Journal of AL-Qadisiyah for computer science and mathematics
Vol.9 No.1 Year 2017

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On coc-r-compact spaces

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Recived : 13\11\2016 **Revised : 25\12\2016** **Accepted : 16\1\2017**

Abstract:

In this work, We will study a new class of popularizer open sets called coc-r-open sets and we will study its properties too, also we will study coc-r-compact, I-Compact space and the relationship between them, finally we have to get some results that show the relationship between these spaces through some of the theorems obtained by using coc-r-open sets.

Keywords:

coc-r-open, r-Interior point, coc-r-Interior point, coc-r-closure coc-r-compact, I-compact, r-compact, compact spaces, coć-r-open, , coć-r-continuous, coc-r-continuous functions.

Mathematics subject classification : 54XX .

1. Introduction

In the year 2011[1] S. Al Gore and S. Samarah provided coc-open sets in the topological spaces, where they studied continuity by using these sets. Later, some researchers have studied those sets and expanded, in1937 [11], regular open sets were introduced and used to define the semi regularization space of a topological space[8],[12], where in the first chapter we will be define coc-r-open set and study its properties, N. Bourbaki [2] introduced the concept of compact space, in [3] D. E. Cameron introduced the concept of I-compact space, where he studied maximal C-compact spaces, maximal QHC spaces, and maximal nearly compact spaces. He also discussed covering property which turns out to be equivalent to *S*-closed and extremally disconnected. In section two introduces the definition of coc-r-compact, compact, I-compact spaces and give salutary characterizations of this concepts.

2. Coc-r-open Sets and its Properties

In this section ,we give some basic definition ,properties and theorems of coc-r-open sets.

Definition (2.1) [1]

A subset A of a space (X,τ) is called cocompact open set (notation : coc-open set) if for every $x \in A$ there exists an open set $U \subseteq X$ and a compact subset K such that $x \in U - K \subseteq A$. the complement of coc-open set is called coc-closed set

Remarks (2.2) [7]

i. Every open set is a coc-open set.
ii. Every closed set is a coc-closed set.
iii.The converse of (i, ii) is not true in general.

Definition: (2.3) [11]

A subset A of a space (X,τ) is called regular open set (notation : r-open set) if $A = \overline{A}^{\circ}$.
The complement of regular open set is called regular closed (r- closed) set and it is easy to see that A is regular closed if $A = \overline{A^{\circ}}$ **[12] .**

Remarks (2.4) [12]

i. Every r-open set is an open set.

ii. Every r-closed set is a closed set.

The converse of (i, ii) is not true in general as the following example shows:

Example (2.5)

Let $X = \{1,2,3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$ be a topology on X. Notice that $\{1,2\}$ is an open set in X, but it is not r-open set and $\{3\}$ is a closed set in X, but it is not r-closed set.

Remarks (2.6) [8]

1) The family of all r - open sets in X is denoted by $RO(X, \tau)$.

2) The family of all r - closed sets in X is denoted by $RC(X, \tau)$.

Definition (2.7)

A subset A of a space (X,τ) is called cocompact regular open set (notation : coc -r-open set) if for every $x \in A$ there exists r-open set $U \subseteq X$ and compact subset K such that $x \in U - K \subseteq A$, the complement of coc-r-open set is called coc -r- closed set .

Remark (2.8)

i. Every coc -r-open set is not necessarily to be open set, ii. Every coc-r-closed set is not necessarily to be closed set .

iii. Every open set is not necessarily to be coc -r-open set.

iv. Every closed set is not necessarily to be coc -r-closed set

As the following examples show:

Examples (2.9)

1- Let $X = \{1,2,3\}, \tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$ be a topology on X, the coc-r-open sets are $\{X, \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$ then $\{3\}$ is a coc-r-open but it is not open and $\{2\}$ is coc-r-closed but it is not closed set.

2- Let $X = \{1,2,3, \ldots\}, \tau = \{G \subseteq X: 1 \in G\} \cup \{\emptyset\}$ be a topology on X, the coc-r-open sets are $\{G \subseteq X: G^c \text{ is finite}\} \cup \{\emptyset\}$. Notice that $\{1\}$ is an open but is not coc-r-open and $\{2,3, \ldots\}$ is a closed but it is not coc-r-closed.

Remarks (2.10)

1- Every r-open set is coc - open set.

2- Every r-closed is coc - closed set.

3- Every r-open set is coc -r-open set.

4- Every r- closed set is coc -r- closed set.

5- Every coc -r-open set is coc-open.

6- Every coc -r- closed set is coc- closed.

Proof :

Clear.

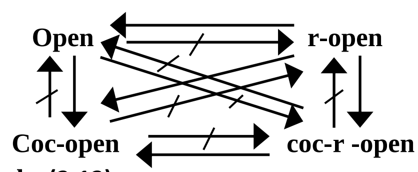
The converse of Remarks (2.10) is not true in general as the following examples show:

Examples (2.11)

1- Let $X = \{1,2,3\}, \tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$ be a topology on X. Notice that $\{1,2\}$ is a coc-open, coc-r-open but it is not r-open and $\{3\}$ is a coc-closed, coc-r- closed but it is not r- closed.

