Single Machine Scheduling To Minimize a Function of Square Completion Time and Maximum Earliness Simultaneously

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Abstract

In this study, to minimize a function of two cost criteria for scheduling n jobs on a single machine , the problem is discussed :

“ **Minimizing a function of total square completion time and maximum Earliness simultaneously”.**

For this problem we proposed some algorithms to find exact(optimal) solution for hierarchical case and efficient (pareto optimal) solutions for simultaneous case. Also we proposed branch and bound algorithm to find exact solution for sum of total square completion time and maximum Earliness ,and present algorithm D to find exact solution in a fast way with respect to (BAB) method. We present computational experience for the (BAB) method and algorithm(D) on a large set of test problems.

المستخلص

في هذه الدراسة ولتصغير دالة الكلفة لمعيارين والحاصلة من جدولة n من الاعمال على ماكنة واحدة درست المسألة:

تصغير الدالة $F(\sum C_i^2 , E_{max})$ حيث ان E_{max} هي irregular measure

في هذه المسألة اقترحنا بعض الخوارزميات لايجاد الحل الامثل في حالة الـ (hierarchical) والحلول الكفوءة في حالة الـ (simultaneous) . وكذلك اقترحنا خوارزمية للـ (BAB) لايجاد الحل الامثل للمسألة (P4) . وقدمنا ايضاً خوارزمية D لايجاد الحل الامثل للمسألة (P4) ولكن بطريقة اسرع من خوارزمية(BAB) . وقدمنا حسابات الاختبارات لخوارزميات BAB و D والتي تم تنفيذها على مجموعة كبيرة من المسائل.

Introduction:

It is well known that the optimal solution of single objective models can be quite different if the objective is different (for instance, for the simplest model of one machine, without any additional constraint, the rule SPT is optimal to minimize flow time but the rule MST is optimal to minimize the maximal earliness E_{max}).

In fact, often each particular decision maker wants to minimize a given criterion.

Recently, research on more than one criterion scheduling has increased. Since real life scheduling problems may require the decision maker to consider a number of criteria before arriving at any decision. Nagar et al. [24] in their detailed literature survey of multiple and bi-criteria problems in scheduling point out the importance of this subject.

Because, the one-machine problem provides a useful laboratory for the development of ideas for heuristics and interactive procedure that may prove to be useful in more general models. We consider the one-machine case in this study.

Multi-Criteria Scheduling Problem and approaches

In general, multiple-criteria scheduling refers to the scheduling problem in which the advantages of a particular schedule are evaluated using more than one performance criterion. The managerial relevance of considering multiple criteria for scheduling has been cited in the production and operations management literature since the 1950's. Smith (1956)[26] shows that the choice of a criterion will affect the characteristics of a "best schedule"; different optimizing criteria will result in very different schedules. Van Wassenhove and Gelders (1980)[29] and provide evidence that a schedule that performs well using a certain criterion might yield a poor result using other criteria. Hence, lack of consideration of various criteria may lead to solutions that are very difficult to implement in practice. Although the importance of multi-criteria scheduling has been recognized for many years (French, 1982[7]; Nelson et al., 1986[25]; George S., and Paul S. 2007[8]), little attention has been given in the literature to this topic. From the problem complexity perspective, the multiple-criteria problem becomes much more complex than related single-criteria counterparts (Lenstra et al., 1975[22]) Nagar et al. (1995)[24] reviews the problem in its general form whereas Lee and Vairaktarakis (1993)[21] review a special version of the problem, where one criterion is set to its best possible value and the other criterion is tried to be optimized under this restriction. Hoogeveen (2005)[11] studies a number of bi-criteria scheduling problems.

