)$, thus by Proposition 2.2, $fcl(f(H))\subset fcl(f(clH))=f(clH)\subset U$,therefore , $fcl(f(H))\subset U$, thus $y\in f(H)\subset fcl(f(H))\subset U$. Hence Y is feebly regular.

Corollary 3.5

If $f:X\to Y$ is a continuous , feebly open and closed surjection from regular space X to a space Y , then Y is feebly regular .

Proof :

The proof is obvious from Theorem 3.4 and a fact that every closed map is feebly closed.

Theorem 3.6

A mapping $f:X\to Y$ is feebly closed if and only if for all B subset of Y and for all U is open subset of X such that $f^{-1}(B)\subset U$, there exists feebly open set $H \subset Y$ such that $B\subset H$ and $f^{-1}(H)\subset U$.

Proof:

Let f is feebly closed and let $B\subset Y$ and U is open set in X such that $f^{-1}(B)\subset U$, let $H=Y-f(X-U)$, since U is open in X , then X-U is closed set in X , since f is feebly closed then $f(X-U)$ is feebly closed in Y , thus $H=Y-f(X-U)$ is feebly open in Y , now we will prove that $B\subset H$, let $y\notin H$ we will prove $y\notin B$ then $y\in f(X-U)$, thus there exists $x\in(X-U)$ such that $f(x)=y$,then $x\notin U$,since $f^{-1}(B)\subset U$, then $x\notin f^{-1}(B)$, therefore ,$y\notin B$,thus $B\subset H$,now to prove f^{-}

$^{1}(H)\subset U$,$H=Y\text{-}f(X\text{-}U)$,then $f^{-1}(H)=f^{-1}[Y\text{-}f(X\text{-}U)] = f^{-1}(Y)\text{-}f^{-1}[f(X\text{-}U)]\subset X\text{-}(X\text{-}U) = U$, thus $f^{-1}(H)\subset U$.Conversely,

Let A is closed set in X , then $U=X\text{-}A$ is open set in X,let $B=Y\text{-}f(A)$ be any set in Y, then

$f^{-1}(B)=f^{-1}[Y\text{-}f(A)] =f^{-1}(Y)\text{-} f^{-1}(f(A))\subset X\text{-}A=U$ and by hypothesis , there exists feebly open set H in Y such that $f^{-1}(H)\subset U$ and $B\subset H$, i.e , $Y\text{-}f(A)\subset H$ and $f^{-1}(H)\subset X\text{-}A$, then

$Y\text{-}f(A)\subset H$ and $H\subset f(X\text{-}A)$,thus $Y\text{-}f(A)\subset H$ and $H\subset Y\text{-}f(A)$,then $H=Y\text{-}f(A)$. Since H is feebly open in Y, then $Y\text{-}f(A)$ is feebly open in Y , then $f(A)$ is feebly closed in Y , thus f is feebly closed.

Theorem 3.7

If $f:X\rightarrow Y$ is a continuous feebly closed and surjection from a normal space X into Y , then Y is feebly normal .

Proof:

Let A and B be disjoint closed sets of Y . Since f is continuous , then $f^{-1}(A)$ and $f^{-1}(B)$ be disjoint closed in X , since X is normal there exist disjoint open sets $U_1$and U_2 of X such that $f^{-1}(A)\subset U_1$ and $f^{-1}(B)\subset U_2$, then by Theorem 3.6 , there exist feebly open sets G and H of Y such that $A\subset G$, $B\subset H$ and $f^{-1}(G)\subset U_1$, $f^{-1}(H)\subset U_2$, then $f^{-1}(G)\cap f^{-1}(H)=\phi$ and hence $G\cap H=\phi$. Thus Y is feebly normal.

4- Basic properties of generalized feebly closed maps

Theorem 4.1

If $f:X\rightarrow Y$ is gf-closed and A is a closed set of X , then $f/A:A\rightarrow Y$ is gf-closed .

Proof :

Let H is closed in A , to prove $f_A(H)$ is gf-closed in Y , since A is closed in X , then H is closed in X and f is gf-closed , then $f(H)$ is gf-closed in Y , but $f_A(H)=f(H)$, thus $f_A(H)$ is gf-closed in Y , hence f/A is gf-closed.

Corollary 4.2

(i) If $f:X\rightarrow Y$ is closed and A is a closed set of X, then $f/A:A\rightarrow Y$ is gf-closed.

