On Automorphisms with Derivations on Semiprime Rings

by

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Abstract:

The main purpose of this paper is to investigate automorphisms and identities with derivations on semiprime ring R, we obtain R contains a non-zero central ideal.

Key words:*Automorphism,identitie, derivation,semprime ring, central ideal.*

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1-Introducation

The history of commuting and centralizing mappings goes back to (1955) when Divinsky [1] proved that a simple Artinian ring is commutative if it has a commuting nontrivial automorphism.Tow years later, Posner [2] has proved that the existence of a non- zero centralizing derivation on prime ring forces the ring to be commutative (Posner's second theorem).Luch [3]generalized the Divinsky result, we have just mentioned above, to arbitrary prime ring. Mayne [4] prove that in case there exists a nontrivial centralizing automorphism on a prime ring, then the ring is commutative (Mayne's theorem). Chung and Luh[5] have shown that every semicommuting automorphism of a prime ring is commuting provided that R has either characteristic different from 3 or non- zero center and thus they proved the commutativity of prime ring having nontrivial semicommuting automorphism except in the indicated cases. Mayne[4] has called d centralizing if $[x,d(x)]\in Z(R)$ *for all* $x\in R$*,where Z(R)is the center of R and d an automorphism of R. Kaya and Koc[6] have shown that let R be a prime ring and d a semicentralizing automorphism of R, then d is a commuting automorphism. A result of Bresar[7] which states that every additive commuting mapping of a prime ring R is of the from* $x\to\lambda x+\zeta(x)$ *where* λ *is an element of C and* $\zeta:R\to C$ *is an additive mapping, should be mentioned. A mapping* $d: R\to R$ *is called skew-centralizing on R if* $d(x)x+xd(x)\in Z(R)$ *holds for all* $x\in R$*, in particular, if* $d(x)x+xd(x)=0$ *holds for all* $x\in R$*, then it is called skew- commuting on R .Vukman [8] have shown that let R be* *a semiprime ring. Suppose that there exist a derivation $d:R \rightarrow R$ and an automorphism α: $R \rightarrow R$ such that $d(x)\ x+x(\alpha(x)-x)=0$ holds for all $x \in R$. In this case, $d=0$ and $\alpha=I$, where I the identity mapping of a ring R. In this paper, we investigate and study the automorphisms and identities with derivations on semiprime ring, we give some results about that.*

2- Preliminaries:

Throughout this paper, R will represent an associative ring with cancellation property. We recall that R is semiprime if $xRx=(0)$ implies $x=0$ and it is prime if $xRy=(0)$ implies $x=0$ or $y=o$ .A prime ring is semiprime but the converse is not true in general. An additive mapping $d: R \rightarrow R$ is called a derivation if $d(xy)=d(x)y +xd(y)$ holds for all $x,y \in R$. Let α be an automorphism of a ring R. An additive mapping $d: R \rightarrow R$ is called an α -derivation if $d\ (xy)=d(x)\alpha(y)+xd(y)$ holds for all pairs $x,y \in R$. Note that the mapping $d=\alpha$-I is an α- derivation, where we denote by I the identity mapping of a ring R,of course , the concept of α- derivation generalizes the concept of derivation. A mapping $d: R \rightarrow R$ is called centralizing of $[d(x),x] \in Z(R)$ for all $x \in R$, in particular, if $[d(x),x]=0$ for all $x \in R$., then it is called commuting and called central if $d(x) \in Z(R)$ for all $x \in R$, where Z(R) the center of R. Every central mapping is obviously commuting but not conversely, in general. We write $[x,y]$ for $xy-yx$ and xoy for $xy+yx$. We will frequently use the identities $[xy,z]=x[y,z]+[x,z]y$ and $[x,yz]= y[x,z]+[x,y]z$ for all $x,y,z \in R$..

To achiever our purpose , we mention the following results.

Lemma 1 [9: Lemma1]

Let R be a semiprime ring. Suppose that the relation $axb+bxc=0$ holds for all $x \in R$ and some $a, b, c \in R$ .In this case, $(a+c)xb=0$ is satisfied for all $x \in R$.

Lemma2[8 : Lemma3]

Let R be a semiprime ring and let $d: R \rightarrow R$ be an additive mapping.If either $d(x)x=0$ or $xd(x)=0$ holds for all $x \in R$, then $d=0$.

Lemma3[10: Main Theorem]

Let R be a semiprime ring, d a non-zero derivation of R, and U a non-zero left ideal of R. If for some positive integers t_0, t_1, ...,t_n and all $x \in U$, the identity$[[\ldots[[d(x^{to}),x^{t1}],x^{t2},\ldots],x^{tn}]=0$ holds either $d(U)=0$ or else $d(U)$ and $d(R)U$ are contained in a non-zero central ideal of R. In particular when R is a prime ring, R is commutative.