2- Let $X = \{1,2,3, \ldots\}$, $\tau = \{G \subseteq X: 1 \in G\} \cup \{\emptyset\}$ be a topology on X, the coc-r-open sets are $\{G \subseteq X: G^c \text{ is finite}\} \cup \{\emptyset\}$, thus $\{1\}$ is a coc-open but it is not coc-r-open and $\{2,3, \ldots\}$ is a coc- closed but it is not coc-r- closed.

The following diagram shows the relation among types of open sets



Remarks (2.12)

1- The intersection of two r-open set is r-open **[12]**.

2- The intersection of r-open set and open set is open .

3- The intersection of two coc -r -open sets is coc -r -open .

4- The union of coc-r-open sets is coc-r-open set .

5- The intersection of coc-r-open set and coc-open set is coc-open .

6- The intersection of two coc-open sets is coc-open**[1]**.

Proof :

2) Clear.

3) Let A, B be coc -r –open, to prove $A \cap B$ coc -r -open set. Suppose $x \in A \cap B$, then $x \in A$ and $x \in B$, since A, B are coc -r - open, thus there exist two r-open sets $U, V \subseteq X$ and two compact subsets K, L such that $x \in U - K \subseteq A$, $x \in V - L \subseteq B$, there fore $x \in (U - K) \cap (V - L) \subseteq A \cap B$ imply that $x \in (U \cap K^c) \cap (V \cap L^c) \subseteq A \cap B$ then $x \in (U \cap V) \cap (K^c \cap L^c) \subseteq A \cap B$ thus we get $x \in (U \cap V) - (K \cup L) \subseteq A \cap B$, by using (1) $U \cap V$ is r-open, since $K \cup L \subseteq X$ is compact set in X. Hence $A \cap B$ is coc -r -open.

4) Let A_α , $\alpha \in \Lambda$ be coc-r-open to prove $\cup_{\alpha\in\Lambda} A_\alpha$ is coc-r-open. Suppose $x \in \cup_{\alpha\in\Lambda} A_\alpha$, then $x \in A_\alpha$ for some $\alpha \in \Lambda$, since A_α is coc-r-open, thus there exist r-open sets $U_\alpha \subseteq X$ and compact subset K_α such that $x \in U_\alpha - K_\alpha \subseteq A_\alpha$ for some $\alpha \in \Lambda$, since $A_\alpha \subseteq \cup_{\alpha\in\Lambda} A_\alpha$. Hence $\cup_{\alpha\in\Lambda} A_\alpha$ is coc-r-open.

5) clear.

Definition (2.13) [10]

Let X be a space and $A \subseteq X$, a point $x \in A$ is called r-Interior point of A if there exists r - open set U in X containing x such that $x \in U \subseteq A$.

The set of all r-Interior points of A is called r-Interior set of A and it is denoted by $A^{\circ r}$.

Remark (2.14) [10]

Let X be a space and $A \subseteq X$, then $A^{\circ r} = \cup \{B : B$ is r-open set in X and $B \subseteq A \}$.

Definition (2.15) [10]

A space X is said to be r - compact if every r - open covering of X has a finite sub covering.

Proposition (2.16) [10]

1) Every compact space is r - compact space.
2) Every r-compact subset of T_2-space is r- closed set.
3) $A^{\circ r} \subseteq A^{\circ}$.

Theorem (2.17)

Let X be T_2-space, $A \subseteq X$, if A is a coc-r-open in X, then $A = A^{\circ r}$.

Proof :

Let A be coc -r-open in X, since $A^{\circ r} \subseteq A^{\circ} \subseteq A$, we need to prove that $A \subseteq A^{\circ r}$. Let $x \in A$, since A is coc-r-open, then there exist r-open U, compact subset K such that $x \in U - K \subseteq A$. Since every compact is r-compact and X be T_2-space, thus K is r- closed set (by using Proposition (2.16), (1), (2)) , so K^c r-open subset in X and $x \in U \cap K^c \subseteq A$ and U, K^c are r-open sets in X, there fore $U \cap K^c$ is r-open in X, hence $x \in A^{\circ r}$.

Remarks (2.18)

1) The coc-r-open sets forms topology on X denoted by τ^{rk} .

2) Every compact subset of T_2-space is closed set. **[9]**

3) A space X is regular space iff for every $x \in X$ and each open set U in X such that $x \in U$ there exists an open set W such that $x \in W \subseteq \overline{W} \subseteq U$. **[4]**

4) A space (X, T) is called T_3-space if X is regular space and T_1-space. **[4]**

5) Every T_3-space is T_2-space. **[4]**

Proposition (2.19) [10]

Let X be regular space, if $A \subseteq X$ is an open then $A \in RO(X, \tau)$.

Corollary (2.20)

Let X be regular space, if $F \subseteq X$ is a closed then $F \in RC(X, \tau)$.

Proof :

Clear.

Theorem (2.21)

Let (X,τ) be a T_2-space, then $\tau^{rk} \subseteq \tau$.

Proof :

Let $A \in \tau^{rk}$ to prove $A \in \tau$.