Also, there are some papers about this object (Cheng et al. 2008[5], and Azizoglu et al. 2003 [1]). In literature there are two approaches for the bi-criteria problems: the hierarchical approach and the simultaneous approach. In the hierarchical approach, one of the two criteria is considered as the primary criterion and the other one is considered as the secondary criterion. The problem is to minimize the primary criterion while breaking ties in favor of the schedule that has the minimum secondary criterion value. The studies by Chang P. and Su L.(2001)[3] and Chen W., et al.(1997)[4] are examples of hierarchical minimization problems with earliness and tardiness costs. The computational complexity results in hierarchical minimization are reviewed in Lee and Vairaktarakis (1993)[21]. In the simultaneous approach there are two types ,the first one typically generates all efficient schedules and selects the one that yields the best composite objective function value of the two criteria .The second is to find sum of these objectives .Several scheduling problems considering the simultaneous minimization of various forms of earliness and tardiness costs have been studied in the literature (see, e.g. Hoogeveen, (1995)[13]; Moslehi, et al. (2005)[23]; Emmons(1975)[6]) .

Basic definitions:

Definition(1):[16] *The term "optimize" in a multi-objective decision making problem refers to a solution around which there is no way of improving any objective without worsening at least one other objective.*

Definition(2):[16] *Suppose we have a problem P ,any schedule* $S \in \delta$ *(where* δ *is the set of all schedules) is said to be feasible if it satisfies the constraints of the problem P.*

Definition(3):[11]. *A schedule S is said to be efficient if there does not exist another schedule* $S^{/}$ *satisfying* $f_i(S^{/}) \le f_i(S)$ *,* $i=1,\dots,k$ *with at least one of the above holding as a strict inequality. Otherwise S is said to be dominated by* $S^{/}$*.*

Definition(4):[24] *A measure of performance is said to be regular if it is a non-decreasing function of job completion times and the scheduling objective is to minimize the performance measure. Examples of regular measures are job flowtime (*$\bar{F}$*), schedule makespan (*C_{max}*) and tardiness based performance measures.*

Definition(5):[24] *A non-regular performance measure is usually not a monotone function of the job completion times. An example of such a measure is job earliness.*

Definition (6):[11] *The function F(f,g) is said to be non-decreasing in both argument ,if for any pair of outcome value (x,y) of the functions f and g ,we have* $F(x,y) \le F(x+A,y+B)$ *for each pair of non-negative value A and B.*

Theorem (1):[11] If the composite objective function F(f,g) is non-decreasing in both argument ,then there exists a pareto optimal schedule that minimize F.■

Basic Scheduling Concepts

We start with introducing some important notation where we concentrate on the performance criteria with out elaborating on the machine environment etc. We assume that there are n jobs, which we denoted by $j_1,\ldots,j_n$ these jobs are to be scheduled on a set of machines that are continuously available from time zero on words and that can handle only one job at a time .

In this paper, we only state here the notation that is used for single machine , jobs $J_i(i=1,\ldots,n)$ has:

N: set of jobs.

n: The number of jobs in a known sequence.

$\mathbf{P}_j$: which means that it has to processed for a period of length p_j .

$\mathbf{d}_j$: a due date ,the date when the jobs should ideally be completed , the completion of job after its due date is allowed ,but a penalty is incurred . When due date is constant for all jobs ,then called common due date.

$\mathbf{r}_j$: a release date of job j ,i.e. the earliness time at which the processing of job can begin.

- The completion time C_j
- The lateness $L_j=C_j -d_j$
- The earliness $E_j=\max\{ 0, d_j-C_j\}$

For a given schedule σ we compute.

- $C_{max}(\sigma) =\max_j(C_j)$
- $L_{max}(\sigma)=\max_j(L_j)$
- $E_{max}(\sigma)=\max_j(E_j)$

Fundamental Results and Algorithms:

Theorem (2)(Smith_1956)[26]. The $1/\ /\ \Sigma C_i$ problem is minimized by sequencing the jobs according to the shortest –processing-time (SPT) rule, that is, in order of non-decreasing p_i.■

Theorem(3)(Jackson 1955)[15]. The $1/\ /\ L_{max}$ problem is minimized by sequencing the jobs according to the earliest-due- date (EDD) rule, that is, in order of non-decreasing d_i.■

Theorem(4)(Lawler 1973)[20].The $1//f_{max}$ problem, f_{max} is minimized as follows: while there are unassigned jobs, assign the job that has minimum cost when scheduled in the last unassigned position in that position.■

Hoogeveen and Van de Velde [13] provide a generalization to the case that the two criteria are $\sum C_j$ and f_{max} where f_{max} is regular cost function.