Lemma 3[11:Lemma 3.1]

Let R be a semiprime ring and $a \in R$ some fixed element. If $a[x,y] = 0$ for all $x,y \in R$, then there exists an ideal U of R such that $a \in U \in Z(R)$ holds .

3-The Main Results

Theorem 3.1

Let R be a semiprime ring and U a non- zero ideal of R. Suppose that there exist a derivation $d: R\rightarrow R$ and an automorphism $\alpha: R\rightarrow R$ such that $d(x)x+x(\alpha(x)-x)=0$ for all $x\in U$. Then R contains a non-zero central ideal.

Proof: *We have the relation*

$d(x)x+xg(x)=0$ *for all* $x\in U$. (1)

where g(x) stands for $\alpha(x)-x$. *The linearization of above relation gives*

$d(x)x+d(x)y+d(y)x+d(y)y+xg(x)+xg(y)+yg(x)+yg(y)=0$ *for all* $x,y\in U$. *According to (1) , we obtain*

$d(x)y + d(y)x +xg(y) + yg(x) =0$ *for all* $x, y\in U$. (2)

In (2) , replacing y by yx, we obtain

$d(x)yx-d(y)\alpha(x)x+yd(x)x+xg(y)\alpha(x)+xyg(x)+yxg(x)=0$ *for all* $x,y\in U$. *According to (1),we get*

$d(x)yx+d(y)x^2+xg(y)\alpha(x)+xyg(x)=0$ *for all* $x, y\in U$. (3)

Right- multiplying (2) by x, we obtain

$d(x) yx+d(y)x^2+xg(y)x+yg(x)x=0$ *for all* $x,y\in U$. (4)

Subtracting (3) and (4) , we get

$xg(y)\alpha(x) +xyg(x)-xg(y)x-yg(x)x=0$ *for all* $x,y\in U$.

$xg(y)(\alpha(x)-x)+xyg(x)-yg(x)x=0$ *for all* $x,y\in U$. *Then*

$xg(y)g(x)+xyg(x)-yg(x)x=0$ *for all* $x,y\in U$. (5)

Replacing y by xy, we obtain

$xg(x)\alpha(y)g(x)+x^2g(y)g(x)+x^2yg(x)-xyg(x)x=0$ *for all* $x,y\in U$.

According to (5) , we get

$xg(x)\alpha(y)g(x)=0$*forall*$x,y\in U$. (6)

Now , since α *is an automorphism,we obtain*

$xg(x) yg(x)=0$*forall*$x,y\in U$.*Replacing y by rx,we get*

$xg(x)=0$ *forallx* $\in U$. *Putting this relation in (1)gives*

$d(x)x=0$ *for all* $x\in U$. (7)

By Lemma2,the relation (7)with left-multiplying by x ,gives

$xd(x)=0$ *for all* $x\in U$. (8)

By subtracting (8)and(7)with using Lemma 4,we obtain R contains a non-zero central ideal.

Theorem 3.2

Let R be a semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[d(x)+\alpha(x),x]=0$ for all $x\in U$. Then R contains a non-zero central ideal.

Proof: *The linearization of the relation*

$[d(x)+\alpha(x),x]=0$ for all $x\in U$. (9)

We obtain

$[d(x)+\alpha(x),y]+[d(y)+\alpha(y),x]=0$ for all $x,y\in U$. (10)

Replacing y by yx, we obtain

$y[d(x)+\alpha(x),x]+[d(x)+\alpha(x),y]x+[d(y)x+yd(x)+\alpha(y)\alpha(x),x]=0$ forall $x,y\in U$.According to (9), we obtain

$[d(x)+\alpha(x),y]x+[d(y),x]x+y[d(x),x]+[y,x]d(x)+\alpha(y)[\alpha(x),x]+[\alpha(y),x]\alpha(x)=0$ for all $x,y\in U$.

Replacing $[d(x)+\alpha(x),y]x+[d(y),x]x$ by $-[\alpha(y),x]x$ and $y[d(x),x]$ by $-y[\alpha(x),x]$which gives

$-[\alpha(y),x]x-y[\alpha(x),x]+[y,x]d(x)+\alpha(y)[\alpha(x),x]+[\alpha(y),x]\alpha(x)=0$ for all $x,y\in U$. Then

$[\alpha(y),x]g(x)+g(y)[\alpha(x),x]+[y,x]d(x)=0$ for all $x,y\in U$. (11)

Where $g(x)$ stands for $\alpha(x)-x$ and $g(y)$ stands for $\alpha(y)-y$.

Putting in above relation xy for y, we get

$[\alpha(xy),x]g(xy)[\alpha(x),x]+[xy,x]d(x)=0$ for all $x,y\in U$.

