Journal of Al-Qadisiyah for Computer Science and Mathematics Vol.17.(1) 2025,pp.Math 130–141

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Blood Flow Analysis in the Influence of a Magnetic Field in a Stenotic Artery

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ARTICLE INFO

Article history:
Received: 13 /2/2025
Rrevised form: 16 /3/2025
Accepted : 20 /3/2025
Available online: 30 /3/2025

Keywords:

Impedance
Catheter
Jeffery fluid
stenotic region
shear stress

ABSTRACT

In this work, the Jeffery fluid is used to model the mathematical analysis of unsteady blood flow through a catheterized artery with composite stenosis. Exact solutions are derived by simplifying the extremely nonlinear momentum equations governing the Jeffery fluid under the assumption of minimal stenosis. Graphs are used to figure out and look at the expressions for velocity, flow impedance, pressure, shear stress, and stream function for a number of interesting physical parameters

MSC..

https://doi.org/10.29304/jqcsm.2025.17.11998

1. Introduction

These compounds build up on the inside walls of blood vessels as a result of unhealthy eating habits, such as consuming carbohydrates and fats and not exercising. These buildups eventually cause fatty plaques to form on the vessel walls, which causes the vessels to progressively harden, narrow, or occasionally become blocked. Because of the narrow passageway or the hardening of the artery walls, blood finds it difficult to control its flow, which prevents the nutrients and oxygen carried by the blood from getting to the different parts of the body. This may lead to long-term damage to these tissues. As a result, numerous research studies have looked at the factors influencing blood flow in constricted arteries [1-10]. In [11] isah A., et al. discussed the impact of heat source and chemical reaction on MHD blood flow through permeable bifurcated arteries with tilted magnetic field in tumor treatments, computer methods in biomechanics and biomedical engineering. One of

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the most important elements that significantly influences blood flow dynamics is the magnetic field. The velocity and direction of blood flow can be altered by magnetic fields' interaction with the fluid. The direction and strength of the applied magnetic field can either improve or impair blood flow in constricted arteries. Therefore, in order to understand how these factors, affect blood flow in the restricted arteries and to design treatment procedures with the purpose of enhancing the health of blood vessels, we will solve a mathematical model in this chapter. A study on the multi-stenosis effect on the Jeffrey fluid model of blood flow was conducted in [12]. Ellahi et al. address analytical solutions of unsteady blood flow of Jeffery fluid through stenosis arteries with permeable walls in [13].

2. Mathematical Formulation

Consider an incompressible Jeffrey fluid flowing through a catheterized artery with a composite stenosis. We are considering cylindrical coordinates (r, θ, z) in such a way that the z -axis is taken along the axis of the artery, r and θ are the radial and circumferential directions, respectively.

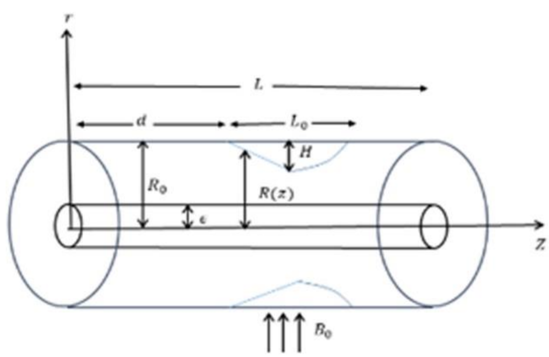

Figure 1: Geometry of a composite stenosis in a catheterized artery the presence of a magnetic field.
As shown in figure 1, see [3], the flow is under the influence of a regular magnetic field that is oriented exactly perpendicular to the artery's direction.

The stenosis artery geometry is defined as:

$$\frac{\bar{R}(\bar{z})}{R_0} = \begin{cases} 1 - \frac{2\bar{H}}{R_0 L_0}(\bar{z} - \bar{d}), & \bar{d} < \bar{z} \leq \bar{d} + \frac{L_0}{2} \\ 1 - \frac{\bar{H}}{2R_0}\left(1 + cos\frac{2\pi}{L_0}(\bar{z} - \bar{d} - \frac{L_0}{2})\right), & \bar{d} + \frac{L_0}{2} < \bar{z} \leq \bar{d} + L_0 \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

where $R(z)$ is the radius of the artery with stenosis in the peripheral region, R_0 is the constant radius of the normal artery in the non-stenotic region, L_0 is the length of overlapping stenosis, $\widetilde{H}$ is the radius of stenosis, ϵ is the radius of the catheter, $\bar{d}$ is the location of the stenosis, $cos\frac{2\pi}{L_0}$ is taken to be the critical height of the overlapping stenosis.

Constitutive Equations

Based on the above consideration, the basic governing equations that describe the flow in the present problem are given by [16]

Equation of mass conservation:

$$\nabla . \bar{V} = 0. \quad (2)$$

motion equations (Navier –Stokes equations)

$$\rho(\bar{V}.\nabla)\bar{V} = \nabla\bar{\sigma} + \bar{J} \times \bar{B} \tag{3}$$

Where $\bar{V}$ is velocity field, ρ is density, $\bar{B} = (0, B_0, 0)$ is the magnetic field, $\bar{\sigma}$ is Cauchy stress tensor, ∇V is fluid velocity gradient.

The velocity vector $\bar{V} = (u, v, w)$ in the cylindrical coordinates should be(r, ϑ, z). The Jeffrey incompressible fluid is considered and the constitutive equations can be defined as:

$$\bar{\sigma} = -\bar{P}\bar{I}. + \bar{S}, \tag{4}$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\bar{\dot{\varkappa}} + \lambda_2\bar{\ddot{\varkappa}}), \tag{5}$$

When $\bar{S}$ is extra stress tensor, μ is viscosity, $\bar{p}$ is pressure , $\bar{I}$ is identity tensor, λ_1 is relaxation ratio with the retardation time, $\bar{\dot{\varkappa}}$ is shear rate, $\bar{\ddot{\varkappa}}$ is material derivative and λ_2 is retardation time.

