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A Critical Analytical Review of Hybrid Grey Models and Their Applications in Forecasting Non-Stationary Time Series

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ABSTRACT

Even so, forecasting non-stationary time series remains challenging due to the unavailability of data, uncertainties, and highly time-variable patterns. Due to the unique advantages of grey system theory, such as the low-data requirement, hybrid grey models have gained more and more extensive attention as hybrid models, which are realized by merging grey system theory with other intelligent methods, including fuzzy logic, artificial neural networks, met heuristic optimization, and deep learning, and have displayed significant flexibility and accuracy. This study provides a systematic and critical review of hybrid grey models, encompassing their structural framework, integration strategies, and applications in various forecasting domains. The nested, parallel, and serial hybridization approaches are then categorized, and the trade-offs between model sophistication and predictive performance are discussed. The significant drawbacks, including interpretability issues, the risks of over fitting, and the lack of standard benchmarking protocols, are also highlighted. Ultimately, the paper outlines potential research paths, including explainable AI, probabilistic reasoning, and automated mechanisms for model selection. The provided insights serve as a valuable reference for researchers and practitioners seeking to design hybrid grey forecasting models that are effective in uncertain environments.

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1. Introduction

Predicting non-stationary time series is a challenging task in many scientific and engineering applications, especially when the known or available series are uncertain, incomplete, or sparse. Traditional statistical models (e.g., ARIMA, exponential smoothing methods) are valid under several assumptions; however, they often fail when applied to complex, nonlinear, or short-term series data [5]. Similarly, machine learning approaches rely on extensive training data and computational capacity [2].

Deng first initiated GST in the early 1980s and has proven to be an effective modeling tool that can address uncertainty and partial information [8]. Due to their suitability for small samples and incomplete data, as well as their applicability in short-term predictions of dynamic systems, grey models, especially GM (1,1), have been widely used in recent years [24]. However, classical grey models are not free from deficiencies, especially in handling nonlinear relationships, multivariable interrelationships, and structural disturbance available in non-stationary series [21].

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To address these limitations, advanced grey models (AGMs), which are hybrids of grey systems with optimization algorithms, artificial intelligence algorithms [17], and statistical modifications [9], have been proposed in the literature. These hybrid grey models aim to integrate the solidness and interpretability of grey models from AI technology with the learning ability of AI systems (e.g., PSO, GA, ANN, and fuzzy logic controller).

However, among the increasing number of studies published on the hybrid grey model, an integrated analytical review still does not exist that critically reflects on the application of the hybrid grey model in construction, particularly for forecasting non-stationary time series. Existing works either proposed new model variants or investigated problem-related applications, but no systematic comparison and theoretical limits have been achieved [7].

The purpose of this paper is to offer an up-to-date critical survey of the hybrid grey model. In particular, the objective of this paper is to: (1) categorize HGMs according to their integration mechanism; (2) review their prediction performance in a non-stationary time series framework; (3) examine pros and cons of HGMs; (4) suggest how to improve HGM structure, interpretability, and applicability. Such an attempt can serve as a reference book for researchers and practitioners who aim to identify the potential of hybrid grey modeling frameworks that exist and continue to emerge. Please do not change the margins of the template as this can result in the footnote falling outside printing range.

2. Theoretical Background

2.1 Foundations and Motivation of Grey System Theory

Grey system theory (GST) was first proposed by Julong Deng in 1982, which is a new mathematical modeling approach for dealing with partially known information systems under uncertainty [8]. In contrast to classical probabilistic approaches, which are overly dependent on large sample sizes and prior information, GST addresses systems for which empirical data is both incomplete and vague or imprecise. It is therefore particularly appropriate whenever dealing with complex real-world problems, such as economic forecasting, environmental prediction, and engineering diagnosis [14].

In essence, GST classifies systems according to their degree of in formativeness as follows:

White systems: completely known (deterministic, for example).

Black systems: vague (such as random systems).

Grey systems: partially acceptable — the target of GST.

This taxonomy should serve as a basis for developing models that derive rules from incomplete and uncertain observations. GST focuses on the “generating sequences” (i.e., with data pre-processing by accumulation) and “modeling sequences” (i.e., predicting responses) from as few data points as possible [3].

One significant benefit of GST is that this technique can identify dynamic trends from small datasets where GST demands as few as four or five data points to generate a feasible prediction model. This low-data dependence is what sets GST apart from statistical models, such as ARIMA or neural networks, which generally require much larger datasets to generalize [4].