$\alpha(x)[\alpha(y),x]g(x)+[\alpha(x),x]\alpha(y)g(x)+g(x)\alpha(y)[\alpha(x),x]+xg(y)[\alpha(x),x]+x[y,x]d(x)=0$ for all $x,y\in U$. (12)

Multiplying (11) from the left sided by x, subtracting the relation so obtained from (12) and replacing $\alpha(y)$ by y, we obtain (note that $[\alpha(x),x]=[g(x),x]$ for all $x\in U$).

$g(x)[y,x]g(x)+[g(x),x]yg(x)+g(x)y[g(x),x]=0$ for all $x,y\in U$.

Which reduces to

$xg(x)yg(x)+g(x)y(-g(x)x)=0$ for all $x,y\in U$.

By Lemma 1, the above relation gives

$[g(x),x]yg(x)=0$ for all $x,y\in U$. (13)

In (13),replacing y by $rg(x)xt[g(x),x]r$ with rigt-multiplying by x,we get

$[g(x),x]rg(x)xt[g(x),x]rg(x)x=0$ for all $x,y\in U,r,t\in R$. (14)

Since R is semiprime ,we obtain

$[g(x),x]rg(x)x=0$ for all $x\in U,r\in R$. (15)

Again from(13),we get

$[g(x),x]rxg(x)=0$ forall $x,y\in U$. (16)

Subtacting (15) and (16)we get

$[g(x),x]=0$ forall $x\in U$. Since $[\alpha(x),x]=[g(x),x]$for all $x\in U$.Then (9) reduced to $[d(x),x]=0$ for all $x\in U$. By Lemma3, R contains a non-zero central ideal.

Theorem 3.3

Let R be a semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\to R$ and an automorphism $\alpha:R\to R$ such that$[d(x)x+x\alpha(x),x]=0$ for all $x\in U$. Then R contains a non-zero central ideal.

Proof: *We have the relation*

$[d(x)x+x\alpha(x),x]=0$ for all $x\in U$. (17)

Replacing x by x+y , we obtain

$[d(x)x+d(x)y+d(y)x+d(y)y+x\alpha(x)+x\alpha(y)+y\alpha(x)+y\alpha(y),x+y]=0$ for all $x,y\in U$.

According to (17) , we get

$[A(x),y]+[d(x)y+d(y)x+x\alpha(y)+y\alpha(x),x]=0$ for all $x,y\in U$. (18)

Where A(x) stands for $d(x)x+x\alpha(x)$. Let in above y by yx, we obtain

$[A(x),yx]+[d(x)yx+d(y)x^2+yd(x)x+x\alpha(y)\alpha(x)+yx\alpha(x),x]=0$ for all $x,y\in U$.

According to (17), we obtain

$[A(x),y]x+[yA(x),x]+[(d(x)y+d(y)x)x,x]+[x\alpha(y)\alpha(x),x]=0$ for all $x,y\in U$. Then according to(17), above reduces to

$[A(x),y]x+[y,x]A(x)+[d(x)y+d(y)x,x]x+x[\alpha(y)\alpha(x),x]=0$ for all $x,y\in U$. (19)

From (18), we get $[A(x),y]+[d(x)y+d(y)x,x]=-[x\alpha(y)+y\alpha(x),x]$ for all $x,y\in U$. (20)

Substituting (19) in (20), we obtain

$-[x\alpha(y)+y\alpha(x),x]x+[y,x]A(x)+x[\alpha(y)\alpha(x),x]=0$ for all $x,y\in U$. Then

$-x[\alpha(y),x]x-y[\alpha(x),x]x-[y,x]\alpha(x)x+[y,x]A(x)+x\alpha(y)[\alpha(x),x]+$
$x[\alpha(y),x]\alpha(x)=0$ for all $x\in U$. Then

$x[\alpha(y),x]g(x)+[y,x](A(x)-\alpha(x)x)+x\alpha(y)[\alpha(x),x]-y[\alpha(x),x]x=0$ for all $x,y\in U$. Where g(x) denote to $\alpha(x)-x$, then

$x[\alpha(y),x]g(x)+[y,x]B(x)+x\alpha(y)[\alpha(x),x]-y[\alpha(x),x]x=0$ for all $x,y\in U$. (21)

Where B(x) stands for $A(x)-\alpha(x)x$ $(d(x)x+[x,\alpha(x)])$, replacing y by xy, we obtain

$x[\alpha(xy),x]g(x)+[xy,x]B(x)+x\alpha(xy)[\alpha(x),x]-xy[\alpha(x),x]x=0$ for all $x,y\in U$. Then

$x\alpha(x)[\alpha(y),x]g(x)+x[\alpha(x),x]\alpha(y)g(x)+x[y,x]B(x)+x\alpha(x)\alpha(y)[\alpha(x),x]-xy[\alpha(x),x]x=0$ for all $x,y\in U$. (22)

Left-multiplying (21) by x, we obtain

$x^2[\alpha(y),x]g(x)+x[y,x]B(x)+x^2\alpha(y)[\alpha(x),x]-xy[\alpha(x),x]x=0$ for all $x,y\in U$. (23)

Subtracting (23) with (22) and replacing $\alpha(y)$ by y, we get $(x\alpha(x)-x^2)[y,x]g(x)+x[\alpha(x),x]yg(x)+x\alpha(x)y[\alpha(x),x]-x^2y[\alpha(x),x]=0$ for all $x,y\in U$.

