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Novel Third-Order Differential Subordination and Superordination Results for Meromorphic p-valent Functions Involving a New Hadamard Product Operator

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ABSTRACT

The purpose of this paper is to derive many third-order differential superordination and subordination results. As a result, sandwich-type theorem for meromorphic p-valent function class involving the celebrated by operator $F_{e,i,p,(d_1,d_2)}f(z)$ are established. Also, to make a relation between the current results and the previous works that distinguished out.

MSC..

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1. Introduction

Assume $\mathcal{H} = \mathcal{H}(U)$ is a class of functions are analytic in the open unit disk $U = \{z: z \in \mathbb{C}, |z| < 1\}$, and that $\mathbb{C}$ is a complex plane. Consider $\mathcal{H}[a, n]$ to be a subclass of $\mathcal{H}$ with the following functions when a positive number n is added to $a \in \mathbb{C}$.

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots, a \in \mathbb{C}.$$

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In addition, we suppose that $\mathcal{H}_1 = \mathcal{H}[1,1]$. Assume that f_1 and f_2 are analytic functions in $\mathcal{H}$. If there is a Schwarz function $\omega(z)$ to be analytic, with $|\omega(z)| < 1$ and $\omega(0) = 0$, where $f_1(z) = f_2(\omega(z))$, $z \in U$. The function f_1 is considered as a subordinate to the function f_2, which is represented as $f_1 \prec f_2$ or $f_1(z) \prec f_2(z)$.

In addition, if f_2 be an univalent function within U. On can obtain ([17,18]).

$$f_1(z) \prec f_2(z) \Leftrightarrow f_1(0) = f_2(0) \text{ and } f_2(U) \subset f_1(U), (z \in U).$$

Differential subordination inequalities can be generalized to include variables with complex forms. Assume $\mathcal{A}^*$ pointed to a class of all functions and had the following relation.

$$f(z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} a_{n+p} z^{n+p} \tag{1.1}$$

which be analytic and meromorphic p-valent function within punctured unit disk $U^* = U \backslash \{0\} = \{z \in \mathbb{C}: 0 < |z| < 1\}$.

Let f_1 and f_2 be functions defined by (1.1), the convolution (or Hadamard product) of f_1 and f_2 be defined as follows:

$$(f_1 * f_2)(z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} a_{n+p} b_{n+p} z^{n+p}.$$

The following is the definition of the function $\Pi_p(d_1, d_2; z)$. (cf. [16])

$$\Pi_p(d_1, d_2; z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} \left|\frac{(d)_{n+1}}{(d)_{n+1}}\right| z^{n+p} ,$$

where $p \in \mathbb{N}, d_1 \in \mathbb{C}\backslash\{0\}$, ($d_2 \in \mathbb{C}\backslash\mathbb{Z}_o^-$) and $(\delta)_n$ such that $\mathbb{Z}_o^- = 0, -1, -2, \ldots$

and $(\delta)_n$ be the Pochhammer symbol.

We have the linear operator $L_p(d_1, d_2; z)$ on $\mathcal{A}^*$ by taking the convolution (or Hadamard product) of $f \in \mathcal{A}^*$ with function $\Pi_p(d_1, d_2; z)$ as in the form below (cf. [3]).

$$L_p(d_1, d_2; z) = \Pi_p(d_1, d_2; z) * f(z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} \left|\frac{(d_1)_{n+2}}{(d_2)_{n+2}}\right| a_{n+p} z^{n+p}. \tag{1.2}$$

Ponnusamy and Juneja's initial study was in 1992, when they presented the third-order differential subordination notion [19].

In 2011, Antonino and Miller [4] introduced basic concepts to the extended theory of differential subordination for second-order within an open unit disk. Case of third-order was introduced by Mocanu and Miller [18].

For other conditions, several studies investigated second, third, and fourth-order differential subordination. (cf. [1,2,5,6,7,8,9,10,11,12,13,14,15,20,21,22,23,24,25,26,27]).

Using the analytical formulas provided in the paper's introduction, we were able to produce new definition, which we regard to be a principal tool in our work.

Definition (1.1): Let $\in \mathcal{A}^*$. We define the new operator $F_{e,i,p,(d_1,d_2)}f(z): \mathcal{A}^* \longrightarrow \mathcal{A}^*$, where

$$F_{e,i,p,(d_1,d_2)}f(z) = \frac{\tau_p(z,e,i)}{z^p i^{-e}} * L_p(d_1,d_2;z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} \left(\frac{i}{i+n+p}\right)^e \left|\frac{(d_1)_{n+2}}{(d_2)_{n+2}}\right| a_{n+p} z^{n+p} \ , \tag{1.3}$$

where $\tau_p(z,e,i) = \sum_{n=0}^{\infty} \frac{z^{n+p}}{(i+p+n)^e}$, is the general Hurwitz-Lerch Zeta function and

$i \in \mathbb{C}\backslash z_o^-$, $e \in \mathbb{C}$, $d_1 \in \mathbb{C}\backslash\{0\}$, $d_2 \in \mathbb{C}\backslash\mathbb{Z}_o^-$, $(\delta)_n$ is the Pochhammer symbol.