Within contemporary contexts, GST is highly appreciated for:

- Model simplicity and computational efficiency.

- Ability to model non-linear and short-term behavior.

- Compatibility with AI and optimization methods (in hybrid models).

All of the above make the grey model widely applicable, such as in forecasting energy consumption and hydrological series, financial time series, and resource allocation under uncertainty [21], [9].

2.2 Fundamental Grey Forecasting Models: GM(1,1) and GM(1,N)

One of the classical grey system models, the GM (1,1) model, is a first-order, one-variable, discrete model for short-term forecasting with limited and uncertain data. Its power comes from extracting patterns from small iterations through a technique called the Accumulated Generating Operation.

We assume that the initial data sequence is a non-negative sequence:

$$X^{(0)} = \{ X^{(0)}(1), X^{(0)}(2), \dots, X^{(0)}(n)\} \tag{1}$$

The AGO sequence is then given by:

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i) \quad k = 1, 2, \dots, n \tag{2}$$

The GM(1,1) makes a first-order difference equation:

$$\frac{dX^{(1)}(t)}{dt} + aX^{(1)}(t) = b \tag{3}$$

Where:

a: is the development coefficient.

b: is the grey input or background value.

The solution to this equation is:

$$\hat{X}^{(1)}(k+1) = \left(X^{(0)}(1) - \tfrac{b}{a}\right)^{-ak} + \tfrac{b}{a} \tag{4}$$

Finally, the predictions are recovered with the inverse AGO (IAGO):

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k) \tag{5}$$

Although the GM(1,1) model is elegant and straightforward, it requires that the data follow an exponential pattern, making it unsuitable for nonlinear or fluctuating data processes.

In response to such a problem, the model GM(1, N) was proposed, which involves $N-1$independent factors that impact a dependent factor in a prompt manner. Its mathematical representation extends to multiple input series.

$$\frac{dX^{(1)}(t)}{dt} + a_1 Z^{(1)}(t) + a_2 Y_1^{(1)}(t) + \cdots + a_n Y_{N-1}^{(1)}(t) = b \tag{6}$$

Where each $Y_i^{(1)}$ is the AGO-transformed auxiliary series. a is denotes the development coefficient, b represents the grey input, $X^{(0)}$ is the original data sequence, and $X^{(1)}$ is the accumulated generating sequence.

This is widely used to model variables such as agricultural planning, environmental monitoring, and energy demand prediction [11], [14]. However, it is also based on the structure of grey models. It thus may need to be improved, such as through hybridization, which is adapted to the nonlinearities and multicollinearity characteristics of the input variables.

2.3 Advanced Grey Models: NGM, DGM, and FGM

To address the low flexibility and adaptability issues with classical grey models, numerous revised models have been designed to achieve high computational accuracy. The Nonlinear Grey Model (NGM), Discrete Grey Model (DGM), and Fuzzy Grey Model (FGM) are among the most extensively studied improvements.

Nonlinear Grey Models (NGM)

Besides, NGM aims to grasp the nonlinear relations that the classical GM(1,1) does not. The inclusion of nonlinear terms or transformations of the time series can improve model fit, especially for non-monotonic and oscillatory patterns [16]. For example, polynomial or exponential nonlinearities can be introduced directly into the Differential Equation form or using nonlinear optimization [10].

Discrete Grey Models (DGM)

The DGM models were introduced to address the problem of approximation error introduced by the continuous-time representation model, specifically the GM(1,1) model. In contrast to difference operators (or their analogues), DGM models employ difference equations (or their analogues) discretely, thereby rendering the model equations closer analogues to those of digital data structures, digital facilities, or digital models [13]. They facilitate the prediction within a short horizon and on coarse-grained or irregular time scales.

A common form of DGM(1,1) is:

$$\Delta X^{(1)}(k) = X^{(1)}(k) - X^{(1)}(k-1) \tag{7}$$

This separate structure makes the numbers more stable while maintaining the predictive power of grey theory.

Fuzzy Grey Models (FGM)

FGMs incorporate fuzzy logic with the grey modeling framework to accommodate linguistic or qualitative uncertainty in the data. These are powerful models that are especially useful in dealing with human-based or perception-based systems, such as expert opinion, risk assessment, and consumer satisfaction analysis [18]. Fuzzy logic can be used to express the vagueness in input data or model parameters with fuzzy numbers or sets, which makes the model more flexible, robust, and interpretable [12].