Let $x \in A$, then there exists r-open set $U \subseteq X$ and compact subset K such that $x \in U - K \subseteq A$, thus $x \in U \cap K^C \subseteq A$. Since K is compact and X is T_2-space, there fore K is closed, so K^C is open. By using remarks (2.18), (2), so $U \cap K^C$ is open set in X. Hence $A \in \tau$

Remarks (2.22)

Let (X,τ) be T_2-space, then

1) Every coc-r-open set is open set.

2) Every coc-r-closed set is closed set.

Proof :

Clear.

Theorem (2.23)

Let (X,τ) be regular-space, then $\tau \subseteq \tau^{rk}$.

Proof :

Clear, by using Proposition (2.19) and Remarks (2.10), (3).

Theorem (2.24)

Let (X,τ) be a T_3-space, then $\tau = \tau^{rk}$.

Proof :Clear

Definition (2.25)

Let X be a space and $A \subseteq X$. The intersection of all coc-r-closed sets of X containing A called coc-r-closure of A and is denoted by $\overline{A}^{coc-r}$, such that $\overline{A}^{coc-r} = \cap \{F : F$ coc-r - closed set in X and $A \subseteq F\}$

Remark (2.26)

$\overline{A}^{coc-r}$ is a smallest coc-r - closed set containing A.

Proposition (2.27)

Let X be a space and $A \subseteq B \subseteq X$. Then:

i. . $\overline{A}^{coc-r}$ is an coc-r - closed set .

ii. A is an coc-r - closed set if and only if $A = \overline{A}^{coc-r}$

iii. $\overline{A}^{coc-r} = \overline{\overline{A}^{coc-r}}^{coc-r}$

iv. $\overline{A}^{coc-r} \subseteq \overline{B}^{coc-r}$

Proof:

Clear.

Proposition (2.28)

Let X be a space and $A \subseteq X$. Then $x \in \overline{A}^{coc-r}$ iff for each coc-r - open set U in X contained point x we have $U \cap A \neq \phi$.

Proof:

Assume that $x \in \overline{A}^{coc-r}$ and U coc-r-open set U in X such that $x \in U$. Let $U \cap A = \phi$, then $A \subseteq U^c$, since U coc-r-open set U in X such that $x \in U$, thus U^c coc-r-closed set in X and $x \notin U^c$, since $\overline{A}^{coc-r}$ smallest coc-r-closed set containing A, there fore $\overline{A}^{coc-r} \subseteq U^c, x \notin U^c$. So $x \notin \overline{A}^{coc-r}$ this contradiction, hence $U \cap A \neq \phi$.

Conversely:

Assume $x \notin \overline{A}^{coc-r}$, then $x \in (\overline{A}^{coc-r})^c$, since $\overline{A}^{coc-r}$ coc-r-closed set in X, thus $(\overline{A}^{coc-r})^c$ coc-r-open set U in X and $\overline{A}^{coc-r} \cap (\overline{A}^{coc-r})^c = \phi$, there fore $A \cap (\overline{A}^{coc-r})^c = \phi$, this complete the proof.

Proposition (2.29)

Let X be space, $A, B \subseteq X$.

1- $\overline{\phi}^{coc-r} = \phi, \overline{X}^{coc-r} = X$.

2- $\overline{A \cup B}^{coc-r} = \overline{A}^{coc-r} \cup \overline{B}^{coc-r}$.

3- $\overline{A \cap B}^{coc-r} \subseteq \overline{A}^{coc-r} \cap \overline{B}^{coc-r}$.

Proof:

Clear.

Definition (2.30)

Let X be a space and $A \subseteq X$. The union of all coc-r-open sets of X containing in A is called coc-r-Interior of A denoted by $A^{\circ coc-r}$, such that $A^{\circ coc-r} = \cup \{U : U$ coc-r - open set in X and $U \subseteq A\}$.

Proposition (2.31)

Let X be a space and $A \subseteq X$, then $A^{\circ coc-r}$ is the largest coc-r-open set containing in A

Proof :

Clear by definition of $A^{\circ coc-r}$.

Proposition (2.32)

Let X be a space and $A \subseteq X$, then $x \in A^{\circ coc-r}$ if and only if there exists coc-r- open set U containing x such that $x \in U \subseteq A$.

Proof :

Let $x \in A^{\circ coc-r}$, then $x \in \cup_{\alpha \in \Lambda} V_\alpha$ such that V_α coc-r-open set and $V_\alpha \subseteq A$, $\alpha \in \Lambda$.Thus $x \in V_\alpha$ for some $\alpha \in \Lambda$, since $V_\alpha \subseteq A$ $\alpha \in \Lambda$, then $x \in U = V_\alpha \subseteq A$ for some $\alpha \in \Lambda$.

Conversely, let there exists U coc-r-open set such that $x \in U \subseteq A$ then $x \in \cup U$, $U \subseteq A$ and U coc-r- open set then $x \in A^{\circ coc-r}$.

Proposition (2.33)

Let X be a space and $A \subseteq B \subseteq X$ then .

1- $A^{\circ coc-r}$ is coc-r- open set.

