On intuitionistic fuzzy K-ideals of semiring

Mohammed Jassim Mohammed
Department of Mathematics, College of Education
University of Thi-Qar

Abstract

In this paper, we introduce the notion of intuitionistic fuzzy ideal and intuitionistic fuzzy K-ideal in semiring and investigate some properties of intuitionistic fuzzy K-ideals of semiring.

1. Introduction:

The concept of fuzzy set μ of a set X was introduced by L. A. Zadch [9] as a function from X in [0,1]. The concept of fuzzy ideals in a ring was introduced by W. L. Liu [8]. T. K. Dutta and B. K. Biswas [3,4] studied fuzzy ideals, fuzzy prime ideals of semirings and they defined fuzzy K-ideals and fuzzy prime K-ideals of semirings. Y. B. Jun, J. Neggers and M. S. Kim [5] extended the concept of an L-fuzzy left (resp. right) ideals of a ring to a semiring. The concept of the idea of intuitionistic fuzzy set was first published by K. T. Atanassor [1,2], as a generalization of the notion of fuzzy set. K.H. Kim and J. G. Lee [6] studied the intuitionistic fuzzification of the concept of several ideals in a semigroups and investigate some properties of such ideals. K. H. Kim [7] introduced the notion of intuitionistic Q-fuzzy semiprimality in a semigroup and investigate some properties of intuitionistic Q-fuzzification of the concept of several ideals.

In this paper we introduce the notion of intuitionistic fuzzy K-ideal of semiring and investigate some properties of intuitionistic fuzzy K-ideal of semirings.

Throughout this paper R is a semiring.

2. Preliminaries:

Let $(R,+,\cdot)$ be a semiring. By a left (right) ideal of R we mean a non-empty subset A of R such that $A+A\subseteq A$ and $RA\subseteq A$ ($AR\subseteq A$). By ideal, we mean a non-empty subset of R which both left and right ideal of R. A left ideal A of R is said to be a left K-ideal if $t\in A$, $x\in R$ and if $t+x\in A$ or $x+t\in A$ then $x\in A$. Right K-ideal is defined dually, and two sided K-ideal or simply a K-ideal is both a left and a right K-ideal.

By a fuzzy set μ of a non-empty set R we mean a function $\mu:R\to[0,1]$, and the complement of μ,denoted by $\bar{\mu}$, is the fuzzy set in R given by $\bar{\mu}(x)=1-\mu(x)$, for all $x\in R$.

A fuzzy set μ in R is called fuzzy left (resp. right) ideal of R if for any $x,y\in R$, $\mu(x+y)\geq\min\{\mu(x), \mu(y)\}$ and $\mu(xy)\geq\mu(y)$ ($\mu(xy)\geq\mu(x)$) and μ is called fuzzy ideal if μ both fuzzy left and right ideal of R.

A fuzzy ideal μ of R is called K-fuzzy ideal of R if for any $x,y\in R$, $\mu(x)\geq\min\{\max\{\mu(x+y), \mu(y+x)\}, \mu(y)\}$.

A intuitionistic fuzzy set (IFS for short) A in a non-empty set R is an object have the form:

$$A=\{(x: \mu_A(x),\lambda_A(x))/\ x\in R\}$$

Where the function $\mu_A:R\rightarrow[0,1]$ and $\lambda_A:R\rightarrow[0,1]$ denoted the degree of membership and the degree of non-membership, respectively, and

$$0\leq\mu_A(x)+\lambda_A(x)\leq 1$$

An intuitionistic fuzzy set $A=\{(x: \mu_A(x),\lambda_A(x))/\ x\in R\}$ in R can be identified to ordered pair (μ_A,λ_A) in $I^R\times I^R$. we shall use the symbol $A=(\mu_A,\lambda_A)$ for the IFS:

$$A=\{(x: \mu_A(x),\lambda_A(x))/\ x\in R\}$$

3. Intuitionistic fuzzy K-ideal:

Definition 3.1:

An IFS $A=(\mu_A,\lambda_A)$in R is called an intuitionistic fuzzy left (resp. right) ideal of R if for all $x,y\in R$:

1- $\mu_A(x+y)\geq\min\{\mu_A(x),\mu_A(y)\}$ and $\mu_A(xy)\geq\mu_A(y)$ (resp. $\mu_A(xy)\geq\mu_A(x)$).