Theorem(5)[12**].** The $1/\ /E_{max}$ problem is solved by sequencing the jobs according to the minimum slack time (MST) rule ,that is ,in order of non-decreasing d_i-p_i .■

Van Wassenhove and Gelder [29]propose a pseudo-polynomial algorithm for finding all efficient schedules with respect to $\sum C_j$ and L_{max} .Their algorithm searches all possible L_{max} values .Since a given L_{max} value imposes job dead line $\bar{d}_j$,the algorithm of Smith [26] is used to solve the corresponding $1/\bar{d}_j/\sum C_j$ problem.

The Problem Classification:

In this study , we adopt the terminology of Graham ,Lawler ,and Rinnooy Kan _(1979) [9] to classify scheduling problems.

Suppose that m machines M_i (i=1,…,m) have to process n jobs J_j $(_{j=1,\dots,n})$. A schedule problem type can be specified using a three-field classification α/β/γ composed of the machine environment, the job characteristics, and the optimality criterion .

Minimizing Total Square Completion Time

This section deals with the Quadratic problem of scheduling jobs on a single machine such that the sum of the square of the weighted completion times of jobs is minimized(i.e. $1 // \sum_{i=1}^{n} w_i C_i^2$ problem) . Relatively little work has been done on problems involving a quadratic measure of performance for scheduling a single machine. The single machine scheduling problem with the objective of minimizing the sum of squares of the job completion times has been studied by Townsend (1978)[28], Bagga and Kalra (1980)[2], Gupta and Sen (1984)[10], and Szwarc, Posner, and Liu (1988)[27]). Townsend [28] first formulated the problem and presented a branch-and-bound search method to solve it. Bagga and Kalra [2] improved the method by providing conditions for precedence among set of jobs. If w_i =1 for every $_i$, then the resulting problem $1 // \sum_{i=1}^{n} C_i^2$ is solved by the following proposition.

Proposition(1):[17] The SPT rule gives an optimal value for $1// \sum_{i=1}^{n} C_i^2$ problem.

Minimizing Total Square Completion Time and Maximum Cost

Now, we will consider the bi-criteria single machine problem concerns the simultaneous minimization of the performance measure total square completion time $\sum_{i=1}^{n} C_i^2$ and maximum cost f_{max} (i.e. $1//F(\sum_{i=1}^{n} C_i^2, f_{max})$ problem).Maximum cost is defined as $\max_{1 \le i \le n}$ $\{f_i(C_i)\}$, where each f_i denotes an arbitrary regular or irregular cost function for job i; regular means that $f_i(C_i)$ does not decrease when C_i is increased .

The $1/ / F (\sum_{i=1}^{n} C_i^2, f_{max})$problem is described as follows. A set of n independent jobs has to be scheduled on a single machine that is continuously available from time zero on wards and that can process at most one job at a time. Each job J_j *(j =1, ..., n)* requires an uninterrupted positive processing time p_j and has a due date d_j. Without loss of generality, we assume that the processing times and due dates are integral. A *schedule σ* specifies for each job when it is executed while observing the machine availability constraints. Hence, a schedule σ defines for each job J_j its square of completion time C_j^2 *(σ),* which we sometimes simply write as C_j^2 .

The bi-criteria problem that we consider concerns the simultaneous minimization of the performance measures ***total square completion time*** and ***maximum cost*** f_{max}.