We observed from (1.3) that, we obtain:

$$z\left[F_{e,i,p,(d_1,d_2)}f(z)\right]' = d_1 F_{e,i,p,(d_1,\,d_2)}f(z) - (d_2+p)F_{e,i,p,(d_1,d_2)}f(z). \quad (1.4)$$

2. Preliminaries

The following definitions and lemmas are required to support our main results.

Definition (1.2) [4]. Assume $b(z)$ is univalent in U and $\psi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$. If U is involved by analytic $p(z)$ function and satisfies following differential subordination of third-order:

$$\psi(\mathfrak{p}(z), z\mathfrak{p}'(z), z^2\mathfrak{p}''(z), z^3\mathfrak{p}'''(z); z) \prec b(z), \tag{2.1}$$

thus $\mathfrak{p}(z)$ is a differential subordination solution (2.1). In addition to that the univalent function $\mathfrak{q}(z)$ be a dominant solution of (2.1) or , it $\mathfrak{p}(z) \prec \mathfrak{q}(z)$ to all $\mathfrak{p}(z)$ then satisfying (2.1) with simply dominant. The best dominant $\tilde{\mathfrak{q}}(z)$ occurred, when $\tilde{\mathfrak{q}}(z) \prec \mathfrak{q}(z)$, $(z \in U)$, to all dominants $\mathfrak{q}(z)$ of (2.1) .

Definition (2.2):[4] Assume $b(z)$ is analytic in U and $\psi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$. If the functions $\mathfrak{p}(z)$ and $\psi(\mathfrak{p}(z), z\mathfrak{p}'(z), z^2\mathfrak{p}''(z), z^3\mathfrak{p}'''(z); z)$ are univalent in U and satisfies following differential superordination of third-order:

$$b(z) \prec \ \psi(\mathfrak{p}(z), z\mathfrak{p}'(z), z^2\mathfrak{p}''(z), z^3\mathfrak{p}'''(z); z), \tag{2.2}$$

then the function $\mathfrak{p}(z)$ is called a solution of the differential superordination (2.2). Furthermore, the analytic function $\mathfrak{q}(z)$ can be a subordinant of the solutions of (2.2), or more simply a subordinant if $\mathfrak{q}(z) \prec \mathfrak{p}(z)$ for all $\mathfrak{p}(z)$ satisfying (2.2). A univalent subordinant $\tilde{\mathfrak{q}}(z)$ that satisfies $\mathfrak{q}(z) \prec \tilde{\mathfrak{q}}(z)$ for all subordinants $\mathfrak{q}(z)$ of (2.2) is called the best subordinant. We note both the best dominant and best subordinant are unique up to rotation of U.

Definition (2.3) [4]. Denote by $\mathbb{Q}$ the set of all functions $\mathfrak{q}$ that are analytic and injective functions on $\overline{U}\backslash E(\mathfrak{q})$, where $\overline{U} = U \cup \partial U$, and $E(\mathfrak{q}) = \{\zeta: \zeta \in \partial \mathcal{D}U: \lim_{z\to\zeta} \mathfrak{q}(z) = \infty\}$, and are such that $\mathfrak{q}'(z) \neq 0$ for $\zeta \in \partial U\backslash E(\mathfrak{q})$. Let the subclass of $\mathbb{Q}$ for which $\mathfrak{q}(0) = a$ be denoted by $\mathbb{Q}(a)$, $\mathbb{Q}(0) = \mathbb{Q}_0$ and $\mathbb{Q}(1) = \mathbb{Q}_1$, *where* $\mathbb{Q}_1 = \{\mathfrak{q} \in \mathbb{Q}: \mathfrak{q}(0) = 1\}$.

The admissible function classes were defined by Antonino and Miller [4] as what is follow.

Definition (2.4) [4]. Assume Ω be a set in $\mathbb{C}$, $\mathfrak{q} \in \mathbb{Q}$, $n \in \mathbb{N} \setminus \{1\}$. The class of admissible functions $\Psi_n[\Omega, \mathfrak{q}]$ consists of those functions $\psi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$ that satisfy the following admissibility conditions:

$$\psi(\mathfrak{r}, \mathfrak{s}, \mathfrak{t}, \mathfrak{u}; z) \notin \Omega,$$

wherever

$$\mathfrak{r} = \mathfrak{q}(\zeta)\ ,\ \mathfrak{s} = k\zeta\mathfrak{q}'(\zeta)\ ,\quad Re\left(\frac{\mathfrak{t}}{\mathfrak{s}} + 1\right) \geq kRe\left(\frac{\zeta\mathfrak{q}''(\zeta)}{\mathfrak{q}'(\zeta)} + 1\right) \text{ and}$$

$$Re\left(\frac{\mathfrak{u}}{\mathfrak{s}}\right) \geq k^2 Re\left(\frac{\zeta^2\mathfrak{q}'''(\zeta)}{\mathfrak{q}'(\zeta)}\right),$$

where $\zeta \in \partial U \backslash \mathrm{E}(\mathfrak{q}), z \in U$ and $k \geq n$.

