Some Results With (σ,τ)-Derivation On Near Rings

Kassim Abdul-Hameed Jassim *Zainab Talib Salman*
College of Science –University of Baghdad

Abstract

Let N be a prime –Near ring , I be a nonzero semigroup ideal .Let d be a (σ,τ)-derivation on N .Then

(i) If d(x+y-x-y)=0 for all x,y ∈I , then (N,+) is abelian .

(ii) If d is a homomorphism or anti- homomorphism on I , then d(I)=0 .

(iii) If d is a homomorphism or anti- homomorphism on I , then d(N)=0 .

Introduction

Through this paper N stands for a prime near ring .We recall that N is a prime ring if aNb=0 , then either a=0 or b=0 in [2].A right near ring is a set N together with two binary operation "+" and "." such that (i) (N,+) is a group (not necessarily abelian) (ii) (N,.) is a semigroup .(iii) For all n_1, n_2 , n_3 ∈N :(n_1+ n_2). n_3 =n_1. n_2 + n_2 . n_3 .(right distributive law).In other hand we can get a left near ring by using a left distributive law such that n_1 . (n_2+ n_3) =n_1. n_2 + n_1 . n_3 see [2].

An additive map d:N→N is a (σ,τ)- derivation if d(xy)=d(x) σ(y)+ τ(x)d(y) for all x,y∈N. where σ,τ:N→N two automorphisms ,see[4].

Nurcan Argac in [3] get the following :- (i)If u+v=v+u for all u,v∈N , then N is abelian.(ii)If d acts a homomorphism or anti-homomorphism on I , then d(I)=0.Where d is a two sided α-derivation ,when α:N →N such that d(xy)=d(x) α(y)+ α(x)d(y)where α is any map.

In this paper we generalized these results from (α,α)-derivation to (σ,τ)-Derivation .In case(ii), we get an extension that if we have d(I)=0 , then we can get d(N)=0

2. Results

Lemma (2.1)[3]

Let N be a prime near ring and I be a nonzero semigroup ideal of N.If u+v=v+u for all u,v,∈I, then (N,+) is abelian .

Lemma(2.2)

Let N be a prime near ring and I be a nonzero semigrop ideal of N. Let d be a (σ,τ)-derivation on N. If x∈N and xd(I)=0 , then x=0.

Proof

Assume xd(I)=0 .Then xd(uy)=0 , for all $y\in N$, $u\in I$. Hence $0=xd(uy)=xd(u)\,\sigma(y)+x\tau(u)d(y)= x\tau(u)d(y)$.

Then $x\tau(u)d(y)=0$. Since τ is an automorphism on N, then $\tau^{-1}(x)u\tau^{-1}(d(y))=0$

Since I is a nonzero semigroup ideal of N and d is a nonzero , then x=0.

Lemma(2.3)

Let N be a prime near ring and I be a nonzero semigroup ideal of N , d be a (σ,τ)-derivation of N . If d(x+y-x-y)=0 , for all $x,y \in N$, then $\tau(x+y-x-y)d(z)=0$, for all $x,y\in N$,,$z\in I$.

Proof

Assume that d(x+y-x-y)=0 for all $x,y\in I$. Let us take yz and xz instead of y and x, where $z \in I$ respectively .Then

$0=d((x+y-x-y)z)$

$=d\,(x+y-x-y)\,\sigma(z)+\tau(x+y-x-y)d(z)$

$0= \tau(x+y-x-y)d(z)$ for all $x,y,z\in I$.

Theorem(2.4)

Let N be a prime near ring and I be a nonzero semigrop of N. Let d be a (σ,τ)-derivation on N .If d(x+y-x-y)=0 for all $x,y \in I$, then (N,+) is abelian .

Proof

Suppose that d(x+y-x-y)=0 , for all $x,y \in I$, so we have $\tau(x+y-x-y)d(z)=0$ for all $x,y,z\in I$ [By Lemma (2.3)] . Hence $\tau(x+y-x-y)d(I)=0$.

Therefore , by Lemma (2.2), we have $\tau(x+y-x-y)=0$. Hence , x+y-x-y=0 for all $x,y\in I$ .So , we have (N,+) is abelian .

Lemma(2.5)

Let N be a prime – Near ring , I be a nonzero semigroup ideal of N . If Ia=0 (or aI=0) , then a=0 .

Proof:

Assume aI=0, then for all $u\in I$, $x\in N$. Hence, axu=0 and this implies aNI=0. then a=0 .

Assume that Ia=0. So, we have, uxa=0, for all $u\in I$, $x\in N$. Then INa=0.Since N is a prime Near ring and I is a nonzero ideal of N, then a=0 .

Theorem(2.6)

Let N be a prime – Near ring , I be a nonzero semigroup ideal of N , d is a (σ,τ)-derivation of N .If d is a homomorphism on I , then d(I)=0 .

Proof:

For all $x,y\in I$, we have

$d(xy)=d(x)d(y)=d(x)\sigma(y)+\tau(x)d(y)$….(1)

Replace y by yx ,

$=d(x)\sigma(y)+\tau(x)d(y)$

$=d(x)\ \sigma(yx)+\tau(x)d(yx)$

$= d(x)\sigma(y)\sigma(x)+\tau(x)d(y)d(x)$.