Hoogeveen and Van de Valde [14] find all efficient solution for $1//F(\sum C_i, E_{max})$ problem on the range $[E_{max}(MST), E_{max}(SPT)]$, Kokasalan and Ahmet [18],Kurz and Canterbury[19] used genetic algorithm (GA) to find all efficient solution for $1//F(\sum C_i, E_{max})$ problem. Let $f_{max} = E_{max}$ in our study ,since criterion E_{max} is a particular case of the function f_{max} .

Note that the minimum slack time (MST) rule ,if no idle time is allowed , E_{max} is minimized by sequencing the jobs in order of non decreasing values of $s_i=d_i-p_i$ .Since E_{max} is irregular function then we choose only the value of E_{max} in the range $[E_{max}(MST),E_{max}(SPT)]$.

Now , consider the following two problems:

$1//Lex(\sum_{i=1}^{n} C_i^2, E_{max})$ problem ,and $1//Lex(E_{max}, \sum_{i=1}^{n} C_i^2)$ problem.

The first problem $1//Lex(\sum_{i=1}^{n} C_i^2, E_{max})$

This problem can be written as:

$$\left.\begin{aligned} &\text{Min } E_{max} \\ &\text{s.t.} \\ &\sum_{i=1}^{n} C_i^2 = C^* \quad \text{,where } C^*=\sum C_i^2 (SPT) \end{aligned}\right\} \ldots(P1)$$

Algorithm for problem(P1):

Step(0): Order the jobs by SPT rule and calculate $\sum_{i=1}^{n} C_i^2$ and E_{max}.

Step(1): If there exist a tie(jobs with the same processing times) order these jobs by MST rule to minimize E_{max} .

The problem (P1) can be written as: $1/ \sum_{i=1}^{n} C_i^2 = C^* / E_{max}$.

Example(1): Consider the problem (P1) with the following data:

i	1	2	3	4	5
P_i	1	1	7	2	9
d_i	18	8	7	6	9

$s_1=17$, $s_2=7$, $s_3=0$,$s_4=4$,$s_5=0$.

It is clear that the SPT_1 gives schedule (1,2,4,3,5) ,and the point (542,17).

But SPT_2(break a tie of job 1 and 2) gives schedule (2,1,4,3,5), and optimal point (542,16).

The second problem 1//Lex(E_{max} , $\sum_{i=1}^{n} C_i^2$ **)**

This problem can be written as:

$$\left.\begin{array}{l} \text{Min } \sum_{i=1}^{n} C_i^2 \\ \text{s.t.} \\ \qquad E_{max}=E^* \text{, where } E^*=E_{max}(MST). \end{array}\right\} \ldots(P2)$$

The problem (P2) can be written as: 1/ $E_{max}=E^*$ / $\sum_{i=1}^{n} C_i^2$,

which is equivalent to the problem 1/r_i/$\sum_{i=1}^{n} C_i^2$ where r_i= max{0, s_i-E*}.

Algorithm (C)for problem (P2):

Step(0): Order the jobs by MST rule and calculate E_{max}(MST)= E^*.

Step(1): Let k=1 ,and calculate $r_j=\max\{s_j - E^*, 0\}$for every job $j \in N=\{1,..,n\}$ of unscheduled jobs, σ =(φ), σ be the schedule jobs.

Step(2): Find a job $j^* \in$ N with minimum r_j such that $r_{j^*} \le C_{k-1}$(if there exist a tie choose the job j^* with smallest p_{j^*},if a tie is still choose the job with smallest indexing),C_0=0 when k=1.

Step(3):N=N-{ j^*}, σ =(σ , $\sigma_{(k)}$). If N= φ go to step(4),else k=k+1 ,go to step(2).

Step(4): Calculate $\sum_{i=1}^{n} C_i^2$ (σ) and E_{max}(σ).

Theorem(6): Algorithm (C) gives best possible schedule for problem(P2).