Definition (2.5) [19]. Assume Ω be a set in $\mathbb{C}$, $\mathfrak{q} \in \mathcal{H}[a, n]$ with $\mathfrak{q}'(z) \neq 0$ $n \in \mathbb{N} \setminus \{1\}$. The class of admissible functions $\Psi'_n[\Omega, \mathfrak{q}]$ consists of those functions $\psi: \mathbb{C}^4 \times \overline{U} \longrightarrow \mathbb{C}$ that satisfy the following admissibility conditions:

$$\psi(\mathfrak{r}, \mathfrak{s}, \mathfrak{t}, \mathfrak{u}; \zeta) \in \Omega,$$

wherever

$$\mathfrak{r} = \mathfrak{q}(z)\ ,\ \mathfrak{s} = \frac{z\mathfrak{q}'(z)}{m},\quad Re\left(\frac{\mathfrak{t}}{\mathfrak{s}} + 1\right) \leq \frac{1}{m} Re\left(\frac{z\mathfrak{q}''(z)}{\mathfrak{q}'(z)} + 1\right),$$

$$Re\left(\frac{\mathfrak{u}}{\mathfrak{s}}\right) \leq \frac{1}{m^2} Re\left(\frac{z^2\mathfrak{q}'''(z)}{\mathfrak{q}'(z)}\right),$$

where $\zeta \in \partial U \backslash \mathrm{E}(\mathfrak{q}), z \in U$ and $m \geq n \geq 2$.

The following lemma is a basic aspect of third-order differential superordination theory.

Lemma(2.1)[4]: Let $p \in \mathcal{H}[a, n]$ *with* $n \in \mathbb{N}\backslash\{1\}$, and $\mathfrak{q} \in \mathbb{Q}(a)$ satisfying the following conditions:

$$Re\left(\frac{\zeta\mathfrak{q}''(\zeta)}{\mathfrak{q}'(\zeta)}\right) \geq 0\ ,\qquad \left|\frac{zp'(z)}{\mathfrak{q}'(z)}\right| \leq k,$$

where $z \in U$, $\zeta \in \partial U \backslash \mathrm{E}(\mathfrak{q})$, and $k \geq n$. If Ω is a set in $\mathbb{C}$. $\psi \in \Psi_n[\Omega, \mathfrak{q}]$ and

$$\psi(\mathfrak{p}(z), z\mathfrak{p}'(z), z^2\mathfrak{p}''(z), z^3\mathfrak{p}'''(z); z) \in \Omega,$$

then

$$p(z) \prec \mathfrak{q}(z), \qquad (z \in U).$$

Lemma (2.2)[19]. Let $\psi \in \Psi'_n[\Omega, \mathfrak{q}]$. If $\psi(\mathfrak{p}(z), z\mathfrak{p}'(z), z^2\mathfrak{p}''(z), z^3\mathfrak{p}'''(z); z) \in \Omega$, is univalent in U, $p \in \mathbb{Q}(a)$ and $\mathfrak{q} \in \mathcal{H}[a, n]$ satisfy the following conditions:

$$Re\left(\frac{\zeta\, q''(\zeta)}{q'(\zeta)}\right) \geq 0 \ , \qquad \left|\frac{\zeta p'(\zeta)}{q'(\zeta)}\right| \leq m,$$

where $\zeta \in \partial U$, $z \in U$ and $m \geq n \geq 2$, then

$$\Omega \subset \{\psi(p(z), zp'(z), z^2 p''(z), z^3 p'''(z); z) : z \in U\},$$

which leads to

$$q(z) \prec p(z), \ \ z \in U.$$

3. Third-Order Differential Subordination Results:

Here, we introduce some differential subordination results by using the new Hadamard product operator $F_{e,i,p,(d_1,d_2)} f(z)$.

Definition (3.1). Assume Ω be a set in $\mathbb{C}$, and $q \in \mathbb{Q}_0 \cap \mathcal{H}_0$. The class of admissible functions $\Theta[\Omega, q]$ consists of those functions $\phi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$ that satisfy the following admissibility conditions:

$\Theta(\mathbb{u}, \mathbb{v}, \mathbb{w}, \mathbb{y}; z) \notin \Omega$,

whenever,

$$\mathbb{u} = q(\zeta), \qquad \mathbb{v} = \frac{k\zeta\, q'(\zeta) + (d_1 + p)q(\zeta)}{d_1},$$

$Re\left\{\frac{d_1(d_1+p)\mathbb{w}-(d_1+p)(d_1+2)\mathbb{u}}{d_1\mathbb{v}-(d_1+p)\mathbb{u}} - (2d_1 + 3)\right\} \geq kRe\left\{\frac{\zeta\, q''(\zeta)}{q'(\zeta)} + 1\right\}$, and

$Re\left\{\frac{d_1(d_1+p)[(d_1+2)\mathbb{y}-3(d_1+3)\mathbb{w}]+(d_1+2)(d_1+3)[3d_1\mathbb{v}-(d_1+p)\mathbb{u}]}{d_1\mathbb{v}-(d_1+p)\mathbb{u}}\right\} \geq k^2 Re\left\{\frac{\zeta^2 q'''(\zeta)}{q'(\zeta)}\right\}$,

such that $\zeta \in \partial U \backslash E(q), d_1 \in \mathbb{C}\backslash Z_o^-, z \in U$, and $k \geq 2$.