From the equation (2) – (3) we get:

$$\frac{\partial\bar{u}}{\partial\bar{r}} + \frac{\bar{u}}{r} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \tag{6}$$

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\bar{S}_{\bar{r}\bar{r}}) + \frac{\partial}{\partial\bar{z}}(\bar{S}_{\bar{r}\bar{z}}) - \frac{\bar{S}_{\bar{\vartheta}\bar{\vartheta}}}{\bar{r}} - \sigma(B_0)^2\bar{u} \tag{7}$$

$$\rho\left(\frac{\partial\bar{w}}{\partial\bar{t}} + u\frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\bar{S}_{\bar{r}\bar{z}}) + \frac{\partial}{\partial\bar{z}}(\bar{S}_{\bar{z}\bar{z}}) - \sigma(B_0)^2\bar{W} \tag{8}$$

Where

$$\bar{S}_{\bar{r}\bar{r}} = \frac{2\mu}{1+\lambda_1}\left(\frac{\partial\bar{u}}{\partial\bar{r}} + \lambda_2\left(\frac{\partial^2 u}{\partial\bar{r}\,\partial\bar{t}} + \bar{u}\frac{\partial^2 u}{\partial\bar{r}^2} + \bar{w}\frac{\partial^2\bar{u}}{\partial\bar{r}\,\partial\bar{z}}\right)\right) \tag{9}$$

$$\bar{S}_{\bar{r}\,\bar{z}} = \frac{\mu}{1+\lambda_1}\left(\frac{\partial\bar{u}}{\partial\bar{z}} + \frac{\partial\bar{w}}{\partial\bar{r}} + \lambda_2\left(\frac{\partial^2\bar{u}}{\partial\bar{z}\,\partial\bar{t}} + \bar{w}\frac{\partial^2 u}{\partial\bar{r}\,\partial\bar{t}}\right) + \lambda_2\left(\bar{u}\left(\frac{\partial^2\bar{u}}{\partial\bar{r}\,\partial\bar{z}} + \frac{\partial^2\bar{w}}{\partial\bar{r}^2}\right) + \bar{w}\left(\frac{\partial^2\bar{u}}{\partial\bar{z}^2} + \frac{\partial^2\bar{w}}{\partial\bar{r}\,\partial\bar{z}}\right)\right)\right) \tag{10}$$

$$\bar{S}_{\bar{z}\,\bar{z}} = \frac{2\mu}{1+\lambda_1}\left(\frac{\partial\bar{w}}{\partial\bar{z}} + \lambda_2\left(\frac{\partial^2\bar{w}}{\partial\bar{z}\,\partial\bar{t}} + \bar{u}\frac{\partial^2\bar{w}}{\partial\bar{r}\,\partial\bar{z}} + \bar{w}\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right)\right) \tag{11}$$

Incorporating the non- dimensional boundary conditions are the standard at the artery and catheter walls:

$$\left.\begin{aligned} &\frac{\partial w}{\partial r} = 0, \text{ at } r = \epsilon \\ &w = w_B \text{ and } \frac{\partial w}{\partial r} = \frac{\alpha}{\sqrt{Da}}(w_B - w_{porous}) \text{ at } r = R(z) \end{aligned}\right\} \tag{12}$$

Where w_B is the slip velocity and $w_{porous} = -Da\frac{\partial p}{\partial z}$, w_{porous} is the velocity in the permeable boundary [13], Da is the Darcy number, and α is dimensionless quantity that depends on the material parameters that describe the structure of the permeable material within the boundary region is known as the slip parameter.

To simplify the problem, take a look at the collection of non-dimensional parameters that are used to display the influencing parameters, eliminate the non-influential ones, and define the problem as follows:

$$\left\{\begin{gathered} u = \frac{\bar{u}\,L_0}{R_0 s}, w = \frac{\bar{w}}{s}, r = \frac{\bar{r}}{R_0},\ z = \frac{\bar{z}}{L_0}, S = \frac{R_0\bar{S}}{\mu\, s},\ \delta = \frac{R_0}{L_0}, \\ , p = \frac{R_0^2\bar{p}}{\mu s L_0}, M^2 = \frac{\sigma R_0^2 B_0^2}{\mu}, R(z) = \frac{\bar{R}(\bar{z})}{R_0} \\ r_1 = \frac{\bar{r_1}}{R_0} = \varepsilon < 1, \emptyset = \frac{b}{R_0}, r_2 = \frac{\bar{r_2}}{R_0} = 1 + \emptyset\, sin(2\pi\bar{z}), \\ Re = \frac{\rho s R_0}{\mu},\ H = \frac{\bar{H}}{R_0}, t = \frac{s\bar{t}}{L_0}, d = \frac{\bar{d}}{L_0} \end{gathered}\right\} \tag{13}$$

The non-dimensional form of the stenosis artery geometry is defined as:

$$R(z) = \begin{cases} 1 - 2H(z-d) & d < z \le d + \frac{1}{2} \\ 1 - \frac{H}{2}\left(1 + \cos 2\pi(z - d - \frac{1}{2})\right), & d + \frac{1}{2} < z \le d + 1 \\ 1 & \text{otherwise} \end{cases} \tag{14}$$

