الخلاصة:

في خطط عينات القبول للفحص التمييزي يعتمد على قرار قبول أو رفض الدفعة المنتجة على معالم خطة المعاينة الناتجة من تصغير معدل الكلفة الكلية المتوقعة للسيطرة النوعية , تحت شروط أن توزيع النوعية هو من النوع المختلط (ثنائي الحدين مختلط) (أو بواسون مختلط) وأن معدل النوعية يتغير من دفعة الى أخرى طبقا إلى توزيع احتمالي معين , من المفروض ان يكون هذا التوزيع قابل للاشتقاق في كل النقاط المجاورة للنقطة الحرجة لمستوى النوعية , إذ يهدف بحثنا الى اشتقاق معالم خطة المعاينة المفردة المثلى (n, c, τ) والتي تمثل(n- حجم العينة c, -عدد القبول, τ- الفترة الزمنية بين حصول الفشلات) , وسوف تعتمد طريقة البحث المتعدد للحصول على النتائج .

Abstract

In attributes sampling acceptance plan the decision to accept or to reject the lot can be made, by using sampling plan obtained by minimizing the average total expected cost, under assumption that the distribution of lot quality is a mixed Binomial or mixed Poisson, i.e. each lot is produced by a process under Binomial or Poisson control, but the average process varies from lot to lot according to a frequency distribution which assumed to be differentiable in the neighborhood of the break- even value. The purpose of this paper is to derive the parameter of optimal sampling plan (n, c, τ), which represent the sample size (n) and acceptance number (c) and time interval (τ) between failures, by minimizing the expected value of total cost function. We use the method of multivariate search technique and partial enumeration procedure. Some auxiliary examples are given.

Introduction:

Quality control is considered as a single system from complete production system in factory. This system is correlated with other system including putting specification and improving the quality, to satisfy this we must have complete plan for quality control in order to give a procedure for testing the product and prevent defective. In this paper we introduce or build a model for total expected cost for quality control, and derived from it the parameters of sampling plan, which are used for testing product and identify quality of product, instead of total inspection. These parameters are (n, c, τ), which they are represent the optimal sample size number (n) and acceptance number (c) and time interval (τ) between successive testing. This model achieve the continuous controlling for product and improving quality always. Our model represent a modification for Schmidt- Taylor (1973) PP (151- 167) model which define P(t) (the proportion of defective within time t) as:

$$P(t) = \frac{P_o t + p_1 (\tau - t)}{\tau} \qquad (1)$$

This formula of percentage of defective at time (t) given by Schmidt- Taylor in (1) is modified in the proposed model to the following:

$$p(t) = P_0\,(T \leq t \mid T < \tau) + p_1\,pr\,(t < T < \tau \mid T < \tau) \qquad (2)$$

Equation (2) represents a general form for percentage of defective at time t, which is appropriate for various probability distributions in application field. We first define all assumptions and notations for the proposed model and then apply it on the distribution for defective units which is Binomial with (n, p) and the distribution of time continued until the failure in quality happens, which is assumed to be negative exponential distribution with mean $\left(\frac{1}{\lambda}\right)$

The optimality procedure used to obtain the optimum values for decision variables (n, c, τ) is a composition from multivariate search techniques and partial enumeration procedure. Some auxiliary tables are given with examples of application. The accuracy of proposed model has been evaluated numerically from relative efficiency equation:

$$e(n,c,\tau,N)=\frac{k_0(n_0,c_0,\tau_0,N_0)}{k_1(n_1,c_1,\tau_1,N_1)}$$

where the numerator represents the value of expected cost due to proposed model and the denominator for Schmidt-Taylor model.

Notation and Assumption of Model

The followings are the notations and assumptions for proposed model:

1. P_0: percentage of defective in normal time and assumed fixed.

2. P_1: percentage of defective under not normal(or abnormal) condition and assumed constant ($P_1 > P_0$).
3. P(t): percentage of defective in lot produced when the break- down happens in production line at time t from starting operation of production of lot N ($t < \tau$).
4. Time spend until failure happen is random variable exponentially distributed with mean $\left(\frac{1}{\lambda}\right)$ i.e

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t} & t > 0 \\ 0 & \text{other wise} \end{cases}$$

5. τ: time interval between successive testing.
6. n: sample size

N: lot size ($N = \Psi T$)

ψ: average of production (unit).

C_I: cost of Inspection per unit.

C_R: cost of Rejecting good unit.

C_A: cost of Accepting of defective unit.