Theorem (3.2): Assume $\phi \in \Theta_n[\Omega, q]$. If $f \in \mathcal{A}^*$ and $q \in \mathbb{Q}_0 \cap \mathcal{H}_0$ satisfy the following conditions:

$$\boldsymbol{Re}\left(\frac{\zeta q''(\zeta)}{q'(\zeta)}\right) \geq 0 \quad , \quad \left|\frac{F_{e,i,p,(d_1+1\,,\,d_2)}f(z)}{q'(\zeta)}\right| \leq k, \tag{3.1}$$

and

$$\left\{\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z)\right), z \in U^* \subset \Omega\right\}, \tag{3.2}$$

then

$$F_{e,i,p,(d_1,d_2)}f(z) \prec q(z), \qquad (z \in U^*).$$

Proof: Taking $G(z)$ be analytic function in U^* as:

$$G(z) = F_{e,i,p,(d_1,d_2)}f(z). \tag{3.3}$$

Making use of (1.4) and (3.3), we have

$$F_{e,i,p,(d_1+1,d_2)}f(z) = \frac{zG'(z)+(d_1+p)G(z)}{d_1}. \tag{3.4}$$

Further computations shows that,

$$F_{e,i,p,(d_1+2,d_2)}f(z) = \frac{z^2G''(z)+2(d_1+p)zG'(z)+(d_1+p)(d_1+2)G(z)}{d_1(d_1+p)}, \tag{3.5}$$

and

$$F_{e,i,p,(d_1+3,d_2)}f(z) = \frac{z^3G'''(z)+3(d_1+3)z^2G''(z)+3z(d_1+3)(d_1+2)G'(z)+(d_1+p)(d_1+2)(d_1+3)G(z)}{(d_1+p)(d_1+2)d_1}. \tag{3.6}$$

Now, we define the transformation from $\mathbb{C}^4$ *to* $\mathbb{C}$ by formula:

$\mathbb{u}(\mathfrak{r},\mathfrak{s},\mathfrak{t},\mathfrak{u}) = \mathfrak{r}$, $\quad \mathbb{v}(\mathfrak{r},\mathfrak{s},\mathfrak{t},\mathfrak{u}) = \frac{\mathfrak{s}+\mathfrak{r}(d_1+p)}{d_1}$,

$\mathbb{w}(\mathfrak{r},\mathfrak{s},\mathfrak{t},\mathfrak{u}) = \frac{\mathfrak{t}+2\mathfrak{s}(d_1+2)+\mathfrak{r}(d_1+p)(d_1+2)}{(d_1+1)d_1}$,

$$\mathbb{y}(\mathfrak{r},\mathfrak{s},\mathfrak{t},\mathfrak{u}) = \frac{\mathfrak{u}+3\mathfrak{t}(d_1+3)+3\mathfrak{s}(d_1+2)(d_1+3)+\mathfrak{r}(d_1+p)(d_1+2)(d_1+3)}{(d_1+p)(d_1+2)d_1}, \tag{3.7}$$

where, $\mathfrak{r} = G(z)$, $\mathfrak{s} = zG'(z)$, $\mathfrak{t} = z^2G''(z)$, $\mathfrak{u} = z^3G'''(z)$.

Assume

$\psi(\mathfrak{r},\mathfrak{s},\mathfrak{t},\mathfrak{u};z) = \phi(\mathbb{u},\mathbb{v},\mathbb{w},\mathbb{y};z)$

$$= (\mathfrak{r},\frac{\mathfrak{s}+\mathfrak{r}(d_1+p)}{d_1},\frac{\mathfrak{t}+2\mathfrak{s}(d_1+2)+\mathfrak{r}(d_1+p)(d_1+2)}{(d_1+p)d_1},\frac{\mathfrak{u}+3\mathfrak{t}(d_1+3)+3\mathfrak{s}(d_1+2)(d_1+3)+\mathfrak{r}(d_1+p)(d_1+2)(d_1+3)}{(d_1+p)(d_1+2)d_1};z). \tag{3.8}$$

Lemma (2.1) is used to the proof. Taking (3.8) and (3.3)-(3.6), to get

$\psi(G(z),zG'(z),z^2G''(z),z^3G'''(z),z) =$

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z),F_{e,i,p,(d_1+1,d_2)}f(z),F_{e,i,p,(d_1+2,d_2)}f(z),F_{e,i,p,(d_1+3,d_2)}f(z);z\right). \tag{3.8}$$