Substituting the non-dimensional problem is given by (13) into equations (6) - (11), we have:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{15}$$

$$Re\,\delta^3\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \delta\frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \delta^2\frac{\partial}{\partial z}(S_{rz}) - M^2\delta^2 u \tag{16}$$

$$Re\delta\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \delta\frac{\partial}{\partial z}(S_{zz}) - M^2 w \tag{17}$$

Where

$$S_{rz} = \frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} + \lambda_2\left(\frac{s}{L_0}\right)\delta\left(\left(\frac{R_0}{L_0}\right)\frac{\partial^2 u}{\partial z\partial t} + s\,w\frac{\partial^2 u}{\partial r\partial t}\right) + \lambda_2\delta\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\left(\delta^2\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right) \tag{18}$$

After taking the required assumptions, we obtained the following equations:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{19}$$

$$\frac{\partial p}{\partial r} = 0 \tag{20}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) - M^2 w \tag{21}$$

Where $S_{rr} = S_{\theta\theta} = S_{zz} = 0$ and $(S_{rz}) = \frac{1}{1+\lambda_1}\left(\frac{\partial w}{\partial r}\right)$ (22)

Replacing S_{rz} from equation (22) in to equation (21), we have:

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\left(\frac{\partial w}{\partial r}\right)\right) - M^2 w \tag{23}$$

4. A momentum Equation Solution

Equation (20), indicates that p only depends on r. Might write the equation (23) as:

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - M^2(1+\lambda_1)w = (1+\lambda_1)\frac{dp}{dz}, \tag{24}$$

Set $A = -M^2(1+\lambda_1)$, equation (21) the form takes:

$$r^2\frac{\partial^2 w}{\partial r^2} + r\frac{\partial w}{\partial r} + Aw = (1+\lambda_1)\frac{dp}{dz} \tag{25}$$

The general solution of equation (23) is

$$w = BesselJ[0, \sqrt{A}r]C_1 + BesselY[0, \sqrt{A}r]C_2 - \frac{1}{M^2}\frac{dp}{dz} \tag{26}$$

Using the boundary conditions described in eq (12), we get

Let $C_1 = -\frac{C_4\left(-\frac{dp}{dz} + Aw_B - \frac{dp}{dz}\lambda_1\right)}{C_3}$ and $C_2 = \frac{C_5\left(-\frac{dp}{dz} + Aw_B - \frac{dp}{dz}\lambda_1\right)}{C_3}$

Where

$$C_3 = A\left(BesselJ[1, \sqrt{A}\epsilon]BesselY\left[0, \sqrt{A}R[z]\right] - BesselJ\left[0, \sqrt{A}R[z]\right]BesselY[1, \sqrt{A}\epsilon]\right)$$

$C_4 = BesselY\left[1, \sqrt{A}\epsilon\right]$, and $C_5 = BesselJ\left[1, \sqrt{A}\epsilon\right]$,

The solution of the differential equation (38) can be written as:

$$w = \left(-\frac{C_4\left(-\frac{dp}{dz}+Aw_B-\frac{dp}{dz}\lambda_1\right)}{C_3}\right)\text{BesselJ}[0,\sqrt{A}r] + \left(\frac{C_5\left(-\frac{dp}{dz}+Aw_B-\frac{dp}{dz}\lambda_1\right)}{C_3}\right)\text{BesselY}[0,\sqrt{A}r] - \frac{1}{M^2}\frac{dp}{dz} \quad (27)$$

Taking the derivative (27) for r and using the boundary conditions described in eq (12), we have:

$$w_B = \left(\frac{\sqrt{\text{Da}}\left(\sqrt{\text{Da}}\frac{dp}{dz}\alpha C_3+\sqrt{A}\frac{dp}{dz}\text{BesselJ}[1,\sqrt{A}r]C_4-\sqrt{A}\frac{dp}{dz}\text{BesselY}[1,\sqrt{A}r]C_5+\sqrt{A}\frac{dp}{dz}\text{BesselJ}[1,\sqrt{A}r]C_4\lambda_1-\sqrt{A}\frac{dp}{dz}\text{BesselY}[1,\sqrt{A}r]C_5\lambda_1\right)}{-\alpha C_3+A^{3/2}\sqrt{\text{Da}}\text{BesselJ}[1,\sqrt{A}r]C_4-A^{3/2}\sqrt{\text{Da}}\text{BesselY}[1,\sqrt{A}r]C_5}\right) \quad (28)$$

So the solution of equation (27) is:

$$w = \frac{\frac{dp}{dz}(1+\lambda_1)}{C_3}\begin{pmatrix} C_4\text{BesselJ}[0,\sqrt{A}r] \\ -C_5\text{BesselY}[0,\sqrt{A}r]\end{pmatrix} + \frac{\frac{dp}{dz}(1+\lambda_1)}{A} + \begin{pmatrix} \frac{CA}{C_3}\text{BesselY}[0,\sqrt{A}r] \\ -\frac{C_4 A}{C_3}\text{BesselJ}[0,\sqrt{A}r]\end{pmatrix}\left(\left(\sqrt{\text{Da}}\begin{pmatrix}\sqrt{\text{Da}}\frac{dp}{dz}\alpha C_3 \\ +\sqrt{A}\frac{dp}{dz}\text{BesselJ}[1,\sqrt{A}r]C_4 \\ -\sqrt{A}\frac{dp}{dz}\text{BesselY}[1,\sqrt{A}r]C_5 \\ +\sqrt{A}\frac{dp}{dz}\text{BesselJ}[1,\sqrt{A}r]C_4\lambda_1 \\ -\sqrt{A}\frac{dp}{dz}\text{BesselY}[1,\sqrt{A}r]C_5\lambda_1\end{pmatrix}\right)\Big/\begin{pmatrix}(-\alpha C_3 \\ +A^{3/2}\sqrt{\text{Da}}\text{BesselJ}[1,\sqrt{A}r]C_4 \\ -A^{3/2}\sqrt{\text{Da}}\text{BesselY}[1,\sqrt{A}r]C_5)\end{pmatrix}\right) \quad (29)$$