(ii) If $f:X\rightarrow Y$ is feebly closed and A is a closed in X , then $f/A:A\rightarrow Y$ is gf-closed .

Proof:

(i) The proof is obvious from Theorem 4.1 and byLemma 2.5 (i).

(ii) The proof is obvious from Theorem 4.1 and by Lemma 2.5(ii) .

Theorem 4.3

Let B be an open and gf-closed subset of Y . If $f:X\rightarrow Y$ is feebly closed then $f/A:A\rightarrow Y$ is gf-closed where $A=f^{-1}(B)$.

Proof:

Let H be a closed set in A then H =$G\cap A$ where G is closed in X , since $f_A(H)=f(H)$ then $f_A(H)=f(G\cap A)=f(G\cap f^{-1}(B))=f(G)\cap B$, since f is feebly closed and G is closed in X, then f(G) is feebly closed in Y , since B is open and gf-closed then by Proposition 2.4,B is feebly closed then by Proposition2.3 , $f(G)\cap B$ is feebly closed , thus $f_A(H)$ is feebly closed then $f_A(H)$ is gf-closed , thus f/A is gf-closed.

Corollary 4.4

Let B be an open and gf- closed subset of Y . If $f:X\to Y$ is closed , then $f/A:A\to Y$ is gf-closed , where $A=f^{-1}(B)$.

Proof :

The proof is obvious from Theorem 4.3 and by Lemma 2.5(i) .

Theorem 4.5

Let $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof:X\to Z$ is a gf- closed map and f is continuous and surjective , then g is gf- closed

Proof :

Let H be a closed set of Y , since f is continuous , then $f^{-1}(H)$ is closed in X , since gof is gf-closed then $(gof)(f^{-1}(H))$ is gf-closed in Z , but f is surjective then $(gof)(f^{-1}(H)) = g(H)$ is gf-closed in Z this implies that g is a gf- closed.

Theorem 4.6

If $f:X\to Y$ is a closed and $g:Y\to Z$ is a gf- closed , then $gof:X\to Z$ is gf-closed.

Proof : *straightforword.*

Theorem 4.7

If $f:X\to Y$ is a gf-closed and Y is T½- space then f is feebly closed.

Proof:

Let H is a closed set in X , since f is gf-closed then f(H) is gf-closed in Y , to prove f(H) is feebly closed , i.e, fcl(f(H))=f(H) , it is clear

$f(H)\subseteq fcl(f(H))$ …………………………… (1)

Let $y\in fcl(f(H))$ and $y\notin f(H)$, then $f(H)\subset Y-\{y\}$, since Y is T½-space then every singleton {y} is open or closed , if {y} is open , then

Y-{y} is closed, then Y-{y} is feebly closed , then $fcl(f(H))\subset Y-\{y\}$, but $y\in fcl(f(H))$,then $y\in Y-\{y\}$ this contradiction , thus $y\in f(H)$, thus

$fcl(f(H))\subseteq f(H)$……………………….. (2)

From(1) and (2) we get fcl(f(H))=f(H) , then by Proposition 2.1 , f(H) is feebly closed . If {y} is closed , then Y-{y} is open , since $f(H) \subset Y-\{y\}$ and

$f(H)$ is gf-closed , then $fcl(f(H))\subset Y-\{y\}$ but $y\in fcl(f(H))$, then $y\in Y-\{y\}$ this contradiction , thus $y\in f(H)$, then

$fcl(f(H))\subset f(H)$…………………………….. (3)

From (1) and (3) we get $fcl(f(H))=f(H)$,then by Proposition 2.1, $f(H)$ is feebly closed , thus f is feebly closed map .

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بعض الخواص للتطبيقا ت المغلقة الضئيلة الاعم
والتطبيقات المغلقة الضئيلة

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المستخلص

الغرض الرئيسي من هذا البحث دراسة وتحري بعض الخواص للتطبيقات المغلقة الضئيلة الاعم و التطبيقات المغلقة الضئيلة وقد تم اعطاء العديد من النتائج حول ذلك باستخدام بعض المفاهيم التبولوجية كالمجموعات المغلقة الضئيلة والمجموعات المفتوحة الضئيلةوالتطبقات المغلقةالضئيلة الاعم والفضاء الطبيعي الضئيل