Proof : Since the problem 1//Lex(Emax , $\sum_{i=1}^{n} C_i^2$) can be written as:

$$\text{Min } \sum_{i=1}^{n} C_i^2 \qquad \ldots(1)$$

S.t

$$E_j = d_j - C_j \le E_{\max}(MST) \quad , \quad j=1,\ldots,n \qquad \ldots(2)$$

Equation (2) means that

$$d_j - p_j - C_{j-1} \leq E_{\max}(MST)$$

$$\Rightarrow d_j - P_j - E_{\max}(MST) \leq C_{j-1} \qquad , \qquad j=2,\dots,n$$

Where C_{j-1} is the completion time for job j-1 .

First notice that any job j^* that chooses in step(2) of algorithm C it must satisfies(2) (i.e. $E_{j^*} \leq E_{\max}(MST)$) . This makes the earliness of the chosen job j^* can not violate the maximum earliness of MST schedule ($E_{\max}(MST)$). Second if there exists a tie (more than one job j^*) then we choose the job j^* with smallest processing time Pj^* which minimize (1) also. Hence any schedule constructed by the algorithm C is optimal.

Example(2): consider the data for problem (P2)

i	1	2	3	4
P_i	3	4	8	7
d_i	12	4	10	7

Since the $s_j=d_j-p_j$ hence $s_1=9$, $s_2=0$, $s_3=2$,$s_4=0$.

Hence the MST rule gives the schedule (2,4,3,1) with (E_{max}, $\sum_{i=1}^{n} C_i^2$)=(0,982) .

Now , $r_1=9$, $r_2=0$,$r_3=2$, $r_4=0$

j	r_j	C_{k-1}	j*
1	9	0	2
2	0	4	4
3	2	11	1
4	0	14	3

Hence the schedule (2,4,1,3) gives the optimal point (E_{max}, $\sum_{i=1}^{n} C_i^2$)=(0,817)

Total Square Completion Time and Maximum Earliness (i.e. F($\sum_{i=1}^{n} C_i^2$,E_{max})).

In this section we will try to find (pareto optimal) efficient solutions for

1/ /F($\sum_{i=1}^{n} C_i^2$,E_{max}) problem.

This problem can be define as:

$$\left.\begin{array}{l} \text{Min } \sum_{i=1}^{n} C_i^2 \\ \text{s.t.} \\ E_{max} \le E \quad , \text{ where } E \in [\, E_{max}(MST), E_{max}(SPT)] \end{array}\right\} \quad \ldots(P3)$$

Proposition(2):[17] *There exists an efficient sequence in the bi-criterion problem (P3) that satisfies the SPT rule.*

Note that an analogous proposition for the MST rule does not hold in general as shown by the following example :

i	1	2	3	4
P_i	3	4	8	7
d_i	12	4	10	7
s_i	9	0	2	0

SPT* sequence (1,2,4,3), $\sum_{i=1}^{n} C_i^2$ (SPT*) = 738, E_{max}(SPT*) = 9

MST sequence(2,4,3,1), $\sum_{i=1}^{n} C_i^2$ (MST)= 982, E_{max}(MST) = 0

SPT* is efficient by Proposition (2.4).

MST is not efficient since it is dominated by sequence (2,4,1,3)

with $\sum_{i=1}^{n} C_i^2$ = 817 and E_{max}=0 .

The next algorithm ,which is similar to algorithm for 1//F($\sum C_i$,E_{max}) problem , is given by Hoogeveen and Van de Valde[13] .

Algorithm(D) for problem(P3):

Step(0): Compute E_{max}(MST) ,and E_{max} (SPT) ;let k=1 ,

E_{max} (SPT)=E**.

Step(1): Solve 1/ $E_{max} \le E^{**}$ /$\sum_{i=1}^{n} C_i^2$ by algorithm(C) for problem (P2) ; this produces the k pareto optimal schedule $\sigma^{(k)}$, and the k pareto optimal point ($\sum_{i=1}^{n} C_i^2 (\sigma^{(k)})$, $E_{max}(\sigma^{(k)})$).

Step(2): E**=E** -1, k=k+1.