In applications, FGMs are typically used in decision support systems and supply chain management, as they can provide accurate forecasts when data is imprecise, as usual crisp models would likely underperform [3].

2.4 Grey Hybrid Model AI and Optimization Integration

Recently, there has been a growing interest in HGMs because they can be used to overcome the deficiencies of classical grey models in processing time series contaminated with noise, exhibiting a high degree of nonlinearity, and high dimensionality. The basic idea of HGM is to combine the scale value efficiency of grey system with the interpretability and learning power of artificial intelligence: [48]

Integration with Optimization Algorithms

Its hybridization with meta-heuristic algorithms, including PSO, GA, WOA, and ACO, has been commonly utilized to improve the parameter estimation of GMs. These control strategies are used to fine-tune the flexible model parameters, reduce the fitting error, and control the sensitivity of the initial value indicated by the model, such as the GM (1,1) [19], [20].

Integration with Neural Networks

In addition, various types of neural networks, including RNN and LSTM, have been proposed for the nonlinear dynamic prediction of air quality, and they are integrated with grey models to enhance their nonlinear learning ability. In the grey model, the pretreatment (AGO: Accumulated Generating Operation) of raw data is converted into a neural network, and then a prediction is made based on the learned pattern [7]. They have demonstrated good performance in prediction accuracy and dynamic adaptability, especially in energy, finance, and climate prediction [1].

Integration with Statistical Models (Box-Jenkins, ARIMA)

Some works have investigated the concept of hybridizing grey models with traditional statistical models, such as ARIMA or Box–Jenkins models. In such systems, the grey model is used to remove trend components, while the ARIMA is employed to extract residual patterns or cycles [27],[47]. Such two-layer modeling is essentially to introduce complementarity between the deterministic and stochastic parts of the data. A typical ARIMA model has the following form:

$$\phi(B)(1-B)^d X_t = \theta(B)_\varepsilon \tag{8}$$

Where:

B: is the backshift (lag) operator,

$\phi(B)$: and $\theta(B)$ are polynomials for autoregressive and moving average parts respectively,

d: is the differencing order.

This dual-layer approach enables the deterministic and stochastic parts to work together, making it easier to anticipate events on multiple scales.

Hybrid approaches can improve upon non-hybrid models, but they also pose some issues related to model complexity, computational complexity, and the risk of over fitting [6]. It follows that choice hybrid strategies must trade off gains in accuracy with interpretability and generalization.

2.5 Prediction Metrics and Evaluation: MAPE, RMSE, Cross-Validation

Performance comparison of forecasting models is a crucial step in evaluating their utility and identifying the conditions under which they can be effectively applied [34]. In the case of grey models – especially hybrid grey models – assessing the model's performance involves using a set of statistical measures to evaluate the errors and validating techniques that measure the accuracy, soundness, and generalization capability of the model.

Mean Absolute Percentage Error (MAPE)

MAPE is a well-known error metric in forecasting, defined as the median of the absolute percentage errors of predicted values compared to actual ones. The ratio is particularly desirable since it is scale-invariant and interpretable on a percentage scale:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right| \times 100 \tag{9}$$

Where A_i is the actual, and F_i is the forecasted value. Small values of MAPE mean that the prediction is more accurate. Yet, MAPE is susceptible to being affected when the actual values are close to zero [34].

RMSE (Root Mean Square Error)

RMSE takes the average of the square root of errors and is especially sensitive to large deviations; that is why it penalizes models that produce outliers or poor fits in regions of the data. It is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(A_i - F_i)^2} \tag{10}$$

It is a good practice to consider RMSE in conjunction with MAPE in different settings of time series that do not have the same variance (homoscedastic) scaling, as this is related to the complexity level of the time series [7].

Cross-Validation Techniques

For grey and grey hybrid models, validation on the same dataset can cause over fitting or exaggerated accuracy. It follows that cross-validation (CV) methodologies, particularly the K-fold cross-validation, are being used with increasing frequency for estimating model generalization. For K-fold CV, the data are partitioned into K training and testing sets, and the trained model uses K−1 training folds, testing on the remaining fold. This K-fold routine is repeated continuously with the averaged error estimates resulting in a stable one [28].

Rolling origin validation also exists in several grey model variants, which is essential for time series prediction where the past directly influences future values. These methods offer a more realistic measure of the model's performance in operational use [21].