$xg(x)[y,x]g(x)+xg(x)y[\alpha(x),x]+x[\alpha(x),x]yg(x)=0$ for all $x,y\in U$. Then

$xg(x)([y,x]g(x)+y[\alpha(x),x])+x[\alpha(x),x]yg(x)=0$ for all $x,y\in U$.

Note that $[\alpha(x),x]=[g(x),x]$ for all $x\in U$. Above relation we can write as

$xg(x)[yg(x),x]+x[g(x),x]yg(x)=0$ for all $x,y\in U$. Then

$xg(x)yg(x)x-x^2g(x)yg(x)=0$ for all $x,y\in U$. Then

$xM(x)=0$ for all $x\in U$.

Where M(x) stands for $g(x)yg(x)x-xg(x)yg(x)$,by Lemma2,we get

$M(x)=0$ for all $x \in U$. Thus
$g(x)yg(x)x-xg(x)yg(x)=0$ for all $x,y \in U$. (24)
For Appling Lemma1 , we rewrite (24) by
$-xg(x)yg(x)+g(x)yg(x)x=0$ for all $x, y \in U$. Then
$(g(x)x-xg(x))yg(x)=0$ for all $x,y \in U$.
$[g(x),x]yg(x)=0$ for all $x,y \in U$. (25)
By same method in Theorem3.2,we obtain
$[d(x),x]x=0$ for all $x \in U$.
Now,we have
$W(x)x=0$ for all $x \in U$. Where $W(x)$ stands for $[d(x),x]$ with using Lemmas(2 and 3), we obtain R contains a non-zero central ideal.

Theorem 3.4

Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R \to R$ and an automorphism $\alpha:R \to R$ such that $[d(x) o \alpha(x),x]=0$ for all $x \in U$. Then R contains a non-zero central ideal.

Proof: We have $[d(x) o \alpha(x),x]=0$ for all $x \in U$. Then
$[d(x)\alpha(x),x]+[\alpha(x)d(x),x]=0$ for all $x \in U$. (27)
Putting x by x+y, we obtain
$[d(x)\alpha(x),x]+[d(x)\alpha(y),x]+[d(y)\alpha(x),x]+[d(y)\alpha(y),x]+[d(x)\alpha(x),y]+[d(x)\alpha(y),y]+[d(y)\alpha(x),y]+[d(y)\alpha(y),y]+[\alpha(x)d(x),x]+[\alpha(y)d(x),x]$
$[\alpha(x)d(y),x]+[\alpha(y)d(y),x]+[\alpha(x)d(x),y]+[\alpha(y)d(x),y]+[\alpha(x)d(y),y]+[\alpha(y)d(y),y]=0$ for all $x,y \in U$. (28)
According to (27) , the relation (28) reduces to
$[d(x)\alpha(y),x]+[d(y)\alpha(x),x]+[d(y)\alpha(y),x]+[d(x)\alpha(x),y]+[d(x)\alpha(y),y]+[d(y)\alpha(x),y]+[\alpha(y)d(x),x]+[\alpha(x)d(y),x]+[\alpha(y)d(y),x]+[\alpha(x)d(x),y]+[\alpha(y)d(x),y]+[\alpha(x)d(y),y]=0$ for all $x,y \in U$.
Replacing y by yx, we obtain after some calculation.
$d(x)\alpha(y)[\alpha(x),x]+d(x)[\alpha(y),x]\alpha(x)+[d(x),x]\alpha(y)\alpha(x)+d(y)x[\alpha(x),x]+[d(y),x]x\alpha(x)+yd(x)[\alpha(x),x]+y[d(x),x]\alpha(x)+[y,x]d(x)\alpha(x)+d(y)x\alpha(y)[\alpha(x),x]+d(y)x[\alpha(y),x]\alpha(x)+[d(y),x]x\alpha(x)+yd(x)\alpha(y)[\alpha(x),x]+yd(x)[\alpha(y),x]\alpha(x)+y[d(x),x]\alpha(y)\alpha(x)+[y,x]d(x)\alpha(y)\alpha(x)+yd(x)[\alpha(y),x]+y[d(x),x]\alpha(y)+d(x)[\alpha(x),y]x+[d(x),y]\alpha(x)x+d(x)\alpha(y)y[\alpha(x),x]+d(x)\alpha(y)[\alpha(x),y]x+d(x)y[\alpha(y),x]\alpha(x)+d(x)[\alpha(y),y]x\alpha(x)+y[d(x),x]\alpha(y)\alpha(x)+[d(x),y]x\alpha(y)\alpha(x)+d(y)xy[\alpha(x),x]+d(y)x[\alpha(x),y]x+d(y)[x,y]x\alpha(x)+y[d(y),x]x\alpha(x)+[d(y),y]x^2\alpha(x)+yd(x)y[\alpha(x),x]+yd(x)[\alpha(x),y]x+y^2[d(x),x]\alpha(x)+y[d(x),y]x\alpha(x)+y[y,x]d(x)\alpha(x)+\alpha(y)\alpha(x)[d(x),x]+\alpha(y)[\alpha(x),x]d(x)+[\alpha(y),x]\alpha(x)d(x)+\alpha(x)[d(x),x]x+[\alpha(x),x]d(x)x+\alpha(x)y[d(x),x]+\alpha(x)[y,x]d(x)+[\alpha(x),x]yd(x)+\alpha(y)\alpha(x)[d(y),x]x+\alpha(y)[\alpha(x),x]d(y)x+[\alpha(y),x]\alpha(x)d(y)x+\alpha(y)\alpha(x)y[d(x),x]+\alpha(y)\alpha(x)[y,x]d(x)$