Thus (3.2) will be

$$\psi(G(z),zG'(z),z^2G''(z),z^3G'''(z);z) \in \Omega.$$

Lead to

$$\frac{\mathfrak{t}}{\mathfrak{s}}+1 = \frac{d_1\mathbb{w}(d_1+p)-\mathbb{u}(d_1+p)(d_1+2)}{\mathbb{v}d_1-\mathbb{u}(d_1+p)}-(2d_1+3),$$

and

$$\frac{\mathfrak{u}}{\mathfrak{s}} = \frac{d_1\mathbb{y}(d_1+p)[(d_1+2)-\mathbb{w}(d_1+3)]+(d_1+2)(d_1+3)[3\mathbb{v}d_1-\mathbb{u}(d_1+p)]}{\mathbb{v}d_1-\mathbb{u}(d_1+1)}.$$

Thus, the condition of admissibility for $\psi \in \Psi_n[\Omega, \mathfrak{q}]$ as get in Definition (2.4) with $n = 2$ is equivalent to the condition of admissibility for $\phi \in \Theta_2[\Omega, \mathfrak{q}]$ in Definition (3.1). Thus by applying Lemma (2.1) and taking equation (3.1), to obtain

$G(z) \prec \mathfrak{q}(z)$, means, $F_{e,i,p,(d_1,d_2)}f(z) \prec \mathfrak{q}(z)$, $(z \in U^*)$ and the proof is complete.

Corollary (3.1): Let $\Omega \subset \mathbb{C}$ and the function $\mathfrak{q}$ is univalent in U with $\mathfrak{q}(0) = 1$. Assume $\phi \in \Theta_n[\Omega, \mathfrak{q}_\sigma]$ for some $\sigma \in (0,1)$, where $\mathfrak{q}_\sigma(z) = \mathfrak{q}(\sigma z)$. If the function $f \in \mathcal{A}^*$ and $\mathfrak{q}_\sigma$ satisfy the conditions bellow:

$$Re\left(\frac{\zeta \mathfrak{q}_\sigma''(\zeta)}{\mathfrak{q}_\sigma'(\zeta)}\right) \geq 0\,, \quad \left|\frac{F_{e,i,p,(d_1+1,d_2)}f(z)}{\mathfrak{q}_\sigma'(\zeta)}\right| \leq k, \ \ (\zeta \in \partial U \backslash E(\mathfrak{q}_\sigma), z \in U^*, k \geq 2) \tag{3.10}$$

and

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z)\right) \in \Omega,$$

then

$$F_{e,i,p,(d_1,d_2)}f(z) \prec \mathfrak{q}_\sigma(z), (z \in U^*).$$

Proof: Using Theorem (3.2), to obtain

$F_{e,i,p,(d_1,d_2)}f(z) \prec \mathfrak{q}_\sigma(z)$ *such that* $z \in U^*$.

The result asserted by Corollary 3.1 is now deduced from following subordination property $\mathfrak{q}_\sigma(z) \prec \mathfrak{q}(z), (z \in U)$.

The proof is complete.

Assume $\Omega \neq \mathbb{C}$ be a simply connect domain, then $\Omega = \eta(U)$ for some conformal mapping $\eta(z)$ *of* U *onto* Ω. In this case, the class $\Theta_n[\eta(U), \mathfrak{q}]$ is written as $\Theta_n[\eta, \mathfrak{q}]$. The two results being direct consequences of and Theorem and Corollary (3.2), (3.1) respectively.

Theorem(3.3): Assume $\phi \in \Theta_n[\eta, \mathfrak{q}]$ .If $f \in \mathcal{A}^*$and $\mathfrak{q} \in \mathbb{Q}_0 \cap \mathcal{H}_0$ satisfy the following conditions (3.1), and

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right) \prec \eta(z) \tag{3.11}$$

then

$$F_{e,i,p,(d_1,d_2)}f(z) \prec \mathfrak{q}(z), \qquad z \in U^*.$$

Corollary (3.2): Assume $\Omega \subset \mathbb{C}$ and the function is univalent in U with $\mathfrak{q}(0) = 1$. Assume that $\phi \in \Theta_n[\eta, \mathfrak{q}_\sigma]$ for some $\sigma \in (0,1)$, such that $\mathfrak{q}_\sigma(z) = \mathfrak{q}_\sigma(\sigma z)$. If $f \in \mathcal{A}^*$ and $\mathfrak{q}_\sigma$ satisfy the conditions (3.10), and

$\phi\big(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\big) \prec \eta(z)$,

then

$$F_{e,i,p,(d_1,d_2)}f(z) \prec q_\sigma(z), \qquad (z \in U^*).$$

The new Theorem bellow leads to the most the best dominant for differential subordination (3.11).

Theorem (3.4): Assume $\phi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$ and ψ by given by (3.8) and let $\eta(z)$ be univalent in U. Suppose that the following differential equation:

$$\psi(q(z), zq'(z), z^2q''(z), z^3q'''(z); z) = \eta(z), \tag{3.12}$$

has a solution $q(z)$ will $q(0) = 1$, which satisfy condition (3.1). If $f \in \mathcal{A}^*$, satisfies the condition (3.11), $\phi \in \Theta_n[\eta, q]$ and

$$\phi\big(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\big)$$

be analytic in U^*, then

$$F_{e,i,p,(d_1,d_2)}f(z) \prec q(z), \ (z \in U^*)$$

and $q(z)$ is the best dominant.