We extract pressure from the equation (29) as a common factor among the terms.

$$w = \frac{dp}{dz}\left(\frac{(1+\lambda_1)}{C_3}\begin{pmatrix} C_4\text{BesselJ}[0,\sqrt{A}r] \\ -C_5\text{BesselY}[0,\sqrt{A}r]\end{pmatrix} + \frac{(1+\lambda_1)}{A} + \begin{pmatrix} \frac{CA}{C_3}\text{BesselY}[0,\sqrt{A}r] \\ -\frac{C_4 A}{C_3}\text{BesselJ}[0,\sqrt{A}r]\end{pmatrix}\left(\left(\sqrt{\text{Da}}\begin{pmatrix}\sqrt{\text{Da}}\alpha C_3 \\ +\sqrt{A}\text{BesselJ}[1,\sqrt{A}r]C_4 \\ -\sqrt{A}\text{BesselY}[1,\sqrt{A}r]C_5 \\ +\sqrt{A}\text{BesselJ}[1,\sqrt{A}r]C_4\lambda_1 \\ -\sqrt{A}\text{BesselY}[1,\sqrt{A}r]C_5\lambda_1\end{pmatrix}\right)\Big/\begin{pmatrix}(-\alpha C_3 \\ +A^{3/2}\sqrt{\text{Da}}\text{BesselJ}[1,\sqrt{A}r]C_4 \\ -A^{3/2}\sqrt{\text{Da}}\text{BesselY}[1,\sqrt{A}r]C_5)\end{pmatrix}\right)\right)$$

5. Rate of Volume Flow:

We integrate the velocity equation to get the flow where the instantaneous volumetric flow rate in fixed coordinate model is given by [14]:

$Q = 2\pi \int_{\epsilon}^{R(z)} rw(r,z,t)\, dr,$

We have:

$$Q = -2\pi\Big(\frac{1}{288A^2\mathrm{Da}C_5{}^2}\frac{dp}{dz}\big(\epsilon^2(144A\mathrm{Da}C_5{}^2(1+\lambda_1) - 9\pi\epsilon^2\alpha^2C_3(2\pi C_4 + C_5 - 4\mathrm{EulerGamma}C_5)(1 + A\mathrm{Da} + \lambda_1) + 16A\sqrt{\mathrm{Da}}\epsilon\alpha C_5(-3\pi C_4 + 2(-1 + 3\mathrm{EulerGamma})C_5)(1 + A\mathrm{Da} + \lambda_1) + 12\epsilon\alpha C_5(3\pi\epsilon\alpha C_3 + 8A\sqrt{\mathrm{Da}}C_5)\mathrm{Log}[\tfrac{\sqrt{A}\epsilon}{2}](1 + A\mathrm{Da} + \lambda_1)) - (R[z]^2(144A\mathrm{Da}C_5{}^2(1+\lambda_1) - 9\pi R[z]^2\alpha^2C_3(2\pi C_4 + C_5 - 4\mathrm{EulerGamma}C_5)(1 + A\mathrm{Da} + \lambda_1) + 16A\sqrt{\mathrm{Da}}R[z]\alpha C_5(-3\pi C_4 + 2(-1 + 3\mathrm{EulerGamma})C_5)(1 + A\mathrm{Da} + \lambda_1) + 12R[z]\alpha C_5(3\pi R[z]\alpha C_3 + 8A\sqrt{\mathrm{Da}}C_5)\mathrm{Log}[\tfrac{\sqrt{A}R[z]}{2}](1 + A\mathrm{Da} + \lambda_1))\big)\Big)\Big) \tag{30}$$

Set

$$Q = -\frac{dp}{dz}F(z) \tag{31}$$

Implies that

$$\frac{dp}{dz} = -\frac{Q}{\left(\begin{array}{c}\frac{2\pi}{288A^2\mathrm{Da}c_5{}^2}(\epsilon^2(144A\mathrm{Da}c_5{}^2(1+\lambda_1) - 9\pi\epsilon^2\alpha^2c_3(2\pi c_4 + c_5 - 4\mathrm{EulerGamma}c_5)(1 + A\mathrm{Da} + \lambda_1) \\ +16A\sqrt{\mathrm{Da}}\epsilon\alpha c_5(-3\pi c_4 + 2(-1 + 3\mathrm{EulerGamma})c_5)(1 + A\mathrm{Da} + \lambda_1) \\ +12\epsilon\alpha c_5(3\pi\epsilon\alpha c_3 + 8A\sqrt{\mathrm{Da}}c_5)\mathrm{Log}[\frac{\sqrt{A}\epsilon}{2}](1 + A\mathrm{Da} + \lambda_1)) \\ -(R[z]^2(144A\mathrm{Da}c_5{}^2(1+\lambda_1) - 9\pi R[z]^2\alpha^2c_3(2\pi c_4 + c_5 \\ -4\mathrm{EulerGamma}c_5)(1 + A\mathrm{Da} + \lambda_1) \\ +16A\sqrt{\mathrm{Da}}R[z]\alpha c_5(-3\pi c_4 + 2(-1 + 3\mathrm{EulerGamma})c_5)(1 + A\mathrm{Da} + \lambda_1) \\ +12R[z]\alpha c_5(3\pi R[z]\alpha c_3 + 8A\sqrt{\mathrm{Da}}c_5)\mathrm{Log}[\frac{\sqrt{A}R[z]}{2}](1 + A\mathrm{Da} + \lambda_1))))\end{array}\right)} \tag{32}$$