Step(3): If E**< E_{max}(MST) stop, else ,go to step (1).

Example(4): Consider the problem (P3) with the following data:

i	1	2	3	4	5
P_i	3	1	7	7	10
d_i	4	12	14	8	10

$s_1=1$, $s_2=11$, $s_3=7$,$s_4=1$,$s_5=0$.

$E_{max}(MST)=0$,$E_{max}(SPT)=11=E^{**}$.

Now, by proposition (2) ,SPT gives efficient schedule(2,1,3,4,5) then the first efficient point (1246,11).

$E^{**}=11-1=10$

Now we will solve $1/\ E_{max}\leq 10\ /\sum_{i=1}^{n} C_i^2$ by algorithm(C) for (P3)

Now , $r_1=0$, $r_2=1$,$r_3=0$, $r_4=0$,$r_5=0$

j	r_j	C_{k-1}	j*
1	0	0	1
2	1	3	2
3	0	4	4
4	0	11	3
5	0	18	5

Hence the schedule (1,2,4,3,5) gives ($\sum_{i=1}^{n} C_i^2$,E_{max})=(1254,8)

$E^{**}=8-1=7$

Now , $r_1=0$, $r_2=4$,$r_3=0$, $r_4=0$,$r_5=0$

j	r_j	C_{k-1}	j*
1	0	0	1
2	4	3	4
3	0	11	2
4	0	12	3
5	0	18	5

Hence the schedule (1,4,2,3,5) gives ($\sum_{i=1}^{n} C_i^2$,E_{max})=(1338,1)

$E^{**}=1-1=0$

$r_1=1$, $r_2=11$,$r_3=7$, $r_4=1$,$r_5=0$

j	r_j	C_{k-1}	j*
1	1	0	5
2	11	10	1
3	7	13	2

4	1	14	4
5	0	21	3

Hence the schedule (5,1,2,4,3) gives ($\sum_{i=1}^{n} C_i^2$,E_{max})=(1690,0).

$E^{**}=0-1=-1<E_{max}(MST)$.Stop

Now consider the following problem:

The $1//\sum_{i=1}^{n} C_i^2$ **+E_{max} problem**:

In this section we decompose the $1//\sum_{i=1}^{n} C_i^2$ +E_{max} problem into two subproblems with a simpler structure , and state some results which help us in solving it.

This problem can be written as:

$$
\left.
\begin{aligned}
& M_2=\min_{\sigma\in s}\ \{\sum_{i=1}^{n} C_i^2 + E_{max}(\sigma)\} \\
& \quad \text{s.t.} \\
& \quad c_{\sigma(i)} \geq p_{\sigma(i)} \qquad i=1,\dots,n \\
& \quad c_{\sigma(i)} = C_{\sigma(i-1)}+P_{\sigma(i)} \qquad i=2,\dots,n \\
& \quad E_{\sigma(i)} \geq d_{\sigma(i)} - C_{\sigma(i)} \qquad i=1,\dots,n \\
& \quad E_{\sigma(i)} \geq 0 \qquad i=1,\dots,n
\end{aligned}
\right\} \quad \text{...(P4)}
$$

This problem can be decomposed into two subproblems (SP3) and (SP4)

$$
\left.
\begin{aligned}
& V_3=\min_{\sigma\in s}\ \sum_{i=1}^{n} C_{\sigma(i)}^2 \\
& \text{s.t.} \\
& \qquad c_{\sigma(i)} \geq p_{\sigma(i)} \ , \qquad i=1,\dots,n \\
& \qquad c_{\sigma(i)} = C_{\sigma(i-1)}+P_{\sigma(i)} \ , \ i=2,\dots,n
\end{aligned}
\right\} \dots\text{(SP3)}
$$

$$\left.\begin{aligned}
&V4=\min_{\sigma\in s}\ \{\max\{E_{\sigma(i)}\}\}\\
&\text{s.t.}\\
&\qquad E_{\sigma(i)}\geq d_{\sigma(i)}-C_{\sigma(i)} \qquad i=1,\ldots,n\\
&\qquad E_{\sigma(i)}\geq 0 \qquad\qquad\qquad\quad i=1,\ldots,n
\end{aligned}\right\}\ldots\text{(SP4)}$$

Theorem(7)[24] :

V1+V2≤ M1 where V1 ,V2 ,and M1 are the minimum objective function values of (SP3),(SP4), and (P4) respectively .■

Some Special Cases for the Problem (P4).