2- $\lambda_A(x+y)\leq\max\{\lambda_A(x),\lambda_A(y)\}$ and $\lambda_A(xy)\leq\lambda_A(y)$ (resp. $\lambda_A(xy)\geq\lambda_A(x)$).

Definition 3.2:

An intuitionistic fuzzy ideal $A=(\mu_A,\lambda_A)$ of R is called an intuitionistic fuzzy K-ideal of R if:

$\mu_A(x)\geq\min\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

$\lambda_A(x)\leq\max\{\min\{\lambda_A(x+y),\lambda_A(y+x)\}, \lambda_A(y)\}$

For all $x,y\in R$.

Theorem 3.3:

Let $A=(\mu_A,\lambda_A)$ an intuitionistic fuzzy set in R such that μ_A is fuzzy K-ideal of R then $dA=(\mu_A,\bar{\mu}_A)$ is an intuitionistic K-ideal of R.

Proof:

1- Let $x,y\in R$, since μ_A is a fuzzy K-ideal of R

$\Rightarrow \mu_A$ is a fuzzy ideal.

So $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\mu_A(xy) \geq \mu_A(y), \mu_A(xy) \geq \mu_A(x)$

$\bar{\mu}(x+y) = 1-\mu_A(x+y) \leq 1-\min\{\mu_A(x), \mu_A(y\} = \max\{1-\mu_A(x), 1-\mu_A(y\}$

$= \max\{\bar{\mu}_A(x), \bar{\mu}_A(y\}$

So $\bar{\mu}(x+y) \leq \max\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$

$\bar{\mu}_A(xy) = 1-\bar{\mu}_A(xy) \leq 1-\bar{\mu}_A(y) = \bar{\mu}_A(y) \Rightarrow \bar{\mu}_A(xy) \leq \bar{\mu}_A(y)$

also $\bar{\mu}_A(xy) \leq \bar{\mu}_A(x)$

therefore $dA=(\mu_A, \bar{\mu}_A)$ is an intuitionistic fuzzy ideal.

2- Let $\mu(x) \geq \min\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

$\bar{\mu}(x) = 1-\mu(x) \leq 1-\min\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

$= \max\{1-\max\{\mu_A(x+y), \mu_A(y+x)\}, 1-\mu_A(y)\}$

$= \max\{\min\{1-\mu_A(x+y), 1-\mu_A(y+x)\}, \bar{\mu}_A(y)\}$

$= \max\{\min\{\bar{\mu}_A(x+y), \bar{\mu}_A(y+x)\}, \bar{\mu}_A(y)\}$

so $\bar{\mu}(x) \leq \max\{\min\{\bar{\mu}_A(x+y), \bar{\mu}_A(y+x)\}, \bar{\mu}_A(y)\}$

Therefore $dA=(\mu_A, \bar{\mu}_A)$ is an intuitionistic K-ideal

Theorem 3.4:

An IFS $A=(\mu_A, \lambda_A)$ is an intuitionistic fuzzy K-ideal of R if and only if the fuzzy sets μ_A and $\bar{\lambda}_A$ are fuzzy K-ideals of R.

Proof :

Suppose that an IFS $A=(\mu_A, \lambda_A)$ is an is an intuitionistic fuzzy K-ideal of R. Clearly μ_A is a fuzzy K-ideal .