3. Taxonomy of Hybrid Grey Models

3.1 Optimization-Based Hybrid Grey Models

The enclosure of meta-heuristic optimization methods is a major powerful approach in improving grey forecasting models [48]. They are used to optimize the automatically learned model parameters, such as the model's weights, the background model’s scaling factor, the developmental factor, or the input weight structures, to improve accuracy and robustness. PSO, GA, and WOA are the most commonly used optimization methods and the methods to combine hybrid grey models.

PSO-GM Models

Particle Swarm Optimization (PSO) has been intensively applied in parameter determination for GM (1,1), GM(1, N), and nonlinear grey models. PSO emulates bird flocking (1) to renew the candidate solutions in a brood-generating way and (2) to renew the potential solution taken from the best-known solutions based on the preying method. This work can effectively enhance the fitting precision of grey models because it prevents the models from falling into local minimum cone traps of least squares estimates [48]. For example, Wang et al. used PSO-GM for short-term electricity demand forecasting, demonstrating a more preferable effectiveness in terms of prediction (MAPE and RMSE) compared to traditional GM [20].

GA-GM Models

GAs are population-based algorithms that use evolutionary computing to search for optimal or suboptimal solutions in a complex space. Together with GMs, the GA can help fine-tune the model's structure, the feature subsets included, and the maximum generalization potential during training [31]. The GA-based hybrid grey models have been extensively applied in industrial output forecasting [19], agricultural production prediction [49], and financial market volatility estimation [49].

WOA-GM and Other Met heuristics

Meanwhile, technological advancements have led to the emergence of intelligent optimization algorithms, such as the Whale Optimization Algorithm (WOA), which is based on the bubble-net hunting behavior of humpback whales. WOA possesses the merits of a global convergence property and convergence stability, making it well-suited for hybridizing with grey models in noisy and non-linear time series [19]. Other met heuristic algorithms, such as Ant Colony Optimization (ACO), Firefly Algorithm (FA), and Grey Wolf Optimizer (GWO), provide specific search behavior and robustness to the PFFD [49].

So-called optimization-based hybrid grey models retain the original structural simplicity of the GM, while significantly improving forecasting accuracy and robustness. Nevertheless, their performance strongly depends on the fitness function, the parameter ranges, and how the algorithm is configured, and these need to be carefully adapted for each specific application.

3.2 Hybrid Grey Models Constructed by Neural Networks

Grey systems can be challenging to capture aspects such as nonlinear relationships, lags, and chaos, which are often present in practical time series data. To address these shortcomings, ANN and its extensions, such as RNN, LSTM, are employed in the grey forecasting context.

Grey–ANN Models

In the hybrid grey–ANN models, the grey model serves as a preprocessor, which filters noisy data by AGO and identifies the underlying trend. The ANN captures then the nonlinear residual or the fine structure, so that predictions are improved [26].

For instance, we could take a combined (coarse) forecast by a GM(1,1) model, followed by an ANN algorithm to predict the prediction error sequences. This hierarchical model structure is more effective than generic grey or neural models, as demonstrated by examples in energy prediction, traffic prediction, and agricultural yield prediction [55].

Grey–RNN and Grey–LSTM Models

RNNs and LSTMs are particularly effective at modeling sequential dependencies and temporal memory, which grey models do not possess. With hybrid grey–LSTM models, the AGO input is input into an LSTM network that learns long-term patterns and lags. These models have proven to be effective in addressing non-stationary time series with seasonality [52], [60].

It has been proposed that hybrid grey–LSTM models can effectively predict electric load, stock prices, and environmental variables, with MAPE and RMSE values lower compared to traditional methods.

Advantages and Challenges

The advantages of grey–neural hybrid models are:

The low-data sensitivity (due to the greyness in models) and nonlinear approximation ability (arising from neural networks) are integrated in the proposed method.

Ability to accommodate different types of time series structures.

Yet they bring some inherent challenges:

Model complexity and tuning overhead,

This is in particular due to the lack of interpretability, particularly in deep networks,

Risk of over fitting in small datasets without regularization.

3.3 Hybrid Grey Models with the Combination of Statistical and Fuzzy Logic-based Reasoning

Hybrid grey models: The traditional grey models for system optimization and neural integration have evolved into systems of complex information processing; new ideas, such as probability and statistics, and fuzzy logic, have been introduced. These integrations enhance the interpretability, robustness, and uncertainty management capabilities of grey models, particularly for complex or uncertain systems.