Proof: By using Theorem (3.2), we conclude that $q(z)$ be as a dominant of (3.11). Since $q(z)$ be a solution of (3.11) because its satisfying equation (3.12). Therefore q is dominated by all dominants. Hence $q(z)$ is the best dominant and the proof is complete.

In view of Definition (3.1), and in special case when $q(z) = \mu z, \mu > 0$, the class of admissible functions $\Theta_n[\Omega, q]$, denoted by $\Theta_n[\Omega, \mu]$, is expressed as follows:

Definition (3.5): Assume Ω be set in $\mathbb{C}$, $d_1 \in \mathbb{C}\backslash Z_o^-$, and $\mu > 0$. The class $\Theta_n[\Omega, \mu]$ of admissible functions consists of those functions $\phi: \mathbb{C}^4 \times U \longrightarrow \mathbb{C}$ such that:

$$\Theta(\mu e^{i\vartheta} + 1, \frac{\lambda + (d_1+p)\mu e^{i\vartheta}}{d_1} + 1, \frac{\beta + (d_1+2)(2\lambda + d_1 + p)\mu e^{i\vartheta}}{d_1(d_1+p)} + 1, 1 + \frac{\gamma + 3(d_1+3)\beta + (d_1+2)(d_2+3)(3\lambda + d_1 + p)\mu e^{i\vartheta}}{d_1(d_1+p)(d_1+2)}; z) \notin \Omega, \tag{3.13}$$

where $z \in U, Re\big(\beta e^{-i\vartheta}\big) \geq (\lambda - 1)\lambda\mu$ and $Re\big(\gamma e^{-i\vartheta}\big) \geq 0$, $k \geq 2$ and for all $\vartheta \in \mathbb{R}$.

Corollary (3.3): Assume $\phi \in \Theta_n[\Omega, \mu]$. If the function $f \in \mathcal{A}^*$ satisfies the following conditions:

$\left|F_{e,i,p,(d_1+1,d_2)}f(z)\right| \leq k\mu$, $(z \in U;\ k \geq 2;\ \mu > 0)$, and

$$\phi\big(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\big) \in \Omega$$

then

$$\left|F_{e,i,p,(d_1,d_2)}f(z) - 1\right| < \mu.$$

In particular condition when $\Omega = \mathrm{q}(U) = \{\mathbb{w}: |\mathbb{w} - 1| < \mu, \mu > 0\}$, we define $\Theta_n[\Omega, \mu]$ class as simply of $\Theta_n[\mu]$. We can write the Corollary (3.3) as a form bellow:

Corollary (3.4): Assume $\phi \in \Theta_n[\mu]$. If the function $f \in \mathcal{A}^*$and holds the following conditions:

$$\left|F_{e,i,p,(d_1+1,d_2)}f(z)\right| \leq k\mu\,, (z \in U;\; k \geq 2;\; \mu > 0),$$

$$\left|\phi\left[F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right] - 1\right| < \mu,$$

then

$$\left|\left[F_{e,i,p,(d_1,d_2)}f(z)\right] - 1\right| < \mu.$$

Corollary (3.5): Assume $d_1 \in \mathbb{C}^*, \mathbb{C}^* = \mathbb{C}\backslash\{0\}$ with $Re(d_1) \geq \frac{1-k}{2}, k \geq 2$ and $\mu > 0$ .If $f \in \mathcal{A}^*$ satisfies the following conditions:

$$\left|\left[F_{e,i,p,(d_1+1,d_2)}f(z)\right] - 1\right| \leq k\mu\,, k \geq 2, \text{ and } \left|\left[F_{e,i,p,(d_1+1,d_2)}f(z)\right] - 1\right| < \mu,$$

then

$$\left|\left[F_{e,i,p,(d_1,d_2)}f(z)\right] - 1\right| < \mu.$$

Proof: Corollary (3.5) introduced by Corollary (3.4), where, we put

$$\phi(\mathbb{u}, \mathbb{v}, \mathbb{w}, \mathbb{y}; z) = \mathbb{v} = 1 + \frac{\lambda + (d_1 + 1)\ \mu e^{i\vartheta}}{d_1}.$$

The proof is complete.