2- A is coc-r-open if and only if $A = A^{\circ coc-r}$.

3- $A^{\circ coc-r} = (A^{\circ coc-r})^{\circ coc-r}$.

4- if $A \subseteq B$ then $A^{\circ coc-r} \subseteq B^{\circ coc-r}$.

5- $A^{\circ coc-r} \cup B^{\circ coc-r} \subseteq (A \cup B)^{\circ coc-r}$.

6- $A^{\circ coc-r} \cap B^{\circ coc-r} = (A \cap B)^{\circ coc-r}$.

Proof :

Clear.

Definition (2.34)

Let Y be subspace of a space X. A subset B of a space X is said to be coc-r-open set in Y if for every $x \in B$ there exists a r-open set U in Y and a compact subset K in Y such that $x \in U - K \subseteq B$.

Theorem (2.35)

Let Y be subspace of a space X. If Y is an open set in X and $U \subseteq Y$, then U is a r-open set in Y if and only if U is a r-open set in X.

Proof :

Let $U \subseteq Y \subseteq X$, Y be an open set in X and U be a r-open set in Y then $U = \overline{U}^{Y^{\circ Y}} = (\overline{U} \cap Y)^{\circ Y} = \overline{U}^{\circ Y} \cap Y^{\circ Y} = \overline{U}^{\circ Y} \cap Y^{\circ} = \overline{U}^{\circ}$, hence U is a r-open set in X. Conversely, let U be a r-open set in X, then $U = \overline{U}^{\circ} = \overline{U}^{\circ Y} \cap Y^{\circ} = \overline{U}^{\circ Y} \cap Y^{\circ Y} = (\overline{U} \cap Y)^{\circ Y} = \overline{U}^{Y^{\circ Y}}$, hence U is a r-open set in Y.

Definition (2.36) [6]

A subset S of a topological space (X, τ) is said to be clopen if it is both open and closed in (X, τ).

Remarks (2.37)

1) In any space, the intersection of compact set with a closed set is compact **[9].**

2) Every clopen set is r-open set **[6].**

Theorem (2.38)

Let Y be a subspace of a space X, $B \subseteq Y$. If Y is a clopen set in X then B is a coc-r-open set in Y if and only if B is a coc-r-open set in X.

Proof :

Let B be a coc-r-open set in Y and $x \in B \subseteq Y$ then there exists a r-open set U_x in Y and a compact subset K_x in Y such that $x \in U_x - K_x \subseteq B$. Since Y is a clopen set in X then Y is an open set in X, thus U_x is a r-open set in X (Theorem (2.35)), therefore $U_x - K_x$ is a coc-r-open set in X for all $x \in B$. Put $V = \cup_{x \in B} (U_x - K_x)$, thus V is a coc-r-open set in X. Now, we need to prove $B = V$, since $U_x - K_x \subseteq B$ for all $x \in B$ then $V \subseteq B$, let $y \in B$, thus there exists a r-open set U_y in Y and a compact subset K_y in Y such that $y \in U_y - K_y \subseteq B$, therefore $y \in \cup_{x \in B} (U_x - K_x) = V$, so that $B \subseteq V$. Hence $B = V$.

Conversely, let $x \in B$ then there exists a r-open set U in X and a compact subset K in X such that $x \in U - K \subseteq B$, since Y is a clopen set in X, then Y is a r-open set in X (Remarks (2.37), (2)), thus $U \cap Y$ is a r-open set in X, since $U \cap Y \subseteq Y$ and Y is an open set in X, there fore $U \cap Y$ is a r-open set in Y (Theorem (2.35)). Now, since K is a compact in X and Y is a closed in X, so $K \cap Y$ is a compact in X (Remarks (2.37), (1)) and $K \cap Y \subseteq Y$, hence $K \cap Y$ is a compact in Y. Since $x \in U - K$ then $x \in U$ but $x \notin K$, thus $x \in U \cap Y$ but $x \notin K \cap Y$, there fore $x \in (U \cap Y) - (K \cap Y) \subseteq (U - K) \cap Y \subseteq B$. Hence B is a coc-r-open set in Y.

Corollary (2.39)

Let Y be a subspace of a space X, $F \subseteq Y$. If Y is a clopen set in X then F is a coc-r-closed set in Y if and only if F is a coc-r-closed set in X.

Proof :

Let F is a coc-r-closed set in Y then F^c is a coc-r-open set in Y, thus F^c is a coc-r-open in X(Theorem (2.38)), there fore F is a coc-r-closed set in X.

Conversely, let F is a coc-r-closed set in X then F^c is a coc-r-open set in X, thus F^c is a coc-r-open in Y (Theorem (2.38)), there fore F is a coc-r-closed set in Y.

3. Coc-r-compact space.

In this section, we are introduces the definition of coc-r-compact, compact, I-compact spaces and give salutary characterizations of this concepts. And the relationship between them.

Definition (3.1) [2]

A space X is said to be compact if every open cover of X has finite sub cover.

Definition (3.2)

A space X is said to be coc-r-compact if every coc-r-open covering of X has a finite sub covering.