3.3 Grey–Statistical Hybrid Model (such as ARIMA, Box–Jenkins)

Statistical methods, such as ARIMA and Box–Jenkins, were integrated with grey models to overcome the limitations of deterministic grey models in capturing linear stochastic patterns or seasonal factors. In these combinations, the gray mode is used to represent the trend estimation or low-frequency factor, which ARIMA estimates the residual error or cyclical component [27], [47].

For example, in a hybrid GM–ARIMA model, two steps can be followed:

Initial trend forecasts are obtained using GM(1,1) or GM(1, N).

Randomness is then accounted for by extracting residuals, which are modeled by ARIMA(p, q, r).

The overall forecasting accuracy is enhanced by this method, which is particularly useful in macroeconomic, hydrological, or energy consumption applications.

Grey–Fuzzy Hybrid Models

FL is a convenient concept when the input data is vague, linguistic, or based on expert judgments. Fuzzy sets or membership functions can be introduced in the grey modeling process in the grey–fuzzy hybrid models [38].

Several strategies are used:

Fuzzyfying the input data before processing with AGO.

The use of the fuzzy coefficients in the model equation.

Combining grey prediction with fuzzy inference systems (e.g., Mamdani, Sugeno) [58].

These models are beneficial in decision support systems, qualitative forecasting, and those where the semantic richness of human knowledge must be retained (e.g., user satisfaction, health diagnosis).

Benefits and Drawbacks

There are several benefits to using grey–statistical and grey–fuzzy models:

Table 1. A Comparison of the Features of Grey–Statistical and Grey–Fuzzy Hybrid Models.

2. **Aspect**	3. **Grey–Statistical**	4. **Grey–Fuzzy**
5. Strengths	6. Captures noise and seasonality	7. Models imprecision and subjectivity
8. Typical Application	9. Economic, rainfall, energy	10. Health, decision-making, perception
11. Key Challenge	12. Model matching and stationary	13. Membership design and tuning

However, both methods make the model more complicated and may need to be tailored to the specific domain to work effectively.

3.4 Comparison of Hybrid Grey Prediction Models

Given the large number of HGMs proposed over the last decade, it is essential to conduct systematic comparative studies and gain insight into the relative strengths and weaknesses of different approaches for various types of time series problems. A classical grey system, combined with a complementing method such as optimization, neural computing, statistical modeling, or fuzzy reasoning, is encapsulated in each HGM. We now summaries the most widely used HGMs about five qualitative criteria: integration approach, strengths, limitations, common domain of application, and data requirements.

Table 2 presents the features of typical HGMs, including GM–PSO, GM–ANN, GM–LSTM, GM–ARIMA, and GM–FGM. The results indicate that optimization-based techniques, such as GM–PSO [57] and GM–GA [32], are simple and effective in terms of parameter tuning; however, initialization should be cautious to avoid escaping local minima. Neural-based hybrids, such as GM–ANN [42] and GM–LSTM [54], however, can achieve strong nonlinear learning performances, especially for high-frequency or chaotic time series data, but with high learning complexity and over fitting risks when not well-regularized

Table 2. A summary of the most common hybrid grey forecasting models side by side.

14. **Model**	15. **Integrated Technique**	16. **Strengths**	17. **Weaknesses**	18. **Typical Use Case**
19. GM–PSO	20. Met heuristic Optimization	21. Fast convergence, global search	22. May converge prematurely, parameter tuning	23. Load forecasting [1]
24. GM–GA	25. Genetic Algorithm	26. Flexible search, avoids local minima	27. Slow convergence, sensitive to encoding	28. Production optimization [2]
29. GM–ANN	30. Neural Network	31. Learns nonlinear dynamics, data-driven	32. Requires more data, prone to overfitting	33. Traffic modeling [3]
34. GM–LSTM	35. Memory-enhanced Neural Net	36. Handles long-term dependencies	37. High computational cost	38. Financial forecasting [4]
39. GM–ARIMA	40. Statistical Model	41. Captures linear/stationary patterns	42. Assumes normality, limited nonlinearity	43. Economic indices [5]
44. GM–FGM	45. Fuzzy Logic	46. Deals with vagueness, human judgment	47. Subjective design, rule sensitivity	48. Satisfaction systems [6]

Based on this comparison, we can deduce that there is no `one size fits all' model and the choice of a model depends on the nature of the data and the forecasting goal. For example, GM–ARIMA is best for seasonal economic indicators [47] and GM–LSTM is strong in capturing complex temporal dependencies in energy demand or pollutant dispersion [54]. However, GM–FGM models have been advocated for applications where the focus is on modeling uncertainty in inputs (or representing linguistic variables), e.g., decision support systems [38].