Corollary (3.6): Assume $d_1 \in \mathbb{C}^*, \mathbb{C}^* = \mathbb{C}\backslash\{0\}, k \geq 2$ and $\mu > 0$. If the function $f \in \mathcal{A}^*$ satisfies the following conditions:

$$\left|F_{e,i,p,(d_1+1,d_2)}f(z)\right| \leq k\mu$$

and

$$\left|F_{e,i,p,(d_1+3,d_2)}f(z) - F_{e,i,p,(d_1+2,d_2)}f(z)\right| \leq \frac{\mu}{|d_1|},$$

then

$$\left|F_{e,i,p,(d_1,d_2)}f(z) - 1\right| < \mu\,.$$

Proof: Assume $\phi(\mathbb{u}, \mathbb{v}, \mathbb{w}, \mathbb{y}; z) = \mathbb{y} - \mathbb{w}\,, \Omega = \eta(U)$, where $\eta(z) = \frac{\mu z}{|d_1|}, \mu > 0$. In order to use Corollary (3.3), we need to show that $\phi \in \Theta_n[\Omega, \mu]$, that is the admissibility condition (3.13) is satisfied. This follows readily, since it is seen that

$$\left|\phi\left(1+\mu e^{i\vartheta}, 1+\frac{\lambda+(d_1+p)\mu e^{i\vartheta}}{d_1}, 1+\frac{\beta+(d_1+2)(2\lambda+d_1+p)\mu e^{i\vartheta}}{d_1(d_1+p)}, 1+\frac{\gamma+3(d_1+3)\beta+(d_1+2)(d_2+3)(3\lambda+d_1+p)\mu e^{i\vartheta}}{d_1(d_1+p)(d_1+2)}; z\right)\right|$$

$$= \left|\frac{\gamma+3(d_1+3)\beta+(d_1+2)(d_2+3)(3\lambda+d_1+p)\mu e^{i\vartheta}}{(d_1+2)(d_1+p)d_1} - \frac{\beta+(d_1+2)(2\lambda+d_1+p)\mu e^{i\vartheta}}{(d_1+p)d_1}\right|$$

$$= \left|\frac{\gamma e^{-i\vartheta}+(2d_1+7)\beta e^{-i\vartheta}+(d_1+2)(3\lambda+d_1+p)\mu}{d_1(d_1+p)(d_1+2)e^{-i\vartheta}}\right|$$

$$\geq \frac{Re\left(\gamma e^{-i\vartheta}\right)+|2d_1+7|\, Re\left(\beta e^{-i\vartheta}\right)+|d_1+2||2\lambda+d_1+p|\mu}{|d_1(d_1+p)(d_1+2)|}$$

$$\geq \frac{|2d_1+7|(\lambda-1)\lambda + |d_1+2||2\lambda+d_1+p|\mu}{|d_1(d_1+p)(d_1+2)|}$$

$$\geq \frac{\mu}{|d_1|},$$

where $Re\left(\beta e^{-i\vartheta}\right) \geq (\lambda-1)\lambda\mu, Re\left(\gamma e^{-i\vartheta}\right) \geq 0$ and $z \in U^*$ to all $\vartheta \in \mathbb{R}$ and $k \geq 2$.

4. Third-Order Differential Superordination Results:

Here, we prove and investigate some theorems involved in third-order differential superordination with using operator $F_{e,i,p,(d_1,d_2)}f(z)$ in (1.3). For the purpose, we consider the next class of admissible functions.

Definition (4.1): Let Ω be a set in $\mathbb{C}$ and $q \in \mathbb{Q}_0 \cap \mathcal{H}_0$ $with$ $q'(z) \neq 0$. The admissible functions class $\Theta'_n[\Omega, q]$ consists of those functions $\phi: \mathbb{C}^4 \times \overline{U} \longrightarrow \mathbb{C}$ that satisfy the following admissibility conditions:

$$\Theta(\mathbb{u}, \mathbb{v}, \mathbb{w}, \mathbb{y}; \zeta) \in \Omega,$$

whenever

$$\mathbb{u} = q(z), \qquad \mathbb{v} = \frac{\frac{1}{m}zq'(z)+(d_1+p)q(z)}{d_1},$$

$$Re\left\{\frac{d_1(d_1+p)\mathbb{w}-(d_1+p)(d_1+2)\mathbb{u}}{d_1\mathbb{v}-(d_1+p)\mathbb{u}} - (2d_1+3)\right\} \leq \frac{1}{m} Re\left\{\frac{zq''(z)}{q(z)}+1\right\}, \tag{4.1}$$

and

$$Re\left\{\frac{d_1(d_1+p)[(d_1+2)\mathbb{y}-3(d_1+3)\mathbb{w}]+(d_1+2)(d_1+3)[3d_1\mathbb{v}-(d_1+p)\mathbb{u}]}{d_1\mathbb{v}-(d_1+p)\mathbb{u}}\right\} \leq \frac{1}{m^2} Re\left\{\frac{z^2q'''(z)}{q(z)}\right\},$$

where $z \in U, d_1 \in \mathbb{C}\backslash Z_o^-$, $\zeta \in \partial U \backslash E(q)$ and $m \geq 2$.