$+\alpha(y)[\alpha(x),x]yd(x)+[\alpha(y),x]\alpha(x)yd(x)+y\alpha(x)[d(x),x]+y[\alpha(x),x]$ $d(x)+\alpha(x)[d(x),y]x+[\alpha(x),y]d(x)x+\alpha(y)\alpha(x)y[d(x),x]+\alpha(y)\alpha(x)[d(x),y]x+\alpha(y)y[\alpha(x),x]d(x)+\alpha(y)[\alpha(x),y]xd(x)+y[\alpha(y),x]\alpha(x)$ $d(x)+[\alpha(y),y]x\alpha(x)d(x)+\alpha(x)d(y)[x,y]x+\alpha(x)y[d(y),x]x+\alpha(x)$ $[d(y),y]x^2+y[\alpha(x),x]d(y)x+[\alpha(x),y]xd(y)x+\alpha(x)y^2[d(x),x]+\alpha(x)y[d(x),y]x+\alpha(x)y[y,x]d(x)+[\alpha(x),x]yd(x)+[\alpha(x),y]xyd(x)=0$ for all $x,y\in U$. (29)

In (29)replacing $\alpha(x)$ and y by x, we obtain

$4[d(x),x]x^2+4x[d(x),x]x+5x[d(x),x]x^2+3[d(x),x]x^3+5x^2[d(x),x]x+3x^3[d(x),x]+2x^2[d(x),x]=0$ for all $x\in U$. (30)

Replacing x by $-x$ and subtracting with (30) , we obtain $4[d(x),x^3]=0$ for all $x\in U$. Since R is 2-torsion free with using Lemma3 , we obtain R contains a non-zero central ideal.

Theorem 3.5

Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[[d(x),\alpha(x)],x]=0$ for all $x\in U$. Then R contains a non-zero central ideal.

Proof: *We have $[[d(x),\alpha(x)],x]=0$ for all $x\in U$.* (31)

Putting x by $x+y$ in above equation and according to (31), we obtain

$[[d(y),\alpha(x)],x]+[[d(x),\alpha(y),y]+[[d(y),\alpha(y)],x]+[[d(x),\alpha(x),y]+[[d(y),\alpha(x)],y]+[[d(x),\alpha(y),y]=0$ for all $x,y\in U$. (32)

In (32) replacing y by yx with using (31), we obtain

$[d(y)[x,\alpha(x)],x]+[[d(y),\alpha(x)],x]x+[y[d(x),\alpha(x)],x]+[[y,\alpha(x)]d(x),x]+[\alpha(y),x][d(x),\alpha(x)]+[d(x),\alpha(y)][\alpha(x),x]+[[d(x),\alpha(y)],x]\alpha(x)+[d(y)[x,\alpha(yx)],x]+[[d(y),\alpha(yx)],x]x+x[[d(y),\alpha(yx),x]+$ $[x,\alpha(yx)][d(y),x]+[[x,\alpha(yx)],x]d(y)+[[d(x),\alpha(x)],y]x+[d(y)[x,\alpha(x)],yx]+[[d(y),\alpha(x)]x,yx]+[y[d(x),\alpha(x)],yx]+[[y,\alpha(x)]d(x),yx]+\alpha(y)[[d(x),\alpha(x)],yx]+[\alpha(y),yx][d(x),\alpha(x)]+[d(x),\alpha(y)][\alpha(x),yx]+[[d(x)\alpha(y)],yx]\alpha(x)=0$ for all $x,y\in U$. After some calculation, we obtain