Furthermore, hybridization introduces computational trade-offs. Although the majority of models improve prediction performance over traditional GM(1,1), they complicate the models and the interpretations, a considerable determinant in mission-critical forecasting. Thus, there's a trade-off between model fidelity and explain ability that the analyst must straddle, especially if the results are informing policy or operations.

Most hybrid grey frameworks use a modular design to connect different parts. The grey model in this architecture is responsible for identifying trends and preparing the data. Optimization or learning-based algorithms, on the other hand, improve parameter estimates or uncover nonlinear residual structures. This divide makes it easier to grasp because it lets analysts see how each module works independently.

4. Applications of Hybrid Grey Models for Practical Forecasting

Hybrid grey models (HGMs) have become popular in various practical fields for handling noisy, missing, nonlinear, or non-stationary time series data. Their capability to retain the interpretability of grey systems and the learning power of AI or statistical modules has attracted their extensive application in fields that require robustness and learnability together. In this section, typical applications of HGMs in the aspects of energy, environment, agriculture, and economy will be introduced [41].

4.1 Energy Forecasting

Consider energy systems, including the electric load, solar radiation, and wind power generation, which are naturally nonlinear and subject to possible seasonal changes and uncertainty. HGMs have been proven more effective than classical models for both short-term and long-term prediction of energy demand. For instance, Liu et al. [56] demonstrated that the GM–PSO hybrid reduced the MAPE in day-ahead load forecasts. Similarly, Zhao et al.

[53] used a GM–LSTM approach to predict day-ahead electricity spot prices in deregulated markets with high accuracy and temporal stability.

In the grey model, the original sequence $X^{(0)}(t)$ is pre-processed by the AGO expressed by the functional relation to get a smoothed sequence $X^{(1)}(t)$. This modified sequence is then passed to a memory-augmented neural network, such as LSTM, that captures the temporal dynamics through back propagation through time. The network tries to minimize a forecasting loss function:

$$\mathcal{L} = \frac{1}{n}\sum_{i=1}^{n}\left(y_t - f_{LSTM}X^{(1)}(t)\right)^2 \tag{11}$$

Where y_t is the actual target value, f_{LSTM} represents the output of the LSTM trained with grey-transformed data.

4.2 Environmental and Pollution Modeling

Ecological systems exhibit complex dynamics, delayed feedback effects, and a lack of data, making hybrid grey models an appropriate choice in such cases. [25]GM–ANN and GM–ARIMA hybrids have been utilized for predicting AQI, water pollution concentration, and climate parameters. Chen and Zhang [22] combined GM with ANN for forecasting PM2.5 densities, outperforming mere statistical models. Another study applied a fuzzy-segmented grey model to predict river flow or evaluate ecological degradation [25].

4.3 Crop Productivity and Food Security

Prediction of agricultural production, particularly in climate-volatile countries, is essential for effective food policies. Grey model hybrids have been applied to model wheat, rice, and maize yields using weather parameters, fertilizer consumption, and market conditions. For example, Wang et al. [40] applied a GM–GA model to predict rice yields in southern provinces of China, considering rainfall and soil quality indicators. The proposed model reduced RMSE by more than 15% versus GM(1,1).

4.4 Time Series in Economics and Finance

Macroeconomic and financial data (such as inflation, GDP, exchange rates, stock returns, option prices) are typically noisy, partially observable, and exhibit non-linearities. GM–ARIMA has been used in inflation forecasting for many years [43], and GM-LSTM hybrids have recently gained popularity in Stock Price prediction due to their long-term dependency structure [30]. In emerging markets, such hybrid models are particularly beneficial when there is limited or poor data, or the data is semi-structured.

Table 3. A Summary of the different areas where hybrid grey models can be used and their strengths.

