Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

Page 47-51

The set of the homogeneous linear reciprocal block maps

Alaa H.H. Al-Ka'bi
Department of Mathematics - College of Since the computer and Mathematics, University of Qadisia

Abstract

In this paper ,we introduce new definition for set of the block maps reciprocal via
block maps linear homogeneous ,inhomogeneous ,odd and even . while The classical definition is $\Im(f)=\{g\in F: g \text{ commute with } f \text{ under impacts } \circ\}$

1-Preliminaries

let (X,T,π) be a topological transformation group,
We adopt the set of symbols $\zeta=\{0,1\}$ as the alphabet of our shift space , n-block means the function $\beta_n : I_p^q \to \zeta$ where $I_p^q=\{i\in Z: p\le i\le q : \mathrm{p,q}\in \mathrm{Z}\}$, B_n means the set all n-blocks, the n-block map f defined by $f:\mathrm{B_n}\to\zeta$[2], I identity block map defined by $I(a_1a_2...a_n)=a_1a_2...a_n \ \forall a_1a_2...a_n\in B_n$,0,1constants block map defined by $0(a_1a_2...a_n)=0, 1(a_1a_2...a_n)=1\forall a_1a_2...a_n\in B_n$, F_n a set of all n-block maps and F a set of all block maps[1],[3].
The alphabet we adopt is $\zeta=\{0,1\}$,and define translation operator(Ψ)as follows $\Psi f(a_0a_1a_2\dots a_n)=f(a_1a_2\dots a_n)$ such $\mathrm{a_i}\in\zeta$,$\theta(f)=\min\{n: f\in F_n\}$ and can written any block map it's say g as form $g=I\cdot\Psi qg+\Psi rg \ni qg, rg\in F_{n-1}$ such that $qg(a_1\dots a_n)=g(0a_1\dots a_n), rg(a_1\dots a_n)=g(0a_1\dots a_n)+g(1a_1\dots a_n)$ $\forall a_i\in\zeta$ and have $q(g\circ f)=qg\circ f, q(f\circ g)=qg \quad \forall f,g\in F$ [5]. We define set of block maps commuting $\Im(f)=\{g\in F: g\circ f=f\circ g\}$. In research our we define the linear block map as follows $f=a_0+\sum_{i=1}^{n}a_i\Psi^{i-1}I \ ,a_i\in\zeta$ and let $\gamma=\{f\in F: f \text{ linear block map}\}$, and it is said for f homogeneous if $a_0=0$
and inhomogeneous if $a_0=1$, and it is said for f even or odd according to value $card\{i\ge 1: a_i=1\}$even or odd . let γ_H be set of all homogeneous linear block map, and let γ_I set all inhomogeneous linear block map. And say for f non-trivial block map if $card\{i\ge 1: a_i=1\}\ge 2$ .We have $(\gamma_H,+,\circ)\cong(Z_2[x],+,.)$ [4][2].
(2)A New set of the homogeneous linear reciprocal block maps

Preliminaries

In this section , we study the relation between block maps linear homogeneous, inhomogeneous ,odd and even and the composition for block maps .

Theorem (2.1): if f,g,h block maps and $f\in\gamma_H$ then

$$f\circ(g+h)=f\circ g+f\circ h\ \ \forall\, g,h$$

Proof

Since $f\in\gamma_H$, then there exists $a_1\ldots a_n=0\text{ or }1$ such that

$$f\circ[g+h]=\sum_{i=1}^{n}a_i\Psi^{i-1}I\circ[\sum_{i=1}^{n}a_i\Psi^{i-1}I\circ g+\sum_{i=1}^{n}a_i\Psi^{i-1}I\circ h]=f\circ g+f\circ h$$

Theorem (2.2): if f is block map and $f\in\gamma_H$ then $\gamma_H\subseteq\Im(f)$.

Proof

Let g be homogeneous linear block map, and Since

$$(\gamma_H,+,\circ)\cong(Z_2[x],+,.)$$

then $(\gamma_H,+,\circ)$ is commuting ring ,and so then $g\circ f=f\circ g$ for all $g\in\gamma_H$

Theorem (2.3)

let f be non-trivial block map and $f\in\gamma$ then $\Im(f)\subseteq\gamma$.

Proof

we will prove by using the induction on value $\theta(f)$

let $g\in\Im(f)$, $g\circ f=f\circ g$ constant , and so $q(g\circ f)=qg$.

Now we can written g as form $g=b.I+\Psi rg$ such that b constant. so

$$g\circ f=b[a_0+\sum_{i=1}^{n}a_i\Psi^{i-1}I]+\Psi rg\circ f$$

$$f\circ g=b\sum_{i=1}^{n}a_i\Psi^{i-1}I+f\circ\Psi rg$$

$$g\circ f+f\circ g=a_0b+\Psi(rg\circ f+f\circ rg)$$

we notice that $rg\circ f+f\circ rg$ constant , and by using the induction then rg constant , this completes the proof .

Theorem (2.4)

let $f\in\gamma_H$ and $g\in\gamma_I$ then

$g\circ f=f\circ g$ if and only if f odd.

