ON RADIAL BASIS FUNCTION NEURAL NETWORKS AND ITS APPLICATIONS

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1. ABSTRACT

In this paper ,we examine the similarities and differences between RBFNNs and compare the performance of learning, then we applied to the interpolation problem by using data of blood pressure disease which taken from health office in diwaniya city .

2. INTRODUCTION

The approximation of functions is one of the most general uses of artificial neural networks. The general framework of the approximation problem is the following: one supposes the existence of a relation between several input variables and one output variable. This relation being unknown, one tries to build an approximator between these inputs and this output. The structure of this approximator must be chosen and the approximator must be calibrated as to best represent the input-output dependence. To realize these different stages, one disposes of a set of inputs-output pairs that constitute the learning data of the approximator.

Radial basis functions were first introduced by Powell to solve the real multivariate interpolation problem. This problem is currently one of the principal fields of research in numerical analysis. In the field of neural networks, radial basis functions were first used by Broomhead and Lowe . Other major contributions to the theory, design, and applications of RBFNs can be found in papers by Moody and Darken, Renals, and Poggio and Girosi . The paper by Poggio and Girosi[4] explains the use of regularization theory applied to this class of neural networks as a method for improved generalization to new data .

The non linear approximators radial basis function networks (RBFN) has the advantage of being much simpler than the other networks while keeping the major property of universal approximation of functions [5].

3. RADIAL FUNCTIONS [2],[3]

Let X be a normed linear space ,a function $f : X \rightarrow R$ is said to be radial if there exists a function $h : R^{+} \rightarrow R$ such that $f(x) = h(\|x\|)$ for all $x \in X$.

A radial basis function is any translate of f; that is a function of the form $g(x) = f(x-\theta) = h(\|x - \theta\|)$, where θ is any prescribed point of X. In other word , Radial functions are a special class of functions show the characteristic feature that their response decreases or increases monotonically with distance from a central point .

4. THE NETWORK ARCHITECTURE

The design of a RBFN consists of three separate layers: the input layer is the set of source nodes ,the second layer is a hidden layer of high dimension and the output layer gives the response of the network to the activation patterns applied to the input layer. The transformation from the input space to the hidden-unit space is nonlinear. On the other hand, the transformation from the hidden space to the output space is linear [1].

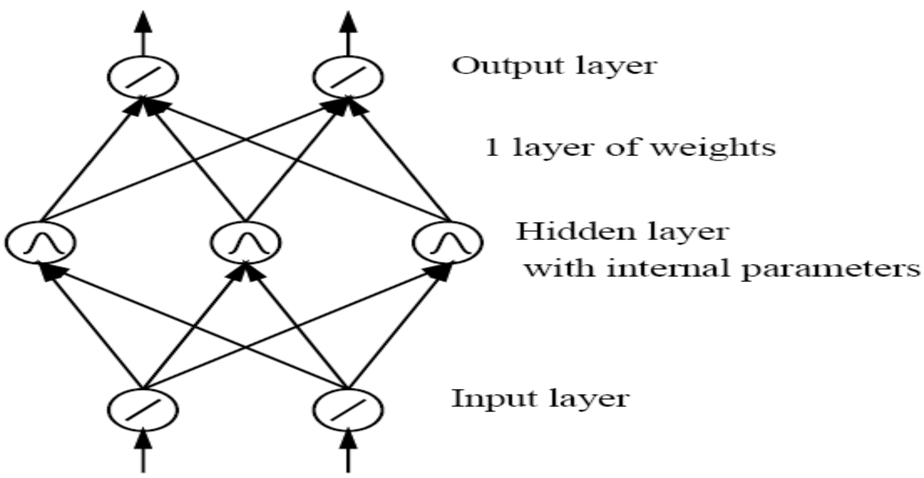


Figure 1: Radial Basis Function Network

Since there are varity radial functions as follows:[1]

(i) $\sigma(r) = r$.

(ii) $\sigma(r) = r^3$.

(iii) $\sigma(r) = r^2 \log(r)$ (thin plate splines) (TPS).

(iv) $\sigma(r) = \exp(-r^2/2)$(Gaussian).(G)

(v) $\sigma(r) = (r^2 + c^2)^{1/2}$ (multiquadric).(MQ)

(vi) $\sigma(r) = (r^2 + c^2)^{-1/2}$ (inverse multiquadric).(IMR)

We obtained to varity types of RBFNNs as follows :

1- newrbe ,which has GRBF as activities function

2- newlrbe , which has LRBF as activities function

3- newrbmqe , which has MQRBF as activities function

4- newrbimqe , which has IMQRBF as activities function

5- newrbpcube , which has PCUBRBF as activities function

6- newrbplse , which has PLSRBF as activities function

5. TRANING OF RBFNN

The training of RBFN can be split into an unsupervised part and a linear supervised part. Unsupervised updating techniques are straightforward and relatively fast. Moreover, the supervised part of the learning consists in solving a linear problem. The training procedure for RBFN can be decomposed naturally into three distinct stages:

(i) RBF centers are determined by some unsupervised/clustering techniques.

