Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

On the Fuzzy $\widetilde{\mathcal{N}-n}$-Quasi-Normal Operators

Ahmed Talib Kareem [a]*

[a] *Ministry of Education, Third Directorate of Rusafa. Email: ahmedmath1221@gmail.com*

ARTICLE INFO

Article history:
Received: 04 /06/2025
Rrevised form: 06 /07/2025
Accepted : 13 /07/2025
Available online: 30 /09/2025

Keywords:

Keywords: ($\tilde{F}$-Vector space), fuzzy adjoint of operator, fuzzy self adjoint operator, fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operator.

ABSTRACT

In this work, we introduce generalizations of fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasinormal operators defined on fuzzy Hilbert spaces over fuzzy vector spaces. We study several fundamental properties of these operators and explore specific operations associated with them.

MSC..

https://doi.org/10.29304/jqcsm.2025.17.32408

Introduction

Many researchers in functional analysis concentrate on operator theory, with particular interest in fuzzy operator theory. In 1965, Zadeh [1] introduced the foundational concept of fuzzy sets, along with their properties and operations, marking a significant development in mathematics. Subsequently, in 1991, Biswas proposed several key definitions in fuzzy mathematics, including the fuzzy inner product and fuzzy norm mappings, accompanied by several theorems [2]. Building on this, in 1993, Keohil and Kumar introduced additional properties of the fuzzy inner product and proposed a concept called the fuzzy co-inner product space [5].
Kohil and Kumar 1995 further developed the theory, by defining fuzzy linear operators on fuzzy inner product spaces and exploring their fundamental properties [4]. Later, in 2009, Goudearzi and Viaezpour examined the concept of fuzzy Hilbert spaces (FH-spaces), presenting several important results related to this structure [5].
In 2018, Radharamani et al. introduced a new class of fuzzy operators known as fuzzy normal operators, defined on fuzzy Hilbert spaces. They provided various characterizations and essential properties of these operators [6]. In 2022 proposed generalizations for the fuzzy $\left((\mu-n)\right)^{*}$ -quasi-normal operators on fuzzy Hilbert spaces and explored their key properties and operations [10].
In the present work, we propose new generalizations of fuzzy operators, specifically the fuzzy$(\widetilde{\mu-n})^{*}$-quasi-normal operators defined on FH-spaces. The primary motivation behind this research is to extend the existing results and investigate new operational behaviors of such operators [7,8]. More specifically, we aim to establish and prove

*Ahmed Talib Kareem

Email addresses: *ahmedmath1221@gmail.com*

several characterizations associated with these concepts in fuzzy operator theory. The main findings are detailed in the following sections of this paper.

2- Preliminaries

Definition 2.1 [1]: The order pair of the sets $\hat{A} = \{(x, \mu_{\hat{A}}(x)) | x \in \mathcal{X} \wedge \mu_{\hat{A}}(x) \in I\}$, is namely fuzzy set $\hat{A}$ over $\mathcal{X}$ where $\mu_{\hat{A}}: \mathcal{X} \longrightarrow I$ is a mapping said to be set characterized of a membership mapping where I = [0,1] and also, $\mu_{\hat{A}}(x)$ is said to be degree of membership of x in $\hat{A}$.

Definition 2.2 [9]: The fuzzy vector space ($\tilde{F}$-space) $\widehat{V(X)}$ is fuzzy set with a couple of operations defined for every fuzzy points $\hat{x}_{\mu(x)}$, $\hat{y}_{\mu(y)}$ in the fuzzy set $\in \widehat{V(X)}$, satisfy

i) $\widehat{(x+y)}_{\mu(x)+\mu(y)} = \hat{x}_{\mu(x)} + y_{\mu(y)}$

ii) $\hat{r}.\hat{x}_{\mu(x)} = \widehat{(r.x)}_{\mu(x)}, \hat{r} \in \hat{R}(A)$.

Now, another essential concept of this work gives by the following definition.