Examples (3.3)

The following are straight forward examples of coc-r-compact spaces.

1) Any finite topological space is coc-r-compact space.

2) Let $X = \{1,2,3, \dots\}, \tau = \{G \subseteq X: 1 \in G\} \cup \{\emptyset\}$, then $\tau^{rk} = \{G \subseteq X: G^c \text{ is finite}\} \cup \{\emptyset\}$, then X is coc-r-compact space.

Remark (3.4)

1) Compact $\not\longrightarrow$ coc-r-compact.

2) Coc-r-compact $\not\longrightarrow$ compact.

Examples (3.5)

1) Let $X = Q$, with indiscrete topology, then $\tau^{rk} = \{A: A \subseteq X\}$, thus X is compact but X is not coc-r-compact.

2) Let $X = \{ 1,2,3, \dots\}, \tau = \{G \subseteq X: 1 \in G\} \cup \{\emptyset\}$, then$\tau^{rk} = \{G \subseteq X: G^c \text{ is finite}\} \cup \{\emptyset\}$, thus X is coc-r-compact but X is not compact.

Proposition (3.6)

If X is T_2-space, then every compact space is coc-r-compact space.

Proof :

It is clear to show that, since in T_2-space every coc-r-open is open set in X.

Proposition (3.7)

If X is regular space, then every coc-r-compact space is compact space.

Proof :

Clear, by using Proposition (2.19), remarks (2.10), (3).

Definition (3.8)

A subset B of a topological space X is said to be coc-r-compact relative to X if every cover of B by coc-r-open sets in X has a finite subcover of B.

The subset B is coc-r-compact in X iff it is coc-r-compact as a subspace.

Definition (3.9)

A space X is called coc-r-T_2-space (coc-r-Hausdorff) if and only if for each $x \neq y \; in \; X$ there exist $U \; and \; V$ disjoint coc-r-open sets such that $x \in U$, $y \in V$.

Remarks (3.10)

1) The space (X, τ) is coc-r-compact iff (X, τ^{rk}) is compact.

2) The space (X, τ) is coc-r-T_2-space iff (X, τ^{rk}) is T_2-space.

3) The subset $B \subseteq X$ is coc-r- closed in (X, τ) iff B closed in (X, τ^{rk}).

Theorem (3.11) [9]

A closed subset of compact space X is compact relative to X.

Proposition (3.12)

1) A coc-r-closed subset of coc-r-compact space X is coc-r-compact relative to X.

2) In any space, the intersection of coc-r-compact set with a coc-r-closed set is coc-r-compact.

3) Every coc-r-compact subset of coc-r-T_2-space is coc-r-closed set.

Proof :
(1) Clear by using remarks (3.10), (1), (3) and theorem (3.11).
(2) Clear by using remarks (3.10), (1), (3) and remarks (2.37), (1).
(3) Clear by using remarks (3.10), (1), (2) and remarks (2.18),(2).

Corollary (3.13)
Every r-closed of coc-r-compact space X is coc-r-compact relative to X.
Proof :
Clear.

Proposition (3.14)
If X is a topological space such that every coc-r-open subset of X is coc-r-compact relative to X, then every subset is coc-r-compact relative to X.
Proof :
Let G be an arbitrary subset of X, $\{U_\alpha : \alpha \in \Lambda\}$ be cover of G by coc-r - open subsets, then the family $\{U_\alpha : \alpha \in \Lambda\}$ is a coc-r-open cover of the coc-r-open set $\cup \{U_\alpha : \alpha \in \Lambda\}$. Thus by assumption there is a finite sub family $\{U_{\alpha i} : i = 1,2, \dots, n\}$ which covers $\cup \{U_\alpha : \alpha \in \Lambda\}$, since $G \subseteq \cup \{U_\alpha : \alpha \in \Lambda\} \subseteq \cup \{U_{\alpha i} : i = 1,2, \dots, n\}$, hence G is coc-r-compact.

Theorem (3.15)
Let Y be a subspace in X, X is coc-r-compact, if Y clopen set, then Y is coc-r- compact.
Proof :
Let Y be a subspace in X, $\{U_\alpha : \alpha \in \Lambda\}$ be cover of Y by coc-r-open subsets of Y such that $Y \subseteq \cup \{U_\alpha : \alpha \in \Lambda\}$, since U_α is coc-r-open in Y, Y is clopen set in X, then U_α is coc-r-open in X for all $\alpha \in \Lambda$ (by using theorem (2.38)). Thus X $= Y \cup Y^c \subseteq \cup \{U_\alpha : \alpha \in \Lambda\} \cup Y^c \subseteq \cup \{U_\alpha \cup Y^c : \alpha \in \Lambda\}$, since Y is clopen set in X, then Y is r-closed, thus Y is coc -r-closed, there fore Y^c is coc -r-open in X. Since X is coc-r-compact, then $X \subseteq \cup \{U_{\alpha i} \cup Y^c : i = 1,2, \dots, n\}$, so that $Y = X \cap Y \subseteq \cup \{U_{\alpha i} \cup Y^c : i = 1,2, \dots, n\} \cap Y = \cup \{U_{\alpha i} \cap Y : i = 1,2, \dots, n\} = \cup \{ U_{\alpha i} : i = 1,2, \dots, n\}$, hence Y is coc-r-compact.