(ii) Spread determined by some techniques

(iii) The network weights between the radial functions layer and the output layer are calculated.

Several algorithms and heuristics are available in the literature regarding the computation of the centers of the radial functions and the weights . However, very few papers only are dedicated to the optimization of the spread.

In this paper we used the simplest and fastest approach which is to randomly select centers from the training data set and keep them constant throughout the training .This is reasonable provided that the training data are well representative of the problem . The widths for all RBFs are also fixed and are the same .This width can be taken as the standard deviation of the Gaussian function , expressed as :

$$\sigma = \frac{d}{\sqrt{2M}}$$

where d is the maximum distance between the selected centers .Such a choice for the standard deviation σ is to ensure that RBF are neither too peaked to cover the whole input space nor too flat to distinguish between dissimilar input patterns .

Then , the only parameters that need to be trained are the weights between the hidden and output layer ,which can be computed directly by solving linear equations . Usually the number of training patterns is much larger than the number of selected centers ,so the resulting linear equations are over determined .A straightforward procedure for solving such equations is to use the pseudo inverse method to obtain a solution with the minimum least square error .

6. PROPERTIES OF RBF

Over the past decades radial basis functions or, more generally, (conditionally) positive definite kernels have very successfully been used for reconstructing multivariate functions from scattered data. This success is mainly based upon the following facts:

(i) Radial basis functions can be used in any space dimension.

(ii) They work for arbitrarily scattered data, bearing no regularity at all.

(iii) They allow interpolants of arbitrary smoothness.

(iv) The interpolants have a simple structure, which makes RBF in particular interesting to users outside mathematics.

However, these positive properties do not come for free. For example, building a smooth interpolant using a smooth basis function leads also to an ill-conditioned linear system that has to be solved. Moreover, since most basis functions are globally supported, a large number of interpolation points leads to an unacceptable complexity concerning both space and time.

For these reasons recent research concentrated on resolving these problems. Fast methods for evaluating and computing an RBF interpolant have been developed and thoroughly investigated. Smoothing techniques have been employed to regularize ill-conditioned systems and to smooth out measurement errors.

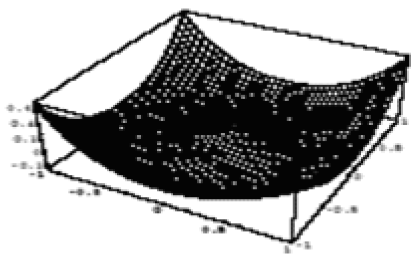
7. CHANGE OF BASIS

So far, we have learned that smoothing is an adequate choice in the situation of highly non-uniform data sets. It also helps in the case of quasi-uniform data sets and infinitely smooth basis functions, like Gaussians and (inverse) multiquadrics, since their associated interpolation matrices are already highly ill-conditioned in that particular situation for moderate separation distances. Unfortunately there exists no theoretical coverage of error estimates in that situation, even if numerical test show promising results.

Our final task, for basis functions of finite smoothness, is to deal with the case of really dense data sets. PTS is piecewise smooth RBF but G, MQ ,IMQ are infinitely smooth RBF. We recall that G, IMQ(inverse multiquadrics) and W_2(wendland compactly supported , i.e. $\phi(r) = (1-r)^4 + (4r+1)$ are positive definite (PD), i.e. the corresponding collocation matrix A is positive definite for every choice of the (distinct) interpolation nodes, while TPS and MQ are conditionally positive definite (CPD). (note: Where A defined in linear system Ac=f (interpolation equations)and is symmetric matrix, usually termed collocation matrix of the RBF (see figure 2) several forms of ϕ are used for RBF models.

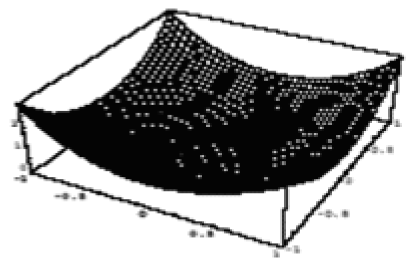
Amongst these, the Gaussian is probably the most popular basis function because it has attractive mathematical properties of universal and best approximation and its hill-like shape is easy to control with the parameter σ.