C_F : loss due to stopping production line.

P_d : probability of failure in production system during production of lot N.

f_b : This event indicates the production unit is in case of failure at starting of producing lot N, this failure is due to random and caused effect which happens during time, which increase P_0 to P_1.

f_t : This event represent failure of production unit at time t, during production of N when (t < T).

f_d: This event represent failure of production unit at some time during production N.

Q_i: probability that production system in case of failure after ith testing.

$P(A/f_b)$: probability of accepting produced lot when failure occurs at starting time of producing lot(N).

$P(A/f_t)$:probability of accepting lot when failure occurs at any time (t) during producing lot(N).

$P(A/f_d)$: probability of accepting lot when failure occurs at any point of time during production lot (N).

$P(A/\overline{f_b}\ \overline{f_d})$: probability of accepting lot, when failure doesn't occur until producing the lot N.

$P_x(x/n,p)$: probability dist. of x defective in a sample of size n.

B: probability (in case of stationarity) that the production system is in case of failure after each operation of testing this mean that the new lot begin on production line where percentage of defective is p_1 i,e:

$$Q_1:\ p(d)P(A/f_d) \tag{4}$$

From value of Q_1 we can find Q_2 which represents the probability of failure after second testing.

$$Q_2 = Q_1\ P(A/f_b) + (1 - Q_1)\ p(d)P(A/f_d) \tag{5}$$

$$= Q_1\ P(A/f_b) + (1 - Q_1)\ Q_1$$

$Q_2 = Q_1[P(A/f_b) + (1 - Q_1)]$ (6)

Similarly

$Q_3 = Q_2\ P(A/f_b) + (1 - Q_2)Q_1$

$= Q_1[P(A/f_b) + (1 - Q_1)]\ P(A/f_b) + Q_1 - Q_1^2[P(A/f_b) + (1 - Q_1)]$

. . . (7)

$= Q_1\{[(P(A/f_b) - Q]^2 + [P(A/f_b) - Q_1] + 1\}$

In same way, we can find that the probability of the production system in case of failure after testing j is:

$$Q_j = Q_1 \sum_{x=0}^{J-1} [p(A/f_b) - Q_1]^x \qquad (8)$$

there for $B = \lim_{j\to\infty} Qj = \dfrac{Q_1}{1 - P(A/f_b) + Q_1}$ (9)

where $Q_1 = \sum_{x=0}^{c} \int_0^{\tau} f_T(t)\, p_x(x \mid n, P(t))\,dt$ (10)

Building The Model

The expected cost function for proposed model consist of four component, which are:

1. Expected inspection cost.
2. Expected rejection cost.
3. Expected acceptance cost.
4. Loss of stopping.

First of all, we simplify the formula of P(t) defined in equation (2) as follows:

1. $$\Pr(T \le t \mid T < \tau) = \frac{\int_0^t \lambda e^{-\lambda x}\, dx}{\int_0^\tau \lambda e^{-\lambda x}\, dx} = \frac{1 - e^{-\lambda t}}{1 - e^{-\lambda \tau}} \qquad (11)$$

2. $$\Pr(t < T < \tau \mid T < \tau) = \frac{\int_t^\tau \lambda e^{-\lambda x}\, dx}{\int_0^\tau \lambda e^{-\lambda x}\, dx} = \frac{e^{-\lambda \tau} - e^{-\lambda t}}{1 - e^{-\lambda \tau}} \qquad (12)$$

After substitute equation 11 and 12 in 2 we find that:

$$P(t) = \frac{e^{-\lambda t}(p_1 - p_0) + (p_0 - P_1 e^{-\lambda \tau})}{(1 - e^{-\lambda \tau})}$$

Now we derive the value of expected total cost function assuming the distribution of number of defective units in the sample is B(n, p).

$$f(x, n, p) = \begin{matrix} C_x^n P^x q^{n-x} & x = 0,1,2..n \\ 0 & o/w \end{matrix} \qquad (14)$$

According to this we define:

1. $P_x(x \mid n, P_0) = C_x^n P_0^x (1 - P_0)^{n-x}$ (15)

2. $P_x(x \mid n, P_1) = C_x^n P_1^x (1 - P_1)^{n-x}$ (16)

3. $P_x(x \mid n, P(t)) = C_x^n (P(t))^x (1 - P(t))^{n-x}$ (17)