The resistance function that depends on the position z along the artery. This function represents the resistance to blood flow in a different part of the artery.

$$F(z) = -\frac{2\pi}{288A^2\mathrm{Da}c_5{}^2}(\epsilon^2(144A\mathrm{Da}c_5{}^2(1+\lambda_1) - 9\pi\epsilon^2\alpha^2c_3(2\pi c_4 + c_5 - 4\mathrm{EulerGamma}c_5)(1 + A\mathrm{Da} + \lambda_1) + 16A\sqrt{\mathrm{Da}}\epsilon\alpha c_5(-3\pi c_4 + 2(-1 + 3\mathrm{EulerGamma})c_5)(1 + A\mathrm{Da} + \lambda_1) + 12\epsilon\alpha c_5(3\pi\epsilon\alpha c_3 + 8A\sqrt{\mathrm{Da}}c_5)\mathrm{Log}[\tfrac{\sqrt{A}\epsilon}{2}](1 + A\mathrm{Da} + \lambda_1)) - (R[z]^2(144A\mathrm{Da}c_5{}^2(1+\lambda_1) - 9\pi R[z]^2\alpha^2c_3(2\pi c_4 + c_5 - 4\mathrm{EulerGamma}c_5)(1 + A\mathrm{Da} + \lambda_1) + 16A\sqrt{\mathrm{Da}}R[z]\alpha c_5(-3\pi c_4 + 2(-1 + 3\mathrm{EulerGamma})c_5)(1 + A\mathrm{Da} + \lambda_1) + 12R[z]\alpha c_5(3\pi R[z]\alpha c_3 + 8A\sqrt{\mathrm{Da}}c_5)\mathrm{Log}[\tfrac{\sqrt{A}R[z]}{2}](1 + A\mathrm{Da} + \lambda_1)))) \tag{33}$$

6. Flow Impedance

A measurement of the resistance to blood flow via the arteries is called impedance. We may determine the total impedance by integrating the impedance of each arterial segment, which varies depending on the size and form of the artery. These computations are helpful in comprehending the effects of blood artery narrowing or shape change on blood flow. So the flow impedance is as follows [13]:

$$\lambda = \frac{1}{Q}\int_{\epsilon}^{R(z)}\left(-\frac{dp}{dz}\right)dz, \tag{34}$$

$$\lambda = \int_0^L G(z)dz, \tag{35}$$

$$G(z)=\frac{1}{F(z)}, \tag{36}$$

Finally, we get the flow impedance equation as it:

$$\lambda=\int_0^d \frac{1}{F_1(z)}dz+\int_d^{d+\frac{1}{2}}\frac{1}{F_2(z)}dz+\int_{d+\frac{1}{2}}^{d+1}\frac{1}{F_3(z)}dz+\int_{d+1}^{L}\frac{1}{F_1(z)}dz \tag{37}$$

7. Stream function

The corresponding stream functions $u=-\frac{1}{r}\frac{\partial\psi}{\partial z}$ *and* $w=\frac{1}{r}\frac{\partial\psi}{\partial r}$ is:

$$\psi=\int rwdr$$

So that

$$\psi=\left(\frac{\begin{array}{c}(\frac{dp}{dz}r(r((\alpha BesselJ[0,\sqrt{A}R[z]]\\ +\sqrt{A}\sqrt{Da}BesselJ[1,\sqrt{A}R[z]])BesselY[1,\sqrt{A}\epsilon]\\ -BesselJ[1,\sqrt{A}\epsilon](\alpha BesselY[0,\sqrt{A}R[z]]\\ +\sqrt{A}\sqrt{Da}BesselY[1,\sqrt{A}R[z]]))(1+\lambda_1)\\ +\frac{2\alpha BesselJ[1,\sqrt{A}\epsilon]BesselY[1,\sqrt{A}r](1+ADa+\lambda_1)}{\sqrt{A}}\\ -r\alpha BesselY[1,\sqrt{A}\epsilon]Hypergeometric0F1Regularized\left[2,-\frac{Ar^2}{4}\right]\\ (1+ADa+\lambda_1)))\end{array}}{\begin{array}{c}(2A((\alpha BesselJ[0,\sqrt{A}R[z]]\\ +\sqrt{A}\sqrt{Da}BesselJ[1,\sqrt{A}R[z]])BesselY[1,\sqrt{A}\epsilon]\\ -BesselJ[1,\sqrt{A}\epsilon](\alpha BesselY[0,\sqrt{A}R[z]]\\ +\sqrt{A}\sqrt{Da}BesselY[1,\sqrt{A}R[z]])))\end{array}}\right) \tag{38}$$

The shear stress is an important characteristic of blood flow and can be calculated as follows [15]:

$$\tau=\frac{1}{1+\lambda_1}(-\sqrt{A}\mathrm{C}_1\mathrm{BesselJ}[1,\sqrt{A}r]-\sqrt{A}\mathrm{C}_2\mathrm{BesselY}[1,\sqrt{A}r]) \tag{39}$$

8. Discussion and The Numerical Results

In this section, the numerical and computational results are discussed for the problem of an incompressible non-Newtonian Jeffrey fluid through a catheterized artery with a composite stenosis under the influence of a magnetic field, indented arterial channel in detail with graphical illustrations. Shows type (1) geometry of a stenosis artery in the presence of a magnetic field. The numerical evaluations of the analytical results and some important results are displayed graphically in figures (2) to (12). MATHEMATICA is used to obtain the numerical results and illustrations. Analytical solution to the momentum equation is obtained by the method of Bessel functions. All the obtained solutions are discussed graphically under the variations of various important parameters in the present section.