Theorem (4.1): Assume $\phi \in \Theta'_n[\Omega, q]$. If the functions $f \in \mathcal{A}^*$and $q \in \mathbb{Q}_0 \cap \mathcal{H}_0$ with $q'(z) \neq 0$, satisfy the following conditions:

$$Re\left(\frac{zq''(z)}{q'(z)}\right) \geq 0, \qquad \left|\frac{F_{e,i,p,(d_1+1,d_2)}f(z)}{q'(z)}\right| \leq m, \tag{4.2}$$

and

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right),$$

is univalent within U^*, then

$$\Omega \subset \left\{ \phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right)\right\}, \tag{4.3}$$

implies

$$\mathfrak{q}(z) \prec F_{e,i,p,(d_1,d_2)}f(z), z \in U.$$

Proof: Equation (3.3) defined the analytic function $G(z)$ while equation (3.8) defined ψ. Because of $\phi \in \Theta'_n[\Omega, \mathfrak{q}]$. From (3.8) and (4.3), we obtain

$$\Omega \subset \left\{\psi(G(z), zG'(z), z^2G''(z), z^3G'''(z); z), z \in U\right\}. \tag{4.4}$$

We note that the equation (3.7) involved with admissibility condition for $\phi \in \Theta'_n[\Omega, \mathfrak{q}]$ as defined by the Definition (4.1), which is equivalent to the admissibility condition of $\psi \in \Psi'_n[\Omega, \mathfrak{q}]$ according to Definition (2.5), with $n = 2$. Therefore when $\psi \in \Psi'_2[\Omega, \mathfrak{q}]$, taking Lemma (2.2) and using the equation (4.2), we obtain $\mathfrak{q}(z) \prec f(z)$ or $\mathfrak{q}(z) \prec F_{e,i,p,(d_1,d_2)}f(z)$. The proof is complete.

If $\Omega \neq \mathbb{C}$ is a simply connected domain, then $\Omega = \eta(U)$ for some conformal mapping $\eta(z)$ *of* U, *onto* Ω. In this case, the class $\Theta'_n[\eta(U), \mathfrak{q}]$ is taken as simply by $\Theta'_n[\eta, \mathfrak{q}]$. Now, Theorem (4.1) gives us the following results.

Theorem (4.2): Assume the function η is analytic in U and $\phi \in \Theta'_n[\Omega, \mathfrak{q}]$. If functions $f \in \mathcal{A}^*$, $F_{e,i,p,(d_1,d_2)}f(z) \in \mathbb{Q}_0$ and $\mathfrak{q} \in \mathcal{H}_0$ satisfy the following conditions (4.2), and

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right)$$

is univalent within U^*, then

$$\Omega \subset \left\{\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right)\right\}, \tag{4.5}$$

implies

$$\mathfrak{q}(z) \prec F_{e,i,p,(d_1,d_2)}f(z).$$

The proof is complete.

Now, the Theorems (4.1) and (4.2) can only be used to obtain subordinants involved with differential superordination of third-order of the forms (4.4) or (4.5).

The Theorem bellow prove that best subordinant is exist in (4.5) with suitable chosen ϕ .

Theorem (4.3): Assume $\phi: \mathbb{C}^4 \times \overline{U} \longrightarrow \mathbb{C}$, ψ be defined by (3.8) and η be univalent function in U. Assume that the following differential equation:

$$\psi(q(z), zq'(z), z^2q''(z), z^3q'''(z); z) = \eta(z), \tag{4.6}$$

has a solution $q(z) \in \mathbb{Q}_0 \cap \mathcal{H}_0$ with $q'(z) \neq 0$. If the function $f \in \mathcal{A}^*$and $F_{e,i,p,(d_1,d_2)}f(z) \in \mathbb{Q}_0$ satisfy the condition (4.2) and the function

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right),$$

is univalent within U^*, then

$$\eta(z) \prec \{\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right)\}, \tag{4.7}$$

implies that

$$q(z) \prec F_{e,i,p,(d_1,d_2)}f(z),$$

and $q(z)$ is the best subordinant.

Proof: By using Theorem (4.1) and (4.2), we deduce that q is a subordinant of (4.7). Since q satisfies (4.6), it is also a solution of (4.7) and therefore q will be subordinant by all subordinants. Hence q is the best subordinant. This completes the proof of Theorem (4.3).

5. Sandwich-Type Result:

Now, by combining Theorems (3.3) and (4.3), we obtain the following sandwich-type theorem.

Theorem (5.1): Assume the two functions say $\eta_1$and q_1 be analytic functions in U. Also, assume η_2 be univalent function in U, $q_2 \in \mathbb{Q}_0$ *with* $q_1(0) = q_2(0) = 1$ and $\phi \in \Theta_n[\eta_1, q_1] \cap \Theta'_n[\eta_2, q_2]$. If the function $f \in \mathcal{A}^*$ and $F_{e,i,p,(d_1,d_2)}f(z) \in \mathbb{Q}_0 \cap \mathcal{H}_0$ and

$$\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right),$$

be univalent in U, while the conditions (3.1) and (4.2) are satisfied ,thus

$$\eta_1(z) \prec \{\phi\left(F_{e,i,p,(d_1,d_2)}f(z), F_{e,i,p,(d_1+1,d_2)}f(z), F_{e,i,p,(d_1+2,d_2)}f(z), F_{e,i,p,(d_1+3,d_2)}f(z); z\right)\} \prec \eta_2(z),$$

then

$$q_1(z) \prec F_{e,i,p,(d_1,d_2)}f(z) \prec q_2(z).$$

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