Let $x,y \in R$ since $A=(\mu_A, \lambda_A)$ is an is an intuitionistic fuzzy K-ideal

$\Rightarrow \lambda_A(x+y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ and $\lambda_A(xy) \leq \lambda_A(y), \lambda_A(xy) \leq \lambda_A(x)$

$\lambda_A(x) \leq \max\{\min\{\lambda_A(\mathbf{x}+\mathbf{y}), \lambda_A(y+x)\}, \lambda_A(y)\}$

$\bar{\lambda}_A(x+y) = 1-\lambda_A(x+y) \geq 1-\max\{\lambda_A(x), \lambda_A(y)\}$

$= \min\{\bar{\lambda}_A(x), \bar{\lambda}_A(y)\}$

so $\bar{\lambda}_A(x+y) \geq \min\{\bar{\lambda}_A(x), \bar{\lambda}_A(y)\}$

$\bar{\lambda}_A(xy) = 1-\lambda_A(xy) \geq 1-\lambda_A(y) = \bar{\lambda}_A(y)$

$\Rightarrow \bar{\lambda}_A(xy) \geq \bar{\lambda}_A(y)$

Also we can get that $\bar{\lambda}_A(xy) \geq \bar{\lambda}_A(x)$

$\bar{\lambda}_A(x) = 1-\lambda_A(x) \geq 1-\max\{\min\{\lambda_A(x+y), \lambda_A(y+x)\}, \lambda_A(y)\}$

$= \min\{1-\min\{\lambda_A(x+y), \lambda_A(y+x)\}, 1-\lambda_A(y)\}$

$= \min\{\max\{1-\lambda_A(x+y), 1-\lambda_A(y+x)\}, \bar{\lambda}_A(y)\}$

$= \min\{\max\{\bar{\lambda}_A(x+y), \bar{\lambda}_A(y+x)\}, \bar{\lambda}_A(y)\}$

So $\bar{\lambda}_A(x) \geq \min\{\max\{\bar{\lambda}_A(x+y), \bar{\lambda}_A(y+x)\}, \bar{\lambda}_A(y)\}$

Therefore $\bar{\lambda}_A$ fuzzy K-ideals of R

Suppose that μ_A and $\bar{\lambda}_A$ are fuzzy K-ideals of R

Let $x,y \in R$

Since μ_A is a fuzzy K-ideals of R

$\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\mu_A(xy) \geq \mu_A(y), \mu_A(xy) \geq \mu_A(x)$

$\Rightarrow \mu_A(x) \geq \min\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

$\lambda_A(x+y) = 1-\bar{\lambda}_A(x+y)$

Since $\bar{\lambda}_A$ is a fuzzy ideals of R we get that

$\lambda_A(x+y) = 1-\min\{\lambda_A(x), \bar{\lambda}_A(y)\}$

$= \max\{1-\bar{\lambda}_A(x), 1-\bar{\lambda}_A(y)\}$

$= \max\{\lambda_A(x), \lambda_A(y)\}$

so $\lambda_A(x+y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

Also we get that $\lambda_A(xy) \leq \lambda_A(y), \lambda_A(xy) \leq \lambda_A(x)$

$\lambda_A(\mathbf{x}) = 1-\bar{\lambda}_A(x)$

Since $\bar{\lambda}_A$ is a fuzzy K-ideals of R we get that

$\lambda_A(x) \leq 1-\min\{\max\{\bar{\lambda}_A(x+y), \bar{\lambda}_A(y+x)\}, \bar{\lambda}_A(y)\}$

$= \max\{1-\max\{\bar{\lambda}_A(x+y), \bar{\lambda}_A(x+y)\}, 1-\bar{\lambda}_A(y)\}$

$= \max\{\min\{1-\bar{\lambda}_A(x+y), 1-\bar{\lambda}_A(x+y)\}, \lambda_A(y)\}$

$= \max\{\min\{\lambda_A(x+y), \lambda_A(y+x)\}, \lambda_A(y)\}$

So $\lambda_A(x) \leq \max\{\min\{\lambda_A(x+y), \lambda_A(y+x)\}, \lambda_A(y)\}$

Therefore $A=(\mu_A, \lambda_A)$ is an is an intuitionistic fuzzy K-ideal

Corollary 3.5 :

An IFS $A=(\mu_A, \lambda_A)$ is an intuitionistic fuzzy K-ideal of R iff $dA=(\mu_A, \bar{\mu}_A)$ and $A^{'}=(\bar{\lambda}_A, \lambda_A)$ are intuitionistic fuzzy K-ideal of R.