Proof

we can written f,g,h as form

$$f=\sum_{i=1}^{n}a_i\Psi^{i-1}I,\ g=1+\sum_{j=1}^{m}b_j\Psi^{j-1}I$$

such that

$a_i, b_j = 0 or 1 \ \forall i = 1,\ldots,n \ \mathrm{j} = 1,\ldots,\mathrm{m}$

and so that $g \circ f = 1 + \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j \Psi^{i+j-2} I$

by using theorem (2.1) $f \circ g = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j \Psi^{i+j-2} I$

and so that $\sum_{i=1}^{n} a_i = 1$ if and only if f odd .

Theorem (2.5)

let $g, h \in \gamma_I$ then $g \circ h = h \circ g$ if and only if g, h either both odd or both even.

Proof

We can written h as form $h = 1 + \sum_{i=1}^{n} c_i \Psi^{i-1} I$ such that $c_i = 0 or 1$ for all $i = 1\ldots n$ and g as in the theorem(2.4).

Since $\sum_{j=1}^{m} b_j \Psi^{j-1} I \in \gamma_H$ and by using theorem(2.1) then

$$g \circ h = 1 + \sum_{j=1}^{m} b_j + \sum_{j=1}^{m}\sum_{i=1}^{n} b_j c_i \Psi^{i+j-2}$$

and $$h \circ g = 1 + \sum_{i=1}^{n} c_i + \sum_{j=1}^{m}\sum_{i=1}^{n} b_j c_i \Psi^{i+j-2}$$

and so that $\sum_{i=1}^{n} c_i = \sum_{j=1}^{m} b_j$ if and only if g, h either both odd or both even.

Theorem(2.6)

if f, g are block maps and $f \in \gamma_H$ and f is non trivial

1. if f odd map then $\Im(f) = \gamma$.
2. if f even map then $\Im(f) = \gamma_H$.

Proof

(1) from theorem (2.2) then $\Im(f) \subseteq \gamma$.

Let $g \in \gamma$, if either $g \in \gamma_I$ and by using theorem(2.5) then $g \circ f = f \circ g$, or $g \in \gamma_H$ and by using theorem(2.2) then $g \in \Im(f)$ this completes the proof .

(2) from theorem(2.2) then $\gamma_H \subseteq \Im(f)$.

Let $g \in \Im(f)$ i.e. $g \circ f = f \circ g$,and will proof by contradiction i.e. $g \notin \gamma_H$ and By using theorem (2.3) then $g \in \gamma_I$ and by using theorem (2.4) then f is odd ,and this contradiction .

Theorem(2.7)

if f, g are block maps and $f \in \gamma_I$ and f is non trivial

1. if f odd map then $\Im(f) = \{g \in \gamma : g \text{ is odd}\}$.
2. if f even map then $\Im(f) = \{g \in \gamma_H : g \text{ is odd}\} \bigcup \{g \in \gamma_I : g \text{ is even}\}$.

proof

(1) let $g \in \Im(f)$ and by using theorem (2.3) then $g \in \gamma$,
and since $g \in \gamma$, either $g \in \gamma_H$ and by using theorem(2.4) then g is odd , or $g \in \gamma_I$ and f is odd and by using theorem(2.5) then g is odd , this completes the proof .

proof (2)

let $g \in \Im(f)$ and by using theorem (2.3) then $g \in \gamma$, either $g \in \gamma_H$ and we have $g \circ f = f \circ g$ and by using theorem (2.4) then g is odd .or $g \in \gamma_I$ and f is even by using theorem(2.5) then g is even, this completes the proof .

Reference

[1] D. Lind and B. Marcus, An Introduction to Symbolic Dynamics and Coding, Cambridge University Press, Cambridge, 1995
[2] G. A. Hedlund, Endomorphisms and automorphisms of the shift dynamical system, Math. System Theory 3, pp(320-375), (1969).
[3] G. C. Mira. and S. Rouabhi, .Two applications of noninvertible maps in communication and information storage,. European Journal of Operational Research, pp 461.468, 2002.
[4] M. Ethan Coven, G. A. Hedlund and Frank Rhodes, The commuting block maps, transaction of Am. Math. Soc. Volume 249, number 1,pp(113-138), April 197,.
[5] M. Nasu, Topological Conjugacy For Sofic systems and Extensions of Automorphisms of finite subsystems of topological markor shifts, Dynamical system: Proceedings, University of Maryland 1986-1987.

Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

مجموعة دوال القطعة الخطية المتجانسة المتبادلة

علاء حسين حمادي الكعبي

قسم الرياضيات – كلية علوم الحاسوب والرياضيات – جامعة القادسية

الخلاصة

في هذا البحث قدمنا تعريف جديد لمجموعة دوال القطعة المتبادلة عن طريق دوال القطعة المتجانسة والغير متجانسة والفردية والزوجية بينما التعريف الاعتيادي هو

$$\Im(f) = \{g \in F : g \text{ commute with } f \text{ under impacts } \circ \}$$