4. $\int_0^\tau P_x[x \mid n, P(t)] = \int_0^\tau C_x^n (p(t))^x (1 - P(t))^{n-x} \lambda e^{-\lambda t} dt$ (18)

since we know that

$$\frac{dP(t)}{dt} = \lambda e^{-\lambda t}(P_0 - P_1)/(1-e^{-\lambda\tau})$$

Also $IB_u(\alpha, B) = \frac{1}{B(\alpha, B)}\int_0^u Z^{\alpha-1}(1-Z)^{B-1}dz$ (19)

According to this formula, the integration in equation (18) simplified to:

$$\int_0^\tau P_x[x \mid n, P(t)\ ]f_T(t)dt$$

$$= \frac{(1-e^{-\lambda\tau})}{(P_0 - P_1)} C_x^n B(x+1, n-x+1)\int_{P_1}^{P_0} \frac{(P(t))^x(1-P(t))^{n-x}}{B(x+1, n-x+1)^{dp(t)}} \qquad (20)$$

$$= \frac{1-e^{-\lambda\tau}}{(P_1 - P_0)} C_x^n B(x+1, n-x+1)\int_{P_0}^{P_1} \frac{[P(t)]^x(1-P(t))^{n-x}}{B(x+1, n-x+1)} dP(t) \qquad (21)$$

Using integral in (19) we find that equation (21) reduced to:

$$= \frac{1-e^{-\lambda\tau}}{P_1 - P_0}\frac{n!}{(n-x)!x!}\frac{x!(n-x)!}{(n+1)!}[IB_{P1}(x+1, n-x+1) - IB_{P0}(x+1, n-x+1)] \qquad (22)$$

Since n, x are integers then we use the relation between incomplete beta and cumulative Binomial function as follows:

$$IB_{P0}(\alpha, B) = \sum_{i=\alpha}^{n} C_i^n P_0^i(1-P_0)^{n-i} \qquad , n = \alpha + B - 1 \qquad (23)$$

$$IB_{P0}(\alpha, B) = \sum_{i=\alpha}^{n} C_i^n P_1^i(1-P_1)^{n-i} \qquad (24)$$

$$IB_{P0}(x+1, n-x+1) = \sum_{i=x+1}^{n+1} C_i^{n+1} P_0^i(1-P_0)^{n+1-i} \qquad (25)$$

$$IB_{P1}(x+1,n-x+1)=\sum_{i=x+1}^{n+1}C_i^{n+1}P_1^i(1-P_1)^{n+1-i} \tag{26}$$

The above relations reduced to:

$$IB_0(x+1,n-x+1)=E(x+1,n+1,P_0) \tag{27}$$

$$IB_1(x+1,n-x+1)=E(x+1,n+1,P_1) \tag{28}$$

Therefore integral in equation (20) become

$$\int_0^{\tau}P_x[x\,|\,n,p(t)]f_T(t)dt$$
$$=\frac{1-e^{-\lambda\tau}}{(P_1-P_0)(n+1)}[E(x+1,n+1,P_1)]-E(x+1,n+1,P_0)] \tag{29}$$

Similarly we can solve the integral

$$\int_0^{\tau}P(t)P_x[x\,|\,n,p(t)]f_T(t)dt$$
$$=\frac{1-e^{-\lambda\tau}(x+1)}{(P_1-P_0)(n+1)(n+2)}[E(x+2,n+2,P_1)]-E(x+2,n+2,P_0)]=S^{**} \tag{30}$$

Another value which must be computed according to binomial distribution is the value of Q_1 (probability of the failure after first testing):

$$Q_1=\sum_{x=0}^{c}\int_0^{\tau}C_x^n(P(t))^x(1-p(t))^{n-x}\lambda e^{-\lambda\tau}dt$$

$$=\frac{1-e^{-\lambda\tau}}{(P_1-P_0)(n+1)}\sum_{x=0}^{c}[E(x+1,n+1,P_1)]-E(x+1,n+1,P_0)] \tag{31}$$

Using Q_1 we find:

$$B=\frac{Q_1}{Q_1+1-P(A\,|\,fb)}$$

$$B=\frac{\sum_{x=0}^{c}[E(x+1,n+1,P_1)]-E(x+1,n+1,P_0)]}{\sum_{x=0}^{c}[E(x+1,n+1,P_1)]-E(x+1,n+1,P_0)]+\frac{(n+1)(P_1-P_0)}{1-e^{-\lambda\tau}}E(c+1,n,P_1)}$$

(32)