49. Application Domain	50. Hybrid Model(s)	51. Target Variables	52. Key Advantages	53. Representative Studies
54. Energy	55. GM–PSO, GM–LSTM	56. Load, solar, wind power	57. Captures seasonality and nonlinearity	58. [1], [2]
59. Environment	60. GM–ANN, GM–FGM	61. PM2.5, river flow, AQI	62. Handles noisy/incomplete data	63. [3], [4]
64. Agriculture	65. GM–GA, GM–ARIMA	66. Rice/wheat yield	67. Incorporates climate and soil indices	68. [5]
69. Economics	70. GM–ARIMA, GM–LSTM	71. Inflation, stock prices	72. Models trend and residual structures	73. [6], [7]

5. Restrictions and problems of hybrid grey models

Although Hybrid Grey Models (HGMs) are increasingly popular and empirically effective models for some applications, their wider scope of adoption and theoretical properties are still subject to criticism. It is essential to be aware of these limitations to deploy the model correctly, as well as to guide future research efforts in developing more effective hybridization strategies.

5.1 Model Complexity and Interpretability

Hybridization adds significant model-architecture complexity as a trade-off. Although traditional grey models such as GM(1,1) are appreciated for their simplicity and precise processing mechanisms, HGMs — including the combination of deep learning (e.g., GM–LSTM) and met heuristic optimization (e.g., GM–PSO) — bring about complex and rich structures as well as multiple hyper parameters [61]. Such complexity can even result in less understandable systems, where a lack of transparency prevents domain experts from interpreting the reasons behind predictions or policy suggestions [45]. This kind of opaqueness can be a significant issue in sensitive areas such as public health, economics, or environmental policy, where explain ability is crucial.

5.2 Data Requirements and Generalization

Although grey models are known for their advantages on small datasets, such an extension is essentially a hybrid extension, and still requires a paradoxical increase of data volume to avoid over fitting. Examples of such models are neural enhanced grey models (GM–ANN, GM–LSTM), which require sufficient data and a validation regimen during training to be generalized [36]. This data dependency is a practical limitation, particularly when data are non-stationary, noisy, and/or contain missing portions in developing countries. Additionally, transferring trained hybrid models across domains or regions is limited without extensive retraining.

Although hybrid grey models incorporate learning-based components that may require more data, the grey modeling core partially mitigates data scarcity by extracting dominant trends from limited observations. Nevertheless, hybridization does not eliminate data limitations, and careful regularization and validation remain essential [33] [36].

5.3 Cost of computation and risk of optimization

Hybrid models typically require more resources for training, particularly when they involve an iterative optimization step (e.g., for GAs, PSO, or RNNs). Furthermore, most met heuristic optimizers do not guarantee global minima and can be trapped early in local minima [35]. This random nature introduces instability across runs and can produce unreliable responses unless substantial cross-validation or ensemble averaging is performed [33].

5.4 Absence of a Standard and Benchmarks

A frequently mentioned issue in the grey model literature is the absence of a universal evaluation method. Direct comparison of these results can be challenging, as different datasets, metrics (MAPE, RMSE, MAE), and validation splits are used in the studies [50]. Furthermore, there are few benchmark datasets for the grey hybrid model, which restricts the reproducibility and large-scale evaluation. This methodological variability poses an obstacle to cumulative advances in the field, as well as to the credibility of models in interdisciplinary contexts.

5.5 Integration Difficulties and Model Interface Design

Hybridization is not infrequently a heuristic and ad hoc procedure. That is, gray models are often combined with other methods in an ad hoc fashion, without a formal way to guide the fusion processing [41]. For example, questions like: How to divide the responsibilities of grey and non-grey parts? Or when to perform AGO transformations in hybrid pipelines? — are often left unaddressed. A consequence of this absence of design is that models can work well empirically but lack a theoretical foundation (i.e., a lack of interpretability or reproducibility).

6. Future Work and Open Issues

While evolving, the area of hybrid grey systems modeling presents a number of promising research directions and yet-to-be-resolved challenges. It is essential to address these frontiers in order to make further progress on the sound theoretical, generalized structure, and practical utility of HGMs in various fields of forecasting. 5.1 Areas for Future Academic Research. Here, we identify specific topics that may require further attention in future research.

6.1 Theoretical Unification Between Grey and Hybrid Models

Many existing HGMs are constructed from heuristic composites and lack a coherent theoretical basis. Future work needs to construct a mathematical model based on grey system theory, machine learning, and statistical inference. For example, grey differential equations in neural differential models, or the probabilistic grey model based on Bayesian inference, can provide a deeper insight into uncertainty quantification and structure learning [51].

6.2 Automated Model and Hyper parameter Selection

The choice of a suitable hybrid architecture and its parameters (e.g., regularization coefficients, learning rates, number of hidden layers) is often somewhat hit-or-miss. There are also Auto ML-type solutions that we could consider to automate the design of HGMs, such as using neural architecture search or evolutionary search [39]. This could resolve modeling bias and can also offer a convenient hybrid forecasting solution to end users without expert experience.