Theorem (3.16)
If X is coc-r- compact space, then every r-open covering of X has a finite sub covering.
Proof :
Clear

Remark (3.17)
The convers of Theorem (3.16) is not true.

Example (3.18)
In Examples (3.5), (1), all r-open covers are $\{\emptyset, X\}$, and it is finite cover of X, but X is not coc-r- compact space.

Theorem (3.19)
If X is T_2-space, then the following statements are equivalent.
i) X is coc-r-compact.
ii) Every cover of X by r- open subsets has a finite subcover.
Proof :
(i) ⟶ (ii) Clear.

(ii) ⟶ (i)
Let $\mathcal{U}$ be coc-r-open cover of X, then $X \subseteq \cup \{U : U \in \mathcal{U}\}$, since X is T_2-space, thus U is equal to the union of r-open sets in X contained in U for each $U \in \mathcal{U}$ (by using theorem (2.17)). There fore all r-open sets in U for each $U \in \mathcal{U}$ are r-open cover of X, this r-open cover has a finite subcover. Since every element of this a finite subcover contained in U for some $U \in \mathcal{U}$, hence $\mathcal{U}$ has a finite subcover.

Theorem (3.20)
If X is T_2-space, then the following statements are equivalent.
i) Every proper r- closed subset of X is coc-r-compact relative to X.
ii) X is coc-r- compact.
iii) X is r- compact.
Proof :
(i) ⟶ (ii)
Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of X by r - open subsets of X such that $X \subseteq \cup \{U_\alpha : \alpha \in \Lambda\}$. If $U_\lambda = X, \lambda \in \Lambda$ then the proof is complete, if $U_\lambda \neq X, \lambda \in \Lambda$ then $X - U_\lambda$ is proper

r- closed subset and $X-U_\lambda \subseteq \cup\{U_\alpha : \alpha \in \Lambda-\{\lambda\}\}$, by the hypothesis there exist a finite subfamily $\{U_{\alpha i} : \alpha i \in \Lambda-\{\lambda\}, i=1,2,\dots,n\}$, such that $X-U_\lambda \subseteq \cup\{U_{\alpha i} : \alpha i \in \Lambda-\{\lambda\}, i=1,2,\dots,n\}$, thus $X \subseteq \cup\{U_{\alpha i} \cup U_\lambda : \alpha i \in \Lambda-\{\lambda\}, i=1,2,\dots,n\}$, hence X is coc-r-compact.

(ii) ⟶ (iii)

Clear, by using Theorem (3.19), Definition (2.16).

(iii) ⟶ (i)

Suppose F be proper r- closed subset of X, then $F \neq X$, let $\{U_\alpha : \alpha \in \Lambda\}$ be cover of F by r - open subsets of X, since F is r- closed subset of X, thus F^c is r-open, since $F \cup F^c \subseteq \cup\{U_\alpha : \alpha \in \Lambda\} \cup F^c$, there fore $\{U_\alpha, F^c : \alpha \in \Lambda\}$ is r-open cover of X and X is r-compact, so $X \subseteq \cup\{U_{\alpha i} : i=1,2,\dots,n\} \cup F^c$, hence $F \subseteq \cup\{U_{\alpha i} : i=1,2,\dots,n\}$.

Definition (3.21) [3]

A space (X, T) is called I-compact if every cover $\mathcal{F}$ of X by r - closed subsets of the space (X, T) contains a finite subcover $\mathcal{L}$ such that $X = \cup\{F^\circ : F \in \mathcal{L}\}$.

Remark (3.22)

coc-r-compact $\not\longleftrightarrow$ I-compact.

Examples (3.23)

1) Let $X = R$, with indiscrete topology, then $\tau^{rk} = \{A : A \subseteq X\}$, thus X is I-compact but X is not coc-r-compact.

2) Let $X = \{1,2,3,\dots\}, \tau = \{G \subseteq X : 1 \notin G\} \cup \{X\}$, then $\tau^{rk} = \{G \subseteq X : 1 \notin G\} \cup \{G \subseteq X : 1 \in G, G^c \text{ is finite}\}$, thus X is coc-r-compact but is not I-compact because $\{\{1, x\} : 1 \in X, x \neq 1\}$ is r-closed cover of X but has not a finite subcover and $\{1, x\}^\circ = \{x\}, x \neq 1$.

Definition (3.24) [5]

A space (X, T) is called extremally disconnected (e.d) if $\overline{U}$ is open for each open set U in X.

Remark (3.25) [5]

A space X is e.d iff for all $U, V \in RO(X, \tau)$ with $U \cap V = \emptyset$, then $\overline{U} \cap \overline{V} = \emptyset$.

Proposition (3.26) [4]

Let X be a topological space, $A \subseteq X$, then:

1- If A a closed set, then A° is a r-open set.

2- If A an open set, then $\overline{A}$ is a r-closed set.

3- If A a r-closed set, then A is closed set.

Remark (3.27) [5]

If a topological space X is e.d space, then every r-closed set in X is an open set.