Definitions 2.3 [4]: A $\tilde{F}$-operator $\tilde{T}: \widetilde{H} \to \widetilde{H}$ is a

1) fuzzy linear operator $\left(\tilde{F}L - operator\right)$ if it satisfies the condition

$\tilde{T}\left(\tilde{a}\tilde{x}_{\mu(x)} + \tilde{b}\tilde{y}_{\mu(y)}\right) = \left(\tilde{a}\tilde{T}\left(\widetilde{\tilde{x}_{\mu(x)}}\right) + \tilde{b}\tilde{T}\left(\widetilde{\tilde{y}_{\mu(y)}}\right)\right)$,

for all $\widetilde{\tilde{x}_{\mu(x)}}$ and $\widetilde{\tilde{y}_{\mu(y)}} \in \widetilde{H}$ and $\tilde{a}, \tilde{b}$ any fuzzy scalars.

2) fuzzy bounded operator $\left(\tilde{F}B - operator\right)$ if it satisfies the condition

$\left\|\tilde{T}\left(\widetilde{\tilde{x}_{\mu(x)}}\right)\right\| \leq \tilde{\alpha} \left\|\widetilde{\tilde{x}_{\mu(x)}}\right\|$

$\forall \tilde{x}_{\mu(x)} \in \widetilde{H}$ and $\exists \tilde{\alpha} \in \tilde{\mathcal{R}}^{+}(A)$.

Definitions 2.4 [3]: Let $\tilde{T}: \widetilde{H} \to \widetilde{H}$ be $\tilde{F}\mathcal{B}L - operator$, the fuzzy adjoint of operator $\tilde{T}^{*}$ is such that $\langle \tilde{x}_{\mu(x)}, \tilde{T}^{*} \tilde{y}_{\mu(y)} \rangle = \langle \tilde{T}\tilde{x}_{\mu(x)}, \tilde{y}_{\mu(y)} \rangle \ \forall \ \tilde{x}_{\mu(x)}, \tilde{y}_{\mu(y)} \in \widetilde{H}$

Definitions 2.5 [1]: An fuzzy operator $\tilde{T}: \widetilde{H} \to \widetilde{H}$ is Fuzzy self adjoint operator $\left(\tilde{F} - self\ adjoint\ operator\right)$ if $\tilde{T} = \tilde{T}^{*}$.

Definitions 2.6 [1]: An fuzzy operator $\tilde{T}: \widetilde{H} \to \widetilde{H}$ is Fuzzy normal operator if $\tilde{T}\tilde{T}^{*} = \tilde{T}^{*}\tilde{T}$ with shortly by $\left(\tilde{F}N - operator\right)$.

Definitions 2.7 [3]: An fuzzy operator $\tilde{T}: \widetilde{H} \to \widetilde{H}$, where $\widetilde{H}$ is fuzzy Hilbert space, is Fuzzy Quasi-normal operator on the if satisfy $\tilde{T}\left(\tilde{T}^{*}\tilde{T}\right) = \left(\tilde{T}^{*}\tilde{T}\right)\tilde{T}$ and shortly by $\left(\tilde{F}QN - operator\right)$.

3. On fuzzy $\widetilde{\mathcal{N} - n}$-quasi-normal operators:

Here, we propose a generalization of the quasi-normal operator, referred to as the fuzzy $(\widetilde{\mathcal{N} - n})^{*}$--quasi-normal operator. We also present several properties associated with this operator. We begin with the main definition.

Definition 3.1: Let $\tilde{T}: \widetilde{H} \to \widetilde{H}$ be fuzzy bounded linear operator defined on fuzzy Hilbert space $\widetilde{H}$, then one can say $\tilde{T}$ is fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal operator if $\tilde{T}^{*}\left(\tilde{T}\tilde{T}^{*^{n}}\right) = \widetilde{\mathcal{N}}\left(\tilde{T}\tilde{T}^{*^{n}}\right)\tilde{T}^{*}$, where $\widetilde{\mathcal{N}}: \widetilde{H} \to \widetilde{H}, n \in N$.

Remark: 3.2:

When $n = 1, \widetilde{\mathcal{N}} = \tilde{I}$, then $\tilde{T}: \widetilde{H} \to \widetilde{H}$ is fuzzy quasi-normal operator.