A semiring R is called intra-regular if for each $a \in R$ there exists $x,y \in R$ such that $a=xa^2y$.

Theorem 3.6:

Let A=(μ_A,λ_A) an intuitionistic fuzzy ideal of intra-regular semi-ring R then A(a)=A(a^2) and λ_A(ab)= λ_A(ba) for all a,b$\in$R.

Proof:

Let a$\in$R, since R intra-regular, there exist x,y$\in$R such that a=xa^2 y

Since A=(μ_A,λ_A) an intuitionistic fuzzy ideal

$\mu_A(a)= \mu_A(xa^2y)\geq \mu_A(a^2y) \geq \mu_A(a^2) \geq \mu_A(a)$

So $\mu_A(a)= \mu_A(a^2)$

$\lambda_A(a)= \lambda_A(xa^2y)\leq \lambda_A(a^2y) \leq \lambda_A(a^2) \leq \lambda_A(a)$

So $\lambda_A(a)= \lambda_A(a^2)$

Hence we have $\mu_A(a)= \mu_A(a^2)$ and $\lambda_A(a)= \lambda_A(a^2)$

Therefore A(a)=A(a^2) for all a$\in$R.

Let a,b$\in$R as above we get

$\mu_A(ab)= \mu_A((ab)^2)\geq \mu_A(a(ba)b) \geq \mu_A(ba)$

$=\mu_A((ba)^2)= \mu_A(b(ab)a) \geq \mu_A(ab)$

So we have $\mu_A(ab)= \mu_A(ba)$

$\lambda_A(ab)= \lambda_A((ab)^2) \leq \lambda_A(a(ba)b) \leq \lambda_A(ba)$

$=\lambda_A((ba)^2)= \lambda_A(b(ab)a) \leq \lambda_A(ab)$

So we have $\lambda_A(ab)= \lambda_A(ba)$

Therefore $\lambda_A(ab)= \lambda_A(ba)$ for all a,b$\in$R.

Theorem 3.7:

An IFS A=(μ_A,λ_A) is an intuitionistic fuzzy K-ideal of R iff for any t$\in$[a,b] such that $(\mu_A)_t\neq\Phi$ and $(\bar{\lambda}A)_t\neq\Phi$. $(\mu_A)_t$ and $(\bar{\lambda}A)_t$ are K-ideal of R, where $(\mu_A)_t=\{x\in R/ \mu_A(x) \geq t\}$.

Proof:

Suppose that IFS A=(μ_A,λ_A) is an intuitionistic fuzzy K-ideal of R

So by theorem (2.3) μ_A and $\bar{\lambda}A$ are fuzzy K-ideal of R

$\Rightarrow\mu_A$ and $\bar{\lambda}A$ are fuzzy ideal of R

By [4] for any t$\in$[0,1] such that $(\mu_A)_t\neq\Phi$ and $(\bar{\lambda}A)_t\neq\Phi$

$(\mu_A)_t$ and $(\bar{\lambda}A)_t$ are ideal of R

Let $x\in(\mu_A)_t$ and y$\in$R and $x+y\in(\mu_A)_t$ or $y+x\in(\mu_A)_t$

$\Rightarrow \mu_A(x) \geq t$ and $\mu_A(x+y) \geq t$ or $\mu_A(y+x) \geq t$

$\Rightarrow \max\{\mu_A(x+y), \mu_A(y+x)\} \geq t$

Since μ_A is fuzzy K-ideal of R

$\mu_A(y) \geq \text{mim}\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(x)\} \geq t$

$\Rightarrow \mu_A(y) \geq t$ so $y \in (\mu_A)_t$

Therefore $(\mu_A)_t$ is K-ideal of R.