8.1 Velocity Distribution

Based on equation (29), figures (2)-(3) illustrate parameter effects δ,α,λ_1 and Q on the velocity distribution w vs. r. We noticed in figures (2)- (3), that the velocity distribution w increases with increasing δ,λ_1 and Q, While it decreases in figure (3) at α.

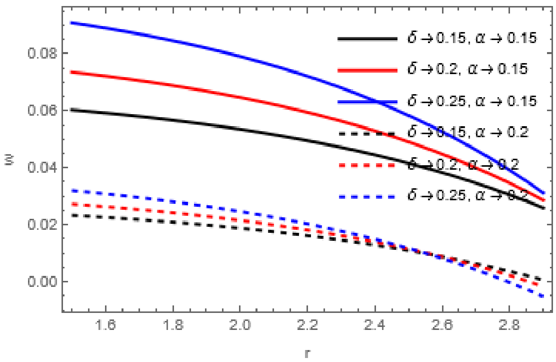

Figure 2: velocity distribution for various values of δ and α with $\mathrm{Da} = 0.9, M = 1.2, \epsilon = 0.2, \lambda_1 = 0.05, d = 0.2, Q = 0.4, z = 0.5$

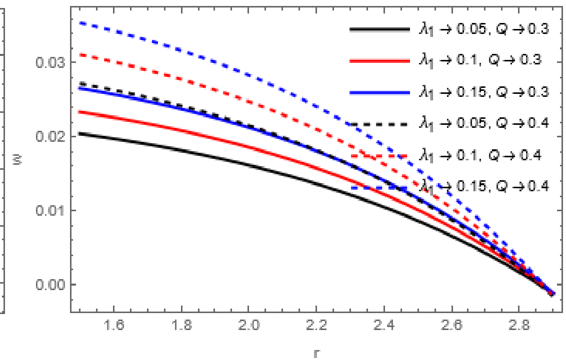

Figure 3: velocity distribution for various values of λ_1 and Q with $\mathrm{Da} = 0.9, M = 1.2, \epsilon = 0.2, \delta = 0.2, d = 0.2, \alpha = 0.2, z = 0.5$

8.2 The Distributed of Pressure

Based on equation (32), figures (4)-(5) illustrate the effect of the parameters ϵ, λ_1, d and Q on the distribution of dp/dz vs. Z. We noticed that dp/dz it starts to increase, and when it reaches a point, $Z = 0.7$ it starts to decrease, and for this, the general shape of the distribution dp/dz is a concave downward curve. Figure (4) illustrates the parameters Q, and d it was found that they dp/dz increase with increasing Q, while with increase d we notice that dp/dz decrease when $Z < 0.7$ and increase when $Z > 0.7$, figure (5) we notice that the dp/dz starts to increase with increasing of ϵ, λ_1.

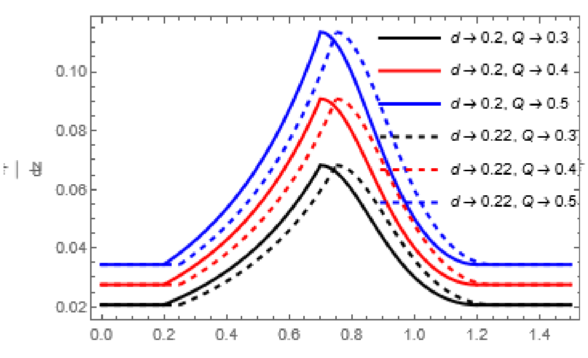

Figure 4: Distribution of $\frac{dp}{dz}$ vs. z for various values of d and Q with $\mathrm{Da} = 0.9, \delta = 0.2, \alpha = 0.2, M = 1.2, \lambda_1 = 0.05, \epsilon = 0.2$

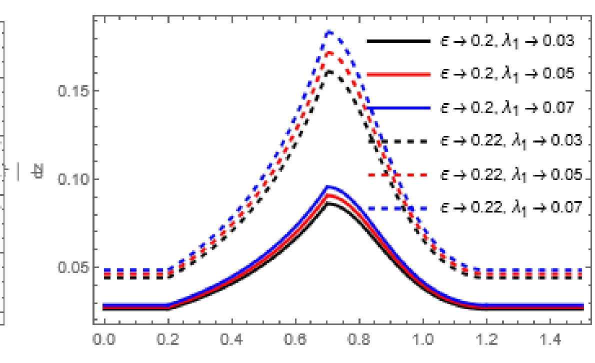

Figure 5: Distribution of $\frac{dp}{dz}$ vs. z for various values of ϵ and λ_1 with $\mathrm{Da} = 0.9, \delta = 0.2, \alpha = 0.2, d = 0.2, M = 1.2, Q = 0.4$

8.3 The flow impedance

It is a measure of how difficult it is for a fluid to flow through a specific channel or vessel, and it depends on several factors such as pressure, speed, and the geometric characteristics of the conduit through which the fluid passes (such as an artery or tube). In the circulatory system, such as blood flow in arteries, flow resistance is calculated to understand how an obstruction or stenosis affects blood flow, and to determine the force needed to push fluid through the vessels.

Based on equation (37), figures (6)-(7) illustrate parameter effects δ, α, M and ϵ on the flow impedance λ. In figure (6) as the parameter ϵ increase then the resistance will increase while α increase then the resistance will decrease. In figure (7) as the magnetic field M increase, the resistance to flow decreases, while when increase δ the resistance to flow increases.