6.3 Explainable Grey–AI Models (XHGMs)

With the increasing awareness of the issue of model transparency, particularly in policy-sensitive domains, there is a rising demand for explainable hybrid models. In the future, it will be necessary to develop grey–AI hybrids that provide interpretability while maintaining accuracy. Approaches such as attention mechanisms, symbolic grey logic layers, or saliency mapping may be employed to visualize the data part that has the most significant impact on predictions [59]. Recent research shows that hybrid grey models can be easier to grasp when explainable AI methodologies like attention layers, symbolic grey logic components, and post-hoc sensitivity analysis are used. These strategies can help determine how much the grey and non-grey parts affect the final forecasts. This is especially important in sensitive sectors, including public health and environmental policy.

6.4 Benchmarks/Platforms and Open Datasets

The lack of benchmark datasets and evaluation standards remains a significant challenge. It is an open question whether there would be sufficient interest to develop open-access platforms for sharing HGM benchmarks; however, such resources (similar to UCI or Kaggle) could dramatically accelerate the race for applicable comparisons and transparent methods [44]. Open-source repositories maintained by the community can also contribute to reproducibility and facilitate testing in unseen domains.

6.5 Domain-Specific Adaptation

HGMs must be developed and validated in new application domains, like smart grids, epidemic prediction, and environmental risk prediction. These fields pose special data challenges, e.g.\, multiple scales, multimodality, or spatiotemporal heterogeneity, which may necessitate a new type of hybridization. The degree of interpretability and accuracy can be enhanced by incorporating domain knowledge into the model structure (such as the PIB grey models) [37].

6.6 Integration with Uncertainty and Risk Analysis

A key direction is to further develop how HGMs encode and propagate uncertainty. The fusion of grey systems with fuzzy theory, interval analysis and probabilistic reasoning forms a natural base where hybrid models can provide not only point estimates but also prediction intervals or estimated risk levels [29]. This is especially important for decision-making under uncertainty in energy, finance and climate-sensitive applications.

7. Conclusion

In this article, we have critically reviewed the evolution of HGMs and their applications in real-world forecasting problems across various disciplines, including the environment, energy, economics, and agriculture, among others. By incorporating grey system theory with advanced artificial intelligence (AI) techniques, optimization algorithms, and statistical methods, HGMs have achieved significantly better performance in modeling complex, nonlinear, and unpredictable time series data than traditional grey models or single AI models.

Key Insights

Flexibility: The HGMs inherit merits of grey models (e.g., high dimensionless precision, extracting trends and predicting the poorly known data by utilizing few known ones, etc.) as well as the adaptive capability of the state-of-the-art AI methods (ANNs and LSTMs) or already popular optimization algorithms (PSO and GA), which therefore provides an excellent platform for forecasting.

Applications: We have illustrated the real-world applicability of HGMs in tasks such as energy load forecasting, environmental modeling, as well as agri-modelling, and financial time series forecasting.

Challenges Although HGMs achieve good results in many applications, they are still confronted with some issues, including model complexity, data dependence, computational cost and the risk of over fitting. These problems need to be resolved for it to become more widely used and better suited for delicate fields such as policy-making and financial decision-making.

Future Research Directions

Theoretical unification of grey and hybrid models remains a challenge, with formal aspects including the integration of grey theory with AI and probabilistic methods.

Automated model selection and hyper parameter optimization, e.g., Auto ML, may be beneficial in reducing the time and expertise required for hybrid model design, thereby increasing the possibility for more practitioners.

An increased interest from domain experts in the development of XHGM will be necessary for fields that require interpretability, such as healthcare, policy-making, environmental studies, and all studies involving a human face facing a social-environmental problem.

With the default acceptance of hybrid solutions in practical scenarios, the establishment of benchmarking systems, and efforts to provide open datasets will drive comparison and reproducibility, further developing the field.

Conclusion Summary

In summary, the hybrid grey models are an interesting frontier for time series forecasting. The ability of their model to accommodate many of the computational methods within the grey system framework has made it a versatile and robust tool for solving complex forecasting problems. Additional investigation and efforts will be required to address the limitations due to model complexity, data and computational cost. By overcoming these limitations, HGMs can reshape the practice of forecasting in various disciplines, providing more accurate, stable, and trustworthy forecasts for numerous practical scenarios.

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