Since $P(A|\ f_b)=\sum_{x=0}^{c}C_x^n p_1^x(1-P_1)^{n-x}=B(c,n,P_1)$

Therefore 1-$P(A|\ f_b)$ = E(c+1, n, P_1)

The final formula for the expected total cost function of quality control under binomial processing, which consists of the sum of four components, A_1, A_2, A_3, A_4, is given by equation

$$T.C=C_I n+C_R\{P_1 nB+n(1-B)[P_0-P_1e^{-\lambda\tau}+$$
$$\frac{1}{2}(P_1-P_0)\frac{1-e^{-2\lambda\tau}}{1-e^{-\lambda\tau}}+P_0e^{-\lambda\tau}]$$
$$+(N-n)[1-BB(c,n,P_1)-(1-B_1)Q_1-(1-B)e^{-\lambda\tau}B(C,n,P_0)]\}$$
$$+CA(N-n)\{P_1BB(c,n,P_1)+(1-B)S^{**}+P_0(1-B)e^{-\lambda\tau}B(C,n,P_0)\}$$
$$+CF\{BE(c+1,n,P_1)+(1-B)(1-Q_1)$$
$$+(1-B)e^{-\lambda\tau}E(C+1,n,P_0)\} \tag{33}$$

where S^{**} are defined in equation (30) and Q_1 by equation (31).

Application of Model

The proposed model and Schmidt- Taylor[3] model, the two were applied on the following data

ψ= 700 units/ hr	CI= 0.009 ID/ unit
λ= 0.1667/ hr	CR= 0.260 ID/ unit

$P_0 = 0.004265$ CA = 0.742 ID/ unit

P1 = 0.05 CF = 288 ID/ shut down

We write the program for equation (33) and apply multivariate search technique and partial enumeration procedure we find the optimum sampling plan according to binomial sampling is:

$n^* = 235$ units, $C^* = 6$ units

$\tau^* = 3.5$ hours, T. cost = 324. 540 ID

Also we apply the same procedure on Schmidt- Taylor model, we obtain the optimal plan is:

$n_1^* = 400\, units,$ $c_1^* = 7\ units$

$\tau_1^* = 8.5\ hours$ $T.C = 386.287\ ID$

then we find the efficiency of proposed model compared with Schmidt – Taylor as:

$$e(n,c,T) = k_0(n_0,c_0,\tau_0)/k_1(n_1,c_1,\tau_1) = \frac{324.540}{386.287} = 85\%$$

this number indicates that the cost of sampling and inspection and loss due to stopping production can be reduced by 15% when we the plan of proposed model instead of Schmidt- Taylor model. Finally we compare the values of total cost for two models for various values of p_0, p_1, λ, and results are tabulated below:

Table (1)
Comparison of cost function for two models according to change in p_0

P_0	Cost of proposed model	Cost of Schmidt-Taylor
0.001	323.955	353.042
0.002	325.679	360.982
0.003	327.340	372.410
0.004	328.940	386.287
0.005	330.480	392.510
0.006	331.958	405.719
0.007	333.374	430.916
0.008	334.725	451.520
0.009	336.013	471.960
0.010	337.238	486.913
0.120	499.657	612.513
0.14	550.630	618.901
0.16	501.624	625.419
0.18	502.630	634.614
0.20	503.643	645.003

Table (2)
Results of comparison of two costs when p_1 varied

P_1	Cost of proposed model	Cost of Schmidt-Taylor
0.01	14.316	78.921
0.02	47.690	120.043
0.03	151.106	187.974
0.04	267.238	294.611
0.05	328.940	386.287
0.06	344.695	414.578
0.07	340.468	462.318
0.08	330.481	498.513
0.09	314.699	518.920
0.10	309.414	577.440
0.20	238.453	583.470
0.30	199.417	618.920

Table 3
Comparison of cost when λ varied

λ	Cost of proposed model	Cost of Schmidt-Taylor
0.136	306.873	375.413
0.1667	328.940	386.287
0.332	382.995	415.917
0.498	402.231	460.203
0.664	411.456	482.998
0.830	416.520	504.376
0.996	419.507	542.004
1.162	421.344	588.612
1.3287	422.508	592.118
2.5	424.534	616.912
4	424.642	663.420
6	424.644	690.555

Conclusion:

Using the proposed model to inspect the product which produced by big lot, when total inspection is very costly, also this model includes continuing processing for product and prevents the defective, also the given model reduces the cost of quality control as compared with Schmidt- Taylor, in table 1 and 2 and 3 the results indicate this fact.

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