$d(y)[[x,\alpha(x)],x]+[d(y),x][x,\alpha(x)]+[[d(y),\alpha(x)],x]x+y[[d(x),\alpha(x)],x]+[y,x][d(x),\alpha(x)]+[y,\alpha(x)][d(x),x]+[[y,\alpha(x)],x]d(x)+[\alpha(y),x][d(x),\alpha(x)]+[d(x),\alpha(y)][\alpha(x),x]+[[d(x),\alpha(y)],x]\alpha(x)+d(y)\alpha(y)[[x,\alpha(x)],x]+d(y)[\alpha(y),x][x,\alpha(x)]+d(y)[x,\alpha(y)][\alpha(x),x]+d(y)[[x,\alpha(y),x]\alpha(x)+[d(y),x]\alpha(y)[x,\alpha(x)]+[d(y),x][x,\alpha(y)]\alpha(x)+\alpha(y)[[d(y),\alpha(x)],x]x+[\alpha(y),x][d(y),\alpha(x)]x+[d(y),\alpha(y)][\alpha(x),x]x+[[d(y),\alpha(y)],x]\alpha(x)x+x\alpha(y)[[d(y),\alpha(x)],x]+x[\alpha(y),x][d(y)],\alpha(x)]+x[d(y),\alpha(y)][\alpha(x),x]+x[[d(y),\alpha(y)],x]\alpha(x)+\alpha(y)[x,\alpha(x)][d(y),x]+[x,\alpha(y)]\alpha(x)[d(y),x]+\alpha(y)[[x,\alpha(x)],x]d(y)+[\alpha(y),x][x,\alpha(x)]d(y)+[x,\alpha(y)][\alpha(x),x]d(y)+[[x,\alpha(y),x]\alpha(x)d(y)+[[d(x),\alpha(x)],y]x+d(y)y[[x,\alpha(x),x]+d(y)[[x,\alpha(x)],y]x+y[d(y),x][x,\alpha(x)]+[d(y),y]x[x,\alpha(x)]+[d(y),\alpha(x)][x,y]x+y[[d(y),\alpha(x)],x]x+[[d($

$y),\alpha(x)],y]x^2+y^2[[d(x),\alpha(x)],x]+y[[d(x),\alpha(x)],y]x+y[y,x][d(x)\alpha(x)]+[y,\alpha(x)]y[d(x),x]+[y,\alpha(x)][d(x),y]x+y[[y,\alpha(x)],x]d(x)+[[y,\alpha(x)],y]xd(x)+\alpha(y)y[[d(x),\alpha(x)],x]+\alpha(y)[[d(x),\alpha(x)],y]x+y[\alpha(y),x][d(x),\alpha(x)]+[\alpha(y),y]x[d(x),\alpha(x)]+[d(x),\alpha(y)]y[\alpha(x),x]+[d(x),\alpha(y)][\alpha(x),x]y+y[[d(x),\alpha(y)],x]\alpha(x)+[[d(x),\alpha(y)],y]x\alpha(x)=0$ *for all* $x,y\in U$. *(33)*

Replacing $\alpha(U)$ *by* U, y *by* x *and* x *by by* $-x$ *with using (31), we get*

$2[[d(x),x],x]x=0$ *for all* $x\in U$. *Since R is 2-torsion free with using Lemma 3 , we obtain R contains a non-zero central.*

Theorem 3.6

Let R be a semiprime ring and U anon-zero ideal of R. Suppose that there exist a derivation $d:R\to R$ *and an automorphism* $\alpha:R\to R$ *suchthat* $[d(x)-x)x\pm x\alpha(x),x]=0$ *for all* $x\in U$. *Then R contains a non-zero central ideal.*

Proof: *We have* $[g(x)x+x\alpha(x)]=0$ *for all* $x\in U$.

Where g(x) stands for d(x)-x, then

$[g(x),x]x+x[\alpha(x),x]=0$ *for all* $x\in U$. *(34)*

Putting in the above relation xy for x, gives

$g(x)[y,x]yxy+x[g(x),y]yxy+[g(x),x]y^2xy+x^2[g(y),y]xy+x[g(y),x]yxy+x[x,y]g(y)xy+xy\alpha(x)x[\alpha(y),y]+xy\alpha(x)[\alpha(y),x]y+xyx[\alpha(x),y]\alpha(y)+xy[\alpha(x),x]y\alpha(y)=0$ *for all* $x,y\in U$. *Replacing* $\alpha(y)$ *by* y, *we obtain*

$g(x)[y,x]yxy+x[g(x),y]yxy+[g(x),x]y^2xy+x^2[g(y),y]xy+x[g(y),x]yxy+x[x,y]g(y)xy+xy\alpha(x)[y,x]y+xyx[\alpha(x),y]y+xy[\alpha(x),x]y^2=0$ *for all* $x,y\in U$. *Replacing y by x, we obtain*