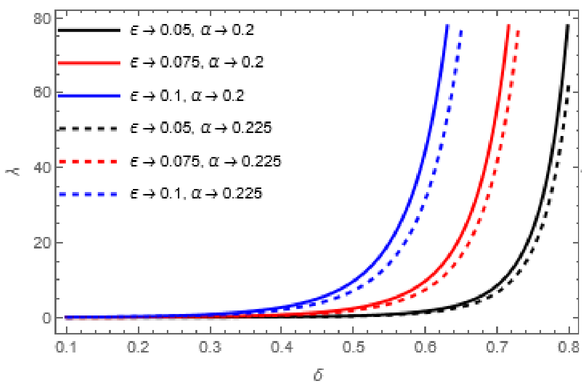

Figure 6: The flow impedance λ vs. δ for various values of ϵ and α with $\mathrm{Da} = 0.8, M = 1.2, \lambda_1 = 0.5$.

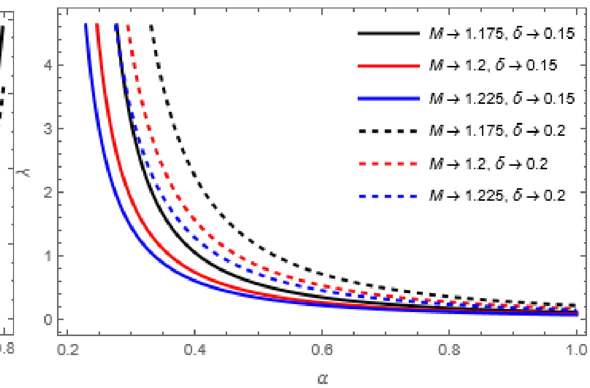

Figure 7: The flow impedance λ vs. α for various values of M and δ with $\mathrm{Da} = 0.8, \epsilon = 0.2, \lambda_1 = 0.5$.

8.4 The Shear Stress

Shear stress is the force exerted on the walls of an ar [illegible] essels, blood flows smoothly, but when stenosis occurs, the flow characteristics change dramatically.

Based on equation (39). In order to observe the effects of α and M. In (8), in the lower channel under the catheter tube, we notice that with increases M, the shear stress τ increases, while τ decreasing with increases α. At figure (9), τ decreases with increases M and τ increases with increases α in the upper channel above the catheter tube.

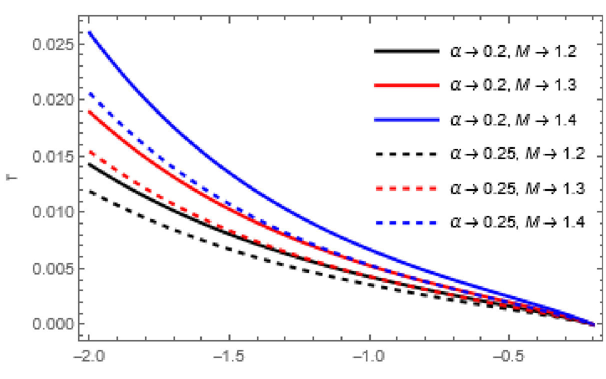

Figure 8: Variation of shear stress at the lower wall in the stenotic region τ vs. r for various values of M and α with $\mathrm{Da} = 0.9, \epsilon = 0.2, \delta = 0.2, \lambda_1 = 0.05, d = 0.2, Q = 0.4, z = 0.5$

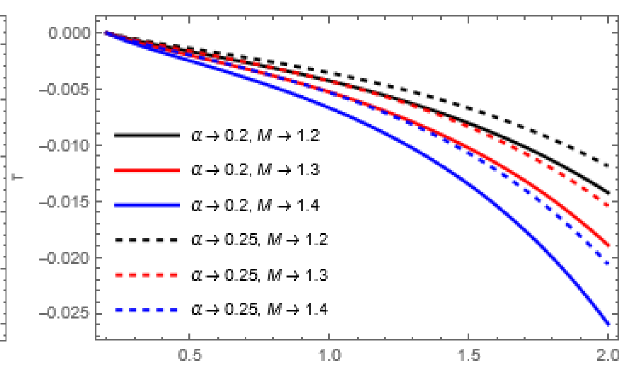

Figure 9: Variation of shear stress at at the upper wall in the stenotic region τ vs. r for various values of M and α with $\mathrm{Da} = 0.9, \epsilon = 0.2, \delta = 0.2, \lambda_1 = 0.05, d = 0.2, Q = 0.4, z = 0.5$

8.5 Phenomena Trapping

The phenomenon of trapping in arterial stenosis refers to the pooling or trapping of blood in the area of stenosis due to dynamic changes in flow, whether due to increased velocity or other effects such as magnetic effects. This phenomenon can lead to increased resistance to flow, increasing pressure within the arteries, causing health problems.

Based on equation (38). In order to observe the effects of α, M and d to the blood flow through stenosis arteries as in (10) – (12), in (10) shows the slip parameter α It is seen here that as we increase the slip parameter, the bolus size in the channel decreases. In Figure (11), two bolus appear at the top and bottom of the channel, and their size increases clearly with the increase of the magnetic field parameter M. Figure (12) shows two bolus above and below the wall moving along the channel when the values of dimension d increase.