Case (1): If for every schedule $C_i \geq d_i$ $\forall$ $i\in N$ then SPT rule gives an optimal value for (P4).

Proof: Since $C_i \geq d_i$ then $E_i=0$ $\forall$ $i\in N \Longrightarrow E_{max}=0$.

Hence the problem (P4) reduce to $1//\sum_{i=1}^{n} C_i^2$ problem .Then by proposition (1) SPT rule gives optimal value .■

Case(2): If $p_i=p$ $\forall$ $i\in N$ then MST rule gives an optimal value for (P4).

Proof: If $p_i=p$ $\forall$ $i\in N$ then $\sum_{i=1}^{n} C_i^2$ is constant for every sequence ,since MST rule gives minimum value for E_{max} , then MST rule gives optimal value for (P4).■

Heuristic method to Calculate Upper Bound (UB) for the Problem (P4).

To calculate upper bound (UB_E)order the jobs by SPT rule and then calculate $\sum_{i=1}^{n} C_i^2$ and E_{max} .

Derivation of lower bound (LB)

To calculate a lower bound (LB) apply theorem (7).The lower bound of each node in the solution search tree are written against the nodes of the tree. To find the optimal solution for (P4), we applied the methods for lower and upper bounds that will be used in BAB algorithm

Example(5): Consider the problem (P4) with the following data:

i	1	2	3	4	5
P_i	3	1	7	7	10
d_i	4	12	14	8	10

$UB_E=\sum_{i=1}^{n} C_i^2$ (SPT)+ E_{max}(SPT)=1246+11=1257

$ILB=\sum_{i=1}^{n} C_i^2$ (SPT)+ E_{max}(MST)=1246+0=1246

The optimal schedule is (2,1,4,3,5) with $\sum_{i=1}^{n} C_i^2$ +E_{max}=1257 is obtaind by (BAB)method.

The lower bound of each node in the solution search tree are written against the nodes of the tree. To find the optimal solution for (P4), we applied the methods for lower and upper bounds that will be used in BAB algorithm. Where (BAB) Branch and bound methods can be used for solving many combinatorial optimization problems. These procedures can be conveniently represented as a search (scheduling, branching) tree whose nodes correspond to subsets of a feasible solution. To minimize an objective function of a particular scheduling problem, first an upper bound UB of the minimum of this objective function is needed. A branching rule is used to partition feasible solutions at a node into subsets and a bounding rule calculates a lower bound LB on the value of each solution in a subset.

Computational experience

An intensive work of numerical experimentations has been performed. We first present how instances (tests problem) can be randomly generated .

There exists in the literature a classical way to randomly generate tests problem of scheduling problems.

- The processing time P_i are uniformly distributed in the interval [1,10].
- The due dates d_i are uniformly distributed in the interval [p(1- TF- RDD/2), p(1+ TF+ RDD/2)] ; where $p=\sum_{i=1}^{n} p_i$, depending on the relative range of due date (RDD) and on the average tardiness factor (TF).

For both parameters, the values 0.2,0.4,0.6,0.8and 1.0, are considered . For each selected value of *n*, one problem was generated for each of five values of parameters producing five problems for each value of *n* .

The BAB and D algorithms were tested by coding them in matlab7 and running on Pentium IV at 2800MHz with Ram 512MB computer. The BAB algorithm that described is tested on problems with size (25,50,75) for problem P(4) .