$x[g(x),x]x^3+[g(x),x]x^4+x^2[g(x),x]x^2+x[g(x),x]x^3 \quad +x^3[\alpha(x),x]x+x^2[\alpha(x),x]x^2=0$ *for all* $x\in U$.*(33)*

From(34),the relation (33)reduces to

$x[g(x),x]\ x^3+[g(x),x]\ x^4=0$ *for all* $x\in U$.*Then*

$[g(x),\ x^2]\ x^3=0$ *for all* $x\in U$.*Thus,we have*

$[d(x),\ x^2]\ x^3=0$ *for all* $x\in U$. *Now,we have*

$W(x)x=0$ *for all* $x\in U$. *Where W(x) stands for* $[d(x),x^2]x^2$ *with using Lemmas(2 and 3), we obtain R contains a non-zero central ideal.*

Similary for $[d(x)-x)x-x\alpha(x),x]=0$ *for all* $x\in U$.

Theorem 3.7

Let R be a semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\to R$ *and an automorphism* $\alpha:R\to R$ *such that* $[x(d(x)-x)\pm\alpha(x)x,x]=0$ *for all* $x\in U$. *Then R contains a non-zero central ideal.*

Proof: *We have* $x[g(x),x]+[\alpha(x),x]x=0$ *for all* $x\in U$. *(36)*

Where g(x) stands for d(x)-x, then replacing x by xy, we obtain

$xyg(x)[y,x]y+xyx[g(x),y]y+xy[g(x),x]y^2+xyx^2[g(y),y]+xyx[g(y),x]y+xyx[x,y]g(x)+\alpha$

$(x)x[\alpha(y),y]xy+\alpha(x)[\alpha(y),x]yxy+x[\alpha(x),y]\alpha(y)xy+[\alpha(x),x]y\alpha(y)xy=0$ for all $x,y\in U$.
Replacing $\alpha(y)$ by y and y by x, we obtain
$x^3[g(x),x]x+x^2[g(x),x]x^2+x^4[g(x),x]+x^3[g(x),x]x+x[\alpha(x),x]x^3+[\alpha(x),x]x^4=0$ for all $x\in U$. (37)
Replacing $\alpha(x)$ by x, we get
$x^3[g(x),x]x+x^2[g(x),x]x^2+x^4[g(x),x]+x^3[g(x),x]x=0$ for all $x\in U$. (38)
Substituting (38) in (37) ,we obtain
$x[\alpha(x),x]x^3+[\alpha(x),x]x^4=0$ for all $x\in U$. (39)
$[\alpha(x), x^2]\, x^3=0$ for all $x\in U$. (40)
$M(x)x=0$ for all $x\in U$.
Where $M(x)$ stands for $[\alpha(x), x^2]\, x^2$,by Lemma2,we get
$M(x)=0$ for all $x\in U$.Thus
$[\alpha(x), x^2]=0$ for all $x\in U$. (41)
From (36),we obtain according to(41)
$xA(x)=0$ for all $x\in U$.
Where $A(x)$ stands for $[g(x),x]-[\alpha(x),x]=0$ for all $x\in U$. ,by Lemma2,we get
$A(x)=0$ for all $x\in U$.Thus
$[\alpha(x),x]=[g(x),x]$ for all $x\in U$. (42)
Substituting (42) in (36) , we obtain
$[d(x),x^2]=0$ for all $x\in U$. By applying Lemma3 ,we get
R contains a non-zero central ideal.
Simililarly for $[x(d(x)-x)-\alpha(x)x,x]=0$ for all $x\in U$.

Theorem 3.8

Let R be a semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[xd(x)\pm x(\alpha(x)-x),x]=0$ for all $x\in U$.Then R contains a non-zero central ideal.

Proof: We have $xM(x)=0$ for all $x\in U$.
Where $M(x)$ stands for $[d(x)+(\alpha(x)-x),x]$, then by using Lemma2, above relation gives
$[d(x),x]+[g(x),x]=0$ for all $x\in U$. (43)
Where $g(x)$ stands for $(\alpha(x)-x)$,the linearization of the relation (43) gives
$[d(x),y]+[d(y),x]+[g(x),y]+[g(y),x]=0$ for all $x, y\in U$, which means that we have
$[d(x)+g(x),y]+[d(y)+g(y),x]=0$ for all $x, y\in U$. (44)
Replacing y by yx with using (43), we obtain
$[d(x)+g(x),y]x+[d(y)+g(y),x]x+y[d(x)+g(x),x]+[y,x](d(x)+g(x)=0$ for all $x\in U$, $y\in R$. According to (43)and (44),we obtain $[y,x]B(x)=0$ for all $x, y\in U$.Where $B(x)$ stands for $(d(x)+g(x))$, by using Lemma 4, we get R contains a non-zero

central ideal.