Theorem (3.28)

If a topological space X is e.d space, then every coc-r-compact is I-compact.

Proof :

Let $\{F_\alpha : \alpha \in \Lambda\}$ be r-closed cover of X, then F_α is closed for each $\alpha \in \Lambda$, thus ${F_\alpha}^\circ$ is r-open for each $\alpha \in \Lambda$ (by using Proposition (3.26), (1,3)). Since F_α is r-closed for each $\alpha \in \Lambda$ and X is e.d space, there fore F_α is open set in X for each $\alpha \in \Lambda$ (by using Remark (3.27)), so F_α is r-open, then F_α is coc- r-open set in X for each $\alpha \in \Lambda$. Since X is coc-r-compact, thus the cover $\{F_\alpha : \alpha \in \Lambda\}$ has a finite subcover such that $X = \cup\{F_{\alpha i} : i=1,2,\dots,n\} = \cup\{{F_{\alpha i}}^\circ : i=1,2,\dots,n\}$. Hence X is I-compact.

Theorem (3.29)

If a topological space X is T_2-space, then every I-compact is coc-r-compact.

Proof :

Let $\{U_\alpha : \alpha \in \Lambda\}$ be r-open cover of X, then U_α is open and $\overline{U_\alpha}$ is a r-closed set in X for each $\alpha \in \Lambda$ (by using Proposition (3.26), (2)), thus $\{\overline{U_\alpha} : \alpha \in \Lambda\}$ is r-closed cover of X and X I- compact, there fore this cover has a finite sub cover such that $X = \cup\left\{\overline{U_{\alpha i}}^\circ : i=1,2,\dots,n\right\} = \cup\{U_{\alpha i} : i=1,2,\dots,n\}$. Hence X is coc-r-compact (by using Theorem (3.19)).

Theorem (3.30)

If X is T_3, e.d space then the following statements are equivalent.

i) X is compact.

ii) X is I-compact.

iii) X is coc-r-compact.

Proof :

(i) ⟶(ii)

Let $\{F_\alpha : \alpha \in \Lambda\}$ be r-closed cover of X, since X e.d space, then $\{F_\alpha : \alpha \in \Lambda\}$ is an open cover of X (by using Remark (3.27)) and X is compact, thus $\{F_\alpha : \alpha \in \Lambda\}$ has finite sub cover such that $X = \cup\{F_{\alpha i} : i = 1,2,\dots,n\} = \cup\{F_{\alpha i}{}^\circ : i = 1,2,\dots,n\}$. Hence X is I-compact.

(ii) ⟶(iii) Clear .

(iii)⟶ (i) Clear.

Proposition (3.31)

If a topological space X is T_2-space, then every r-closed set of I-compact space is coc-r-compact relative to X.

Proof :

It is clear by using theorem (3.29),Corollary (3.13).

Definition (3.32)

A subset B of a topological space X is said to be I-compact relative to X if every cover $\mathcal{F}$ of B by r- closed sets in X has a finite subcover $\mathcal{L}$ such that $B \subseteq \cup\{F^\circ : F \in \mathcal{L}\}$.

Proposition (3.33)

If a topological space X is e.d space, then every r-open set of I-compact space is I-compact relative to X.

Proof :

Let X be e.d space, U be r-open in X and $\{F_\alpha : \alpha \in \Lambda\}$ cover of U by r-closed subsets of X such that $U \subseteq \cup\{F_\alpha : \alpha \in \Lambda\}$, then $U \cup U^c \subseteq \cup\{F_\alpha \cup U^c : \alpha \in \Lambda\}$, thus $X \subseteq \cup\{F_\alpha \cup U^c : \alpha \in \Lambda\}$, U^c is r-closed. Since X is I-compact space, there fore the cover $\{F_\alpha \cup U^c : \alpha \in \Lambda\}$ has a finite sub cover such that $X \subseteq \cup\{(F_{\alpha i} \cup U^c)^\circ : i = 1,2,\dots,n\}$, since X be e.d space, so $F_{\alpha i}$, U^c is open set (Remark (3.27)) for each $i = 1,2,\dots,n$, then $X \subseteq \cup\{F_{\alpha i} \cup U^c : i = 1,2,\dots,n\}$, thus $U \subseteq \cup\{F_{\alpha i} \cap U : i = 1,2,\dots,n\} \subseteq \cup\{F_{\alpha i} : i = 1,2,\dots,n\} = \cup\{F_{\alpha i}{}^\circ : i = 1,2,\dots,n\}$. Hence U is I-compact relative.

Corollary (3.34)

If a topological space X is e.d space, then every r-open set of coc-r-compact space is coc-r-compact relative.

Proof :

It is clear by using theorem (3.28), Proposition (3.33).

Definition (3.35)

Let $f: X \longrightarrow Y$ be a function of a space X into a space Y, then f is called coc-r- irresolute (coć-r-continuous) function if $f^{-1}(U)$ coc-r-open set in X for each coc-r-open set U in Y.

Definition (3.36)

Let $f: X \longrightarrow Y$ be a function of space X into space Y, then f is called coć-r-open function if f(U) is coc-r-open set in Y for each coc-r-open set U in X.