Similarly we can prove that $(\bar{\lambda}_A)_t$ is K-ideal of R.

Suppose that for any $t \in [0,1]$ such $(\mu_A)_t \neq \Phi$ and $(\bar{\lambda}_A)_t \neq \Phi$, $(\mu_A)_t$ and $(\bar{\lambda}_A)_t$ are K-ideal of R.

So $(\mu_A)_t$ and $(\bar{\lambda}_A)_t$ are ideal of R.

By [4] μ_A and $\bar{\lambda}_A$ are fuzzy ideal of R.

Let $x,y \in R$ and $\mu_A(y)=r_1$, $\mu_A(x+y)=r_2$, $\mu_A(y+x)=r_3$, $(r_i \in [0,1])$

Let $t=\min\{\max\{r_1,r_3\},r_1\}$

$\Rightarrow y \in (\mu_A)_t$ and $x+y \in (\mu_A)_t$ or $y+x \in (\mu_A)_t$

Since $(\mu_A)_t$ is K-ideal of R.

So $x \in (\mu_A)_t \Rightarrow \mu_A(x) \geq t$

$\mu_A(x) \geq \text{mim}\{\max\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

Therefore μ_A is fuzzy K-ideal of R

Similarly we can prove that $\bar{\lambda}_A$ is fuzzy K-ideal of R

By theorem (2.5) we get that $A=(\mu_A, \lambda_A)$is an intuitionistic fuzzy K-ideal of R

Recall a function f from a semi-ring R into semi-ring T homomorphism if $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$ for any $x,y \in R$

Let f be a function from a set X into a set Y respectively, then the image of A under f, denoted by f(A) and the preimage of B under f, denoted by $f^{-1}(B)$, are IFS_s in X and Y respectively and defined by :

$f(A)=(f(\mu_A), f(\lambda_A))$, $f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\lambda_B))$

Theorem 3.8 :

Let $f:R \rightarrow T$ be onto homomorphism of semi-rings. If $A=(\mu_A, \lambda_A)$ is an intuitionistic fuzzy set such that μ_A is a fuzzy K-ideal then the $df(A)=(f(\mu_A), \overline{f(\mu_A)})$ is an intuitionistic fuzzy of R

Proof:

Let $(f(\mu_A))_t$ be anon empty level subset of $f(\mu_A)$ for any $t \in [0,1]$, if t=0 then $(f(\mu_A))_t=R$, if $t \neq 0$ by [4] $(f(\mu_A))_t = \bigcap_{0<s<t} (f(\mu_A)_{t-s})$

So $(f(\mu_A)_{t-s})$ is a non empty for any $0<s<t$

$\Rightarrow(\mu_A)_{t-s}$ is anon empty level subset of μ_A for any $0<s<t$
Since μ_A is a fuzzy K-ideal
Clearly $(\mu_A)_{t-s}$ is K-ideal
Since f is an onto homomorphism, so $(f(\mu_A)_{t-s})$ is K-ideal of T for any $0<s<t$
Since $(f(\mu_A))_t = \bigcap_{0<s<t} (f(\mu_A)_{t-s})$
So $(f(\mu_A))_t$ is K-ideal
Therefore $f(\mu_A)$ fuzzy is K-ideal
By theorem (2.3) $df(A)=(f(\mu_A), \overline{f(\mu_A)})$ is an intuitionistic fuzzy K-ideal.