Similary for $[xd(x)-x(\alpha.(x)-x),x]=0$ for all $x\in U$.

By same method we can be prove the following theorem.

Theorem 3.9

Let R be a semiprime ring and U a non-zero of R . Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[xd(x)\pm x(\alpha(x)-x),x]=0$ for all $x\in R$. Then R is commutative.

Theorem 3.10

Let R be a 2-torsion free semiprime ring and U a non-zero of R . Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[[d(x),x]\pm\alpha(x),x]=0$ for all $x\in U$. Then R contains a non-zero central ideal.

Proof: *We have*

$[[d(x),x],x]+[\alpha(x),x]=0$ *for all* $x\in U$. (45)

The linearization of the above relation gives

$[[d(x),x],x]+[[d(x),y],x]+[[d(x),x],y]+[[d(x),y],y]+[[d(y),x],x]+[[d(y),y],x]+[[d(y),x],y]+[[d(y),y],y]+[\alpha(x),x]+[\alpha(x),y]+[\alpha(y),x]+[\alpha(y),y]=0$ *for all* $x\in U, y\in R$.

Replacing y by x and according to (45),we get

$4[[d(x),x],x]=0$ for all $x\in U$.Applying that R has a 2-torsion free with using Lemma 3, we completes the

proof of the theorem.

Similary for $[[d(x),x]-\alpha(x),x]=0$ for all $x\in U$.

Theorem 3.11

Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R . Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[d(x)\pm[\alpha(x),x],x]=0$ for all $x\in U$. Then R is contains a non-zero central ideal.

Proof*: We have*

$[d(x),x]+[[\alpha(x),x],x]=0$ for all $x\in U$. Which means that we have

$[d(x),x]+[\alpha(x),x]x-x[\alpha(x),x]=0$ *for all* $x\in U$. (46)

The linearization of the above relation with using (46) after some calculation gives

$[d(x),y]+[d(y),x]+[\alpha(x),x]y+[\alpha(x),y]x+[\alpha(x),y]y+[\alpha(y),x]x+[\alpha(y),x]y+[\alpha(y),y]x-y[\alpha(x),x]-x[\alpha(x),y]-y[\alpha(x),y]-x[\alpha(y),x]-y[\alpha(y),x]-x[\alpha(y),y]=0$ *for all* $x\in U$, $y\in R$.

Replacing y by x, we obtain

$2([d(x),x]+[\alpha(x),x]x-x[\alpha(x),x])+4([\alpha(x),x]x-x[\alpha(x),x])=0$ for all $x\in U$. According to (46) the above relation reduce to

$4([\alpha(x),x]x-x[\alpha(x),x])=0$ *for all* $x\in U$. (47)

Applying that R has a2-torsion free on (47) and subtituting the results in (46), gives $[d(x),x]=0$ for all $x\in U$.Applying Lemma 3, we completes the proof of theorem.

Similarly for $[d(x)-[\alpha(x),x],x]=0$ for all $x\in U$.

Theorem 3.12

Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R. Suppose that there exist a derivation $d:R\rightarrow R$ and an automorphism $\alpha:R\rightarrow R$ such that $[[d(x)\pm\alpha(x),x],x]=0$ for all $x\in U$. Then R is contains a non-zero central ideal.

***Proof**: We have $[[d(x)+\alpha(x),x],x]=0$ for all $x\in U$. Then*

$$[[d(x),x],x]+[[\alpha(x),x],x]=0 \text{ for all } x\in U. \quad (48)$$

Putting x by x+y , we obtain after some calculation,

$[[d(x),x],x]+[[d(x),x],y]+[[d(x),y],x]+[[d(x),y],y]+[[d(y),x],x]$ $[[d(y),x],y]+[[d(y),y],x]+[[d(y),y],y]+[[\alpha(x),x],x]+[[\alpha(x),x],y]+[[\alpha(x),y],x]+[[\alpha(x),y],y]+[[\alpha(y),x],x]+[[\alpha(y),x]+[[\alpha(y),y],x]+[[\alpha(y),y],y]=0$ *for all* $x,y\in U$.*According to (48) , replacing $\alpha(y)$ by y and y by x, we obtain*

$3[[d(x),x],x]=0$ for all $x\in U$. Since R is a2-tosion free with applying Lemma3 ,we completes the proof.

Similarty for $[[d(x)-\alpha(x),x],x]=0$ for all $x\in U$.

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حول الاوتومورفزمات مع الاشتقاقات على الحلقات شبة الاولية

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الملخص:الغرض الرئيسي من البحث تحري الاوتومورفزمات والاحادية مع الاشتقاقات على الحلقات شبة الاولية R , وسوف نحصل على R تحوي على مثالي مركزي غير صفري .