From the other hand, so we have

$d(x)d(yx)=d(xy)d(x)$

$= d(x)\sigma(y)d(x)+\tau(x)d(y)d(x)$.then

$d(x)\sigma(y)d(x)= d(x)\sigma(y)\sigma(x)$ for all $x,y\in I$. Then $d(x)\sigma(y)d(x) - d(x)\sigma(y)\sigma(x)=0$. Also,

$d(x)\sigma(y)[d(x) - \sigma(x)]=0$. Take ny , $n\in N$ instead of y ,then $0=d(x)\sigma(ny)[d(x)-\sigma(x)]$

$0=d(x)\sigma(n)\sigma(y)[d(x) - \sigma(x)]$

Then , we have either

$d(I)=0$ or $\sigma(y)[d(x) - \sigma(x)]=0$.

If $\sigma(y)[d(x) - \sigma(x)]=0$, then

$y\ \sigma^{-1}([d(x) - \sigma(x)])=0$.

Hence , $I\ \sigma^{-1}([d(x) - \sigma(x)])=0$.Therefor ,

by Lemma(2.5) we have $d(x) - \sigma(x)=0$ and so $d(x) = \sigma(x)$ for all $x\in I$ .From (1) we have $\tau(x)d(y)=0$ for all $x,y\in I$.Replace x by xn ,$n\in N$, so $\tau(xn)d(y)=0$ $xn\tau^{-1}(d(y))=0$. Hence , $IN\ \tau^{-1}(d(I))=0$, so we have $d(I)=0$.

Corollary (2.7)

Let N be a prime – Near ring , I be a nonzero semigroup ideal of N , d is a (σ,τ)-derivation of N .If d is a homomorphism on I , then d(N)=0 .

Proof:

By the previous Theorem , we have d(I)=0 . So , for all $u\in I$, $x\in N$ $0=d(ux)$.Also,

$0= d(ux)= d(u)\sigma(x)+\tau(u)d(x)$

$=\tau(u)d(x)$ .Then $\tau(u)d(x)=0$ for all $u\in I$, $x\in N$ .Since τ is an atomorphism , then

$u\tau^{-1}(d(x))=0$ and hence $d(x)=0$ for all $x\in N$ by Lemma (2.5).Therefore , $d(N)=0$.

Theorem(2.8)

Let N be a prime – Near ring , I be a nonzero ideal semigroup ideal of N , d is a (σ,τ)-derivation of N .If d is an anti- homomorphism on I , then d(I)=0 .

Proof:

For all $x,y\in I$, we have

$d(xy)=d(y)d(x)=d(x)\sigma(y)+\tau(x)d(y)$.Take xy instead of y, so we have

$d(x)\sigma(x)\sigma(y)+\tau(x)d(y)d(x)$

$= d(x)\sigma(xy)+ \tau(x)d(xy)=d(xy)d(x)$

$= d(x)\sigma(y) d(x)+\tau(x)d(y)d(x)$.Then

$d(x)\sigma(y) d(x)= d(x)\sigma(x)\sigma(y)$ for all $x,y\in I$.

Replace y by yt ,$t\in I$. then we have

$d(x)\sigma(y)\sigma(t)d(x) = d(x)\sigma(x)\sigma(y)\sigma(t)$

$= d(x)\sigma(y)d(x)\sigma(t)$

$d(x)\sigma(y)\sigma(t)d(x) = d(x)\sigma(y)d(x)\sigma(t)$

$d(x)\sigma(y)[\sigma(t),d(x)]=0$ for all $x,y, t\in I$.

Since σ is an automorphism , then

$\sigma^{-1}(d(x))\, y\, \sigma^{-1}([\sigma(t),d(x)])=0$.Since I is a nonzero ideal and a primeness of N, we have either $d(x)=0$ or $[\sigma(t),d(x)]=0$. If $d(x)=0$ for all $x \in I$, hence $d(I)=0$.If $[\sigma(t),d(x)]=0$, so taking rt instead of t ,$r\in N$,then $0=[\sigma(rt),d(x)]= [\sigma(r)\sigma(t),d(x)]$

$=\sigma(r) [\sigma(t),d(x)]+ [\sigma(r),d(x)]\sigma(t)$

$=[\sigma(r),d(x)]\sigma(t)$ for all $x,y, t\in I$.

Also , we have

$=\sigma^{-1}([\sigma(r),d(x)]\, t)$ and hence

$\sigma^{-1}([\sigma(r),d(x)])I=0$.By Lemma(2.5) , we have $[\sigma(r),d(x)]=0$ for all $x\in I$, $r\in N$. Therefore , we have $d(I)\subset Z(R)$.So , we have d is a homomorphism on I , then we have by Theorem (2.6) d(I)=0.

Corollary (2.9)

Let N be a prime – Near ring , I be a nonzero ideal semigroup ideal of N , d is a (σ,τ)-derivation of N .If d is an anti- homomorphism on I , then d(N)=0 .

Proof

By the previous Theorem , we have d(I)=0 . So , for all $u\in I$, $x\in N$ $0=d(ux)$.Also,

$0= d(ux)= d(u)\sigma(x)+\tau(u)d(x)$

$=\tau(u)d(x)$ .Then $\tau(u)d(x)=0$ for all $u\in I$, $x\in N$ .Since τ is an atomorphism , then

$u\tau^{-1}(d(x))=0$ and hence $d(x)=0$ for all $x\in N$ by Lemma (2.5).Therefore , d(N)=0 .

References

1.Gunter Pliz,,Near ring the theory and applications,.North –Holland publishing Company -1977.
2. Herstein, I.N. Topics in ring theory.Univ.of Chicago press,Chicago,1969 .
3. Nurcan Argac, On Near –rings with two sided α-derivation .Turk-J. of Math 28(2004),195-204.
4. Yasuyuki Hirano and Hisao Tominga, (1984).Some Commutativity Theorems for prime rings with derivations and deferentially semiprime rings .Math. J.Okayama Univ. 26. 101-108.