For problems that are not solved to optimally because the execution time exceed 30 minutes, the optimal solution for these unsolved problems found by our algorithm D .

Table(1) shows the results for problem (P4) obtained by BAB algorithm. The first column “n” refers to the number of jobs, the second column “EX” refers to the number of example for each instance n, the third column “optimal” refers to the optimal value obtained by BAB algorithm for problem (P4), the fourth column “UB” refers to the upper bound , the fifth column “ILB” refers to the initial lower bound , the sixth column “nodes” refers to the number of nodes , the seventh column “time” refers to the time cost ‘by seconds’ to solve the problem, the last column “status” refers to the problem solved ‘0’ or not ‘1’. The symbol “*” refers to the optimal=UB, we stopped when the sum of status’ column ≥3.

Table(2) , show the results for problem (P4) obtained by algorithm (D). The first two columns as the same columns in table(1), the third column “value” refers to the minimum value that we get by algorithms B, and the last column “time” refers to the time cost ‘by seconds’ to solve the problem .

Table (3)compare between *BAB and algorithm (D) to solve a problem(P4)(time by seconds).* It is clear from table (3) that the BAB method can not solved problems with n ≥ 75.

Table(1): ***The performance of initial lower bound, upper bound and computational time of BAB algorithm for (P4).***

n	EX	optimal	UB	ILB	nodes	time	status
25	1	111967	111969	111965	324	0.095103	0
	2	104115	104116	104095	623	0.052337	0
	3	139657	139657*	139630	632	0.056231	0
	4	92083	92085	91980	2288	0.177016	0
	5	183234	183235	183173	646	0.05464	0
50	1	798166	798170	798150	12995	1.10369	0
	2	983380	983380*	983186	22740736	1800.008	1
	3	809433	809433*	809260	96331	7.641318	0
	4	1208294	1208298	1208029	40797	3.169203	0
	5	964878	964878*	964633	385790	30.44505	0
75	1	2994568	2994569	2994535	20945	1	0
	2	2419151	2419151*	2418829	21254609	1800	1
	3	2422335	2422344	2422236	4660939	410	0
	4	2270098	2270102	2269810	22453697	1800	1
	5	3305425	3305435	3305242	21554412	1800	1

Table(2): ***Results of algorithm (D) for (P4).***

n	ex	value	time
25	1	111967	0.05163197
	2	104115	0.046552197
	3	139657	0.053125805
	4	92083	0.169790367
	5	183234	0.115045191
50	1	798166	0.226852784
	2	983380	1.134018331
	3	809433	1.069051694
	4	1208294	1.620483505
	5	964878	1.631177439
75	1	2994568	0.663017256
	2	2419149	4.178856421
	3	2422335	1.853932184
	4	2270098	4.138179355
	5	3305425	2.829241991
100	1	5563309	5.920008116
	2	7155468	8.233671951
	3	5906704	10.7733986
	4	5826135	8.979650719
	5	7526972	13.31796964

Table (3): ***BAB Vs algorithm (D) to solve a problem(P4)(time by seconds).***

n	BAB	D
25	**0.09**	**0.07**
50	**10**	**0.9**
75	**1800**	**2.8**
100	**1800**	**8.5**
200	**1800**	**60**
300	**1800**	**160**
400	**1800**	**475**
500	**1800**	**1263**

200	1	46373589	76.42389982
	2	55521422	59.2898395
	3	52408910	71.99848927
	4	43624815	38.55980683
	5	45304972	49.51272606
300	1	180480504	43.04842239
	2	189082887	203.9020447
	3	183436529	103.2003436
	4	157887962	186.3504953
	5	164143083	266.5680996
400	1	421800114	257.5486245
	2	423348548	740.1145268
	3	430916378	743.7059773
	4	394604568	336.3430679
	5	413687818	402.8218975
500	1	712946453	1045
	2	763797716	1378.468031
	3	755495456	1208.093472
	4	782833408	1443.172362
	5	750245272	1240.52467

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