Proposition (3.37)

Let $f: X \longrightarrow Y$ be a coć-r-continuous function, onto, if X is coc-r-compact then Y coc-r-compact.

Proof :

Let $\{U_\alpha : \alpha \in \Lambda\}$ be coc-r-open cover of Y, since f is a coć-r-continuous function, then $f^{-1}(U_\alpha)$ is coc-r - open in X for each $\alpha \in \Lambda$, but $Y \subseteq \cup_{\alpha\in\Lambda} U_\alpha$, thus $X = f^{-1}(Y) \subseteq \cup_{\alpha\in\Lambda} f^{-1}(U_\alpha)$, since X is coc-r-compact and $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ forms a cover of X, therefore the cover $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ has a finite subcover such that $X \subseteq \cup\{f^{-1}(U_{\alpha i}), : i = 1,2,\dots,n\}$, since f onto, so $f(X) = Y \subseteq \cup\{f(f^{-1}(U_{\alpha i})) : i = 1,2,\dots,n\} \subseteq \cup\{U_{\alpha i} : i = 1,2,\dots,n\}$. Hence Y coc-r-compact.

Proposition (3.38)

Let $f: X \longrightarrow Y$ be a coć-r-open function and bijective, if Y is coc-r-compact then X coc-r-compact.

Proof :

Let $\{U_\alpha : \alpha \in \Lambda\}$ be coc-r-open cover of X, since f is a coć-r-open function, then $f(U_\alpha)$ is coc-r - open in Y for each $\alpha \in \Lambda$, but $X \subseteq \cup_{\alpha\in\Lambda} U_\alpha$, there fore $Y = f(X) \subseteq \cup_{\alpha\in\Lambda} f(U_\alpha)$, so $\{f(U_\alpha) : \alpha \in \Lambda\}$ forms a cover of Y, since Y is coc-r-compact, then the cover $\{f(U_\alpha) : \alpha \in \Lambda\}$ has a finite subcover such that $Y \subseteq \cup\{f(U_{\alpha i}) : i = 1,2,\dots,n\}$, thus $X = f^{-1}(Y) \subseteq \cup\left\{f^{-1}\left(f(U_{\alpha i})\right) : i = 1,2,\dots,n\right\} = \cup\{U_{\alpha i} : i = 1,2,\dots,n\}$. Hence X coc-r-compact.

Definition (3.39)

Let $f: X \longrightarrow Y$ be a function of a space X into a space Y, then f is called coc-r-continuous function if $f^{-1}(U)$ coc-r-open set in X for each open set U in Y.

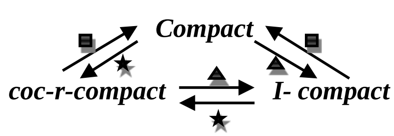
Proposition (3.40)

Let $f: X \longrightarrow Y$ be a coc-r-continuous function, onto and Y be e.d space, if X is coc-r-compact then Y I-compact.

Proof :

Let $\{F_\alpha : \alpha \in \Lambda\}$ be r-closed cover of Y and Y be e.d, then F_α is open in Y for each $\alpha \in \Lambda$ (Remark (3.27)), since f is a coc-r-continuous function, thus $f^{-1}(F_\alpha)$ is coc-r - open in X for each $\alpha \in \Lambda$, but $Y \subseteq \cup_{\alpha\in\Lambda} F_\alpha$, thus $X = f^{-1}(Y) \subseteq \cup_{\alpha\in\Lambda} f^{-1}(F_\alpha)$, since X is coc-r-compact and $\{f^{-1}(F_\alpha) : \alpha \in \Lambda\}$ forms a cover of X, there fore the cover $\{f^{-1}(F_\alpha): \alpha \in \Lambda\}$ has a finite subcover suchthat $X \subseteq \cup \{f^{-1}(F_{\alpha i}), : i = 1,2, \dots , n \}$, since f onto, so $f(X) = Y \subseteq \cup \left\{f\left(f^{-1}(F_{\alpha i})\right): i = 1,2, \dots , n \right\} \subseteq \cup \{F_{\alpha i} : i = 1,2, \dots , n \} = \cup \{F_{\alpha i}{}^{\circ} : i = 1,2, \dots , n \}$. Hence Y I-compact.

The following diagram explains the relationship among these types of compact spaces.



▲ *e.d space*

■ *Regular space*

★ *T_2-space*

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Journal of AL-Qadisiyah for computer science and mathematics
Vol.9 No.1 Year 2017

حول الفضاءات coc-r-compact

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كلية علوم الحاسوب وتكنولوجيا المعلومات

المستخلص :

في ورقة العمل هذه، سوف ندرس مجموعه جديدة من المجموعات مفتوحة تسمى مجموعة coc-r-open وسندرس أيضا خصائصها، كما سندرس coc-r-compact, I-Compact والعلاقة بينهما، حيث حصلنا على بعض النتائج التي تظهر العلاقة بين تلك الفضاءات من خلال بعض النظريات التي تم الحصول عليها باستخدام مجموعات coc-r-open .