Also Gaussian basis Functions are quasi-orthogonal , the product of two basis functions, whose centers far away from each other with respect to their spreads , is almost zero.



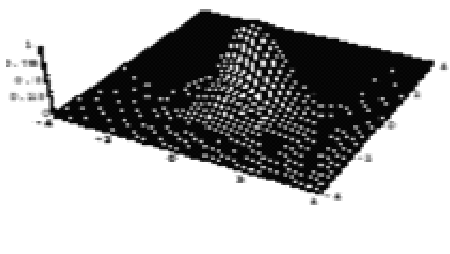
a) Thin-Plate(2-d)

$\phi(r) = r^2 \log r$



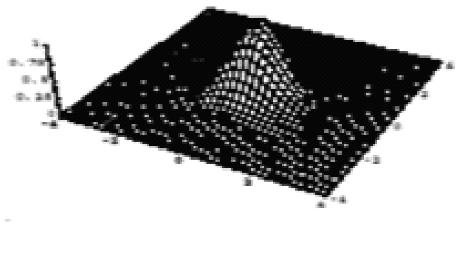
b) Thin-Plate (3-d)

$\phi(r) = r^3$



c) Gaussian

$\phi(r) = e^{r^2/\sigma^2}$



d) Compactly Supported

$\phi(r) = (1-r)^4 + (4r+1)$

Figure (2): Comparison of different types of radial basis function .

While the thin-plate spline embedding function does indeed minimize bending energy, it has the following drawbacks in computation and usefulness for user interaction :

1. $O(n^2)$ computation in required to build the system of equations .
2. $O(n^2)$ storage is required (for the nearly-full matrix) to represent the system.
3. $O(n^2)$ computation is required to solve the system of equations.
4. $O(n)$ computation is required per evaluation
5. Because every known point affects the result, a small change in even one constraint is felt throughout the entire resulting interpolated surface ,an undesirable property for shape modeling.

8. APPLICATION AND RESULTS

We obtained data about blood pressure disease , divided according to months of year , age and sex as table (1)

Age	0-4		5-14		15-44		45-65		65-	
Sex / Month	male	female	male	female	male	female	male	female	male	female
1	0	0	0	0	16	14	19	17	14	13
2	0	0	0	0	34	20	42	55	37	43
3	0	0	0	0	51	49	43	80	102	85
4	0	0	0	0	38	33	82	82	85	69
5	0	0	0	0	34	39	75	67	65	60
6	0	0	0	0	40	36	66	78	82	72
7	0	0	0	0	36	40	60	60	84	77

Table (1) : Data of blood pressure disease

In order to obtain the function f (month , age ,sex) = number of sick , we split the data set in to two parts according to given ratio 75:25 for training and test , respectively and trained all RBFNN as in section 5 .

The mean square error (MSE) of RBFNN with different types of RBF , introduced in table (2),we conclude that RBFNN when we use IMQRBF ,GRBF and LRBF give best accurate interpolation among all types of RBF.

Networks	MSE of training set	MSE of testing set
Newrbe	1.5529e-027	7.1204
Newlrbe	5.2849e-026	11.1944
Newrbimqe	4.7073e-027	7.0011
Newrbmqe	8.8349e-025	14.4156
Newrbpcube	3.2859e-023	16.4996
Newrbplse	2.1033e-026	26.1259

Table (2): MSE of RBFNN with different types of RBF

9. CONCLUSIONS AND RECOMMENDATIONS

In this research We reached to that results of theoretical study similarly to results of practical study and from this we Conclusion , we can say that GRBF introduced best results then IMQRBF and then LRBF compared by results of other types ,therefore we recommend to :

1- Using GRBF neural networks in approximation problems , in particular in case of expectation .

2- We can generalize the design of network in solution of expectation problems for another cases of disease , and no restrict to blood pressure disease . This useful to ministry of health to ensuring sufficient remedy and sufficient medical cadre beforehand and without squander.

10. REFERENCES

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حول الشبكات العصبية ذات دوال الأساس الشعاعية وتطبيقاتها

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المـــستخلص

يتضمن البحث اختبار ودراسة التشابه والفروقات بين أنواع مـــن الشـــبكات العصـــبية ذات دوال الأساس الشعاعية من حيث الأداء والتعلم الخاصة بمسائل الاندراج .

كما إن هذا البحث يساعد القارئ على اختيار دوال الأساس المناسبة والكفوءة لمعالجة مسألة معينة.

ولقد قمنا بتطبيق نتائج البحث على المصابين بمرض ضغط الدم من خلال البيانات التي حصلنا عليها من دائرة صحة مدينة الديوانية .