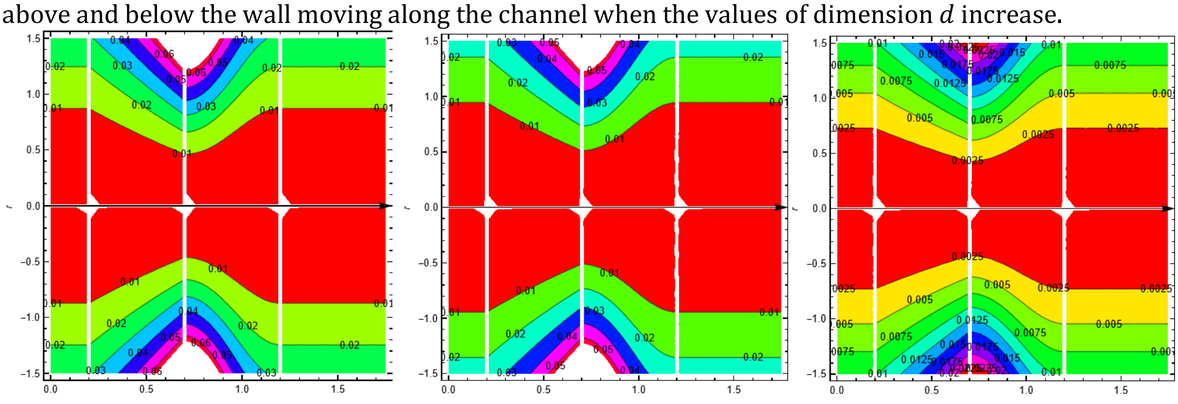

Figure 10: Streamlines for various values of $\alpha = \{0.15, 0.16, 0.17\}$ at $Da = 0.9, \delta = 0.2, M = 1.2, \epsilon = 0.2, \lambda_1 = 0.05, d = 0.2, Q = 0.4$.

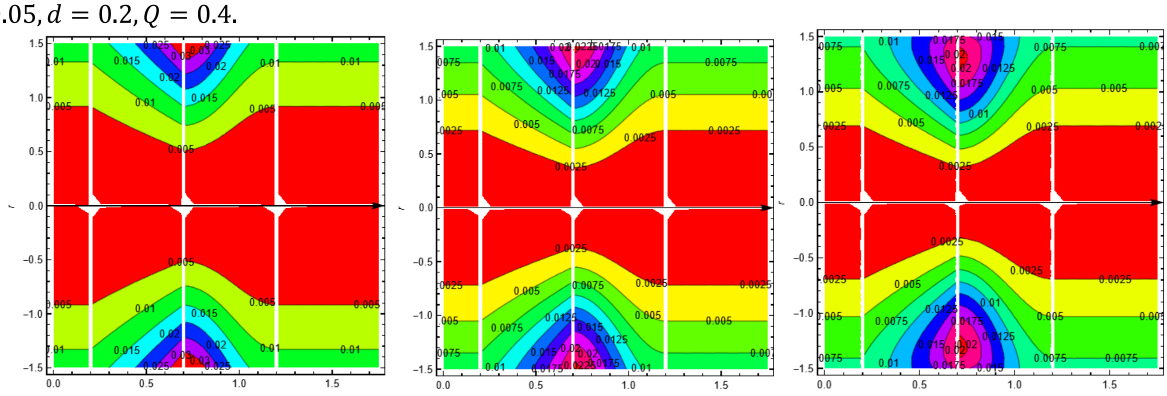

Figure 11: Streamlines for various values of $M = \{1, 1.1, 1.2\}$ at $Da = 0.9, \delta = 0.2,$
$\alpha = 0.2, \epsilon = 0.2, \lambda_1 = 0.05, d = 0.2, Q = 0.4$.

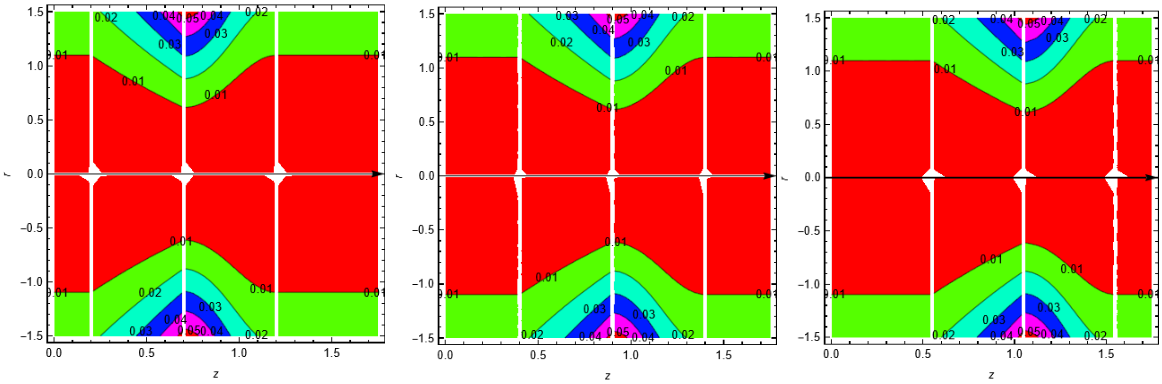

Figure 12: Streamlines for various values of $d = \{0.05, 0.1, 0.15\}$ at $Da = 0.9, \delta = 0.2, \alpha = 0.2, \epsilon = 0.2, \lambda_1 = 0.05, M = 1.2, Q = 0.4$.

9. Conclusions

Assuming that the blood flowing through a composite stenosis behaves as a non-Newtonian fluid, the flow through the stenosis has been analyzed to estimate the increase in velocity, resistance, shear stress, pressure, and flow during artery catheterization.

- It is crucial to note that impedance rises with increasing catheter size and is highly dependent on the height of the stenosis.
- As the artery gets closer to the stenosis in narrower areas, its velocity increases, raising the pressure.
- Additionally, the resistance to flow rises as the available area for flow diminishes.
- In the stenotic region, impedance, flow (stream), and shear stress all rise with increasing velocity.

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