Theorem 3.9 :
Let $f:R\rightarrow T$ be onto homomorphism of semirings. If $B=(\mu_B,\lambda_B)$ is an intuitionistic fuzzy K-ideal of T then the preimage $f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\lambda_B))$ of B under f is an intuitionistic fuzzy K-ideal of R.
Proof :
1- let $x,y\in R$

$f^{-1}(\mu_B)(x+y)= \mu_B(f(x+y))= \mu_B(f(x)+f(y))$
$\geq \min\{\mu_B(f(x)), \mu_B(f(y))\}=\min\{ f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)\}$

So $f^{-1}(\mu_B)(x+y) \geq \min\{ f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)\}$

$f^{-1}(\mu_B)(xy)= \mu_B(f(xy))= \mu_B(f(x)f(y)) \geq \mu_B(f(y))= f^{-1}(\mu_B)(y)$

So $f^{-1}(\mu_B)(xy)\geq f^{-1}(\mu_B)(y)$

Also $f^{-1}(\mu_B)(xy)\geq f^{-1}(\mu_B)(x)$

$f^{-1}(\lambda_B)(x+y)= \lambda_B(f(x+y))= \lambda_B(f(x)+f(y))$
$\leq \max\{\lambda_B(f(x)), \lambda_B(f(y))\}=\max\{ f^{-1}(\lambda_B)(x), f^{-1}(\lambda_B)(y)\}$

So $f^{-1}(\lambda_B)(x+y) \leq \max \{ f^{-1}(\lambda_B)(x), f^{-1}(\lambda_B)(y)\}$

$f^{-1}(\lambda_B)(xy)= \lambda_B(f(xy))= \lambda_B(f(x)f(y)) \leq \lambda_B(f(y))= f^{-1}(\lambda_B)(y)$

So $f^{-1}(\lambda_B)(xy)\geq f^{-1}(\lambda_B)(y)$

Also $f^{-1}(\lambda_B)(xy)\geq f^{-1}(\lambda_B)(x)$

2- let $x,y\in R\Rightarrow f(x),f(y)\in T$

$f^{-1}(\mu_B)(x)\geq \mu_B(f(x))\geq\min\{\max\{\mu_B(f(x)+f(y)),\mu_B(f(y)+f(x))\}, \mu_B(f(y))\}$
$= \min\{\max\{ f^{-1}(\mu_B)(x+y), f^{-1}(\mu_B)(y+x)\}, f^{-1}(\mu_B)(y)\}$

$f^{-1}(\lambda_B)(x)=\lambda_B(f(x)) \leq\max\{\min\{\lambda_B(f(x)+f(y)), \lambda_B(f(y)+f(x))\}, \lambda_B(f(y))\}$
$= \max\{\min\{ f^{-1}(\lambda_B)(x+y), f^{-1}(\lambda_B)(y+x)\}, f^{-1}(\lambda_B)(y)\}$

Therefore $f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\lambda_B))$ is an intuitionistic fuzzy K-ideal of R.

References

1- K. T. Atanassov, **Intuitionistic fuzzy sets**, Fuzzy sets and systems, 20(1986), 87-96.

2- K. T. Atanassov, **New operations defined over the Intuitionistic fuzzy sets,** Fuzzy sets and systems 61(1994), 137-142.

3- T. K. Dutta and B. K. Biswas, **Fuzzy prime ideals of semiring,** Bull Malaysian Math. Soc.17(1994), 9-16.

4- T. K. Dutta and B. K. Biswas, **Fuzzy k-ideals of semirings,** Bull Malaysian Math. Soc.87(1995), 91-96.

5- Y. B. Jun., J. Neggers and M.s.kim, **On L-fuzzy ideals in semirings I,** Czech. Math.J.48(1998).669-675.

6- K. H. Kim and J. G.Lee, **On Intuitionistic fuzzy Bi-ideals of semigroup**, Turk J. Math. 20(2005),201-210.

7- K. H. Kim and J. G. Lee, **On Intuitionistic Q-fuzzy semiprime ideals in semigroups,** Advances in fuzzy Mathematics, Vol.1 No.1(2006),15-21.

8- W. J. Liu, **Fuzzy invariants subgroups and fuzzy ideals,** Fuzzy sets and systems 8(1987),133-139.

9- L.A. Zadch, **Fuzzy sets,** Information and contral 8(1965),338-353.