Proposition 3.3: Let $\tilde{T}:\tilde{H} \to \tilde{H}$ be fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operator defined on fuzzy Hilbert space $\tilde{H}$, then, $[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]^{m} = [\tilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}]^{m}$.

Proof :

By using mathematical induction

if $m = 1$,

$$\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n}) = [\tilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}].$$

if it is true when $m = k$

$$[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]^{k} = [\tilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}]^{k}$$

To prove it when $m = k + 1$

$[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]^{k+1} = [\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]^{k}[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]$,

So that

$$[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})]^{k}[\tilde{T}^{*}(\tilde{T}\tilde{T}^{*n})] = [\tilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}]^{k}[\tilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}]$$

$$= [\widetilde{\mathcal{N}}(\tilde{T}\tilde{T}^{*n})\tilde{T}^{*}]^{k+1}.$$

Theorem 3.4: Let $\widetilde{T_0}, \widetilde{T_1}:\tilde{H} \to \tilde{H}$ be fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$ quasi-normal operator which satisfy the conditions

$\widetilde{T_0}^{*}\widetilde{T_1}^{*} = \widetilde{T_0}^{*}\widetilde{T_1} = \widetilde{T_1}\widetilde{T_0}^{*} = \widetilde{T_0}\widetilde{T_1}^{*} = \tilde{0}$, then $\widetilde{T_0} + \widetilde{T_1}$ is a fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operator.

Proof:

$$(\widetilde{T_0} + \widetilde{T_1})^{*} \ (\widetilde{T_0} + \widetilde{T_1}) \ (\widetilde{T_0} + \widetilde{T_1})^{*n}$$

$$= (\widetilde{T_0} + \widetilde{T_1})^{*}((\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*} + \widetilde{T_1}^{*})^{n})$$

$$= (\widetilde{T_0} + \widetilde{T_1})^{*}(\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*n} + n\left(\widetilde{T_0}^{*n-0}\right)\widetilde{T_1}^{*} + \cdots + \widetilde{T_1}^{*n})$$

$$= (\widetilde{T_0}^{*} + \widetilde{T_1}^{*})(\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*n} + \widetilde{T_1}^{*n})$$

$$= (\widetilde{T_0}^{*} + \widetilde{T_1}^{*})(\widetilde{T_0}\widetilde{T_0}^{*n} + \widetilde{T_1}\widetilde{T_0}^{*n} + \widetilde{T_0}\widetilde{T_1}^{*n} + \widetilde{T_1}\widetilde{T_1}^{*n})$$

$$= \widetilde{T_0}^{*}\widetilde{T_0}\widetilde{T_0}^{*n} + \widetilde{T_0}^{*}\widetilde{T_1}\widetilde{T_0}^{*n} + \widetilde{T_0}^{*}\widetilde{T_0}\widetilde{T_1}^{*n} + \widetilde{T_0}^{*}\widetilde{T_1}\widetilde{T_1}^{*n} + \widetilde{T_1}^{*}\widetilde{T_0}\widetilde{T_0}^{*n} + \widetilde{T_1}^{*}\widetilde{T_1}\widetilde{T_0}^{*n} + \widetilde{T_1}^{*}\widetilde{T_0}\widetilde{T_1}^{*n} + \widetilde{T_1}^{*}\widetilde{T_1}\widetilde{T_1}^{*n}$$

$$= \widetilde{T_0}^{*}\widetilde{T_0}\widetilde{T_0}^{*n} + \widetilde{T_1}^{*}\widetilde{T_1}\widetilde{T_1}^{*n}$$

$$= \tilde{\mathcal{N}}(\widetilde{T_0}\widetilde{T_0}^{*n})\widetilde{T_0}^{*} + \tilde{\mathcal{N}}(\widetilde{T_1}\widetilde{T_1}^{*n})\widetilde{T_1}^{*} \ldots.(1)$$

$$\tilde{\mathcal{N}}\left((\widetilde{T_0} + \widetilde{T_1}) \ (\widetilde{T_0} + \widetilde{T_1})^{*n}\right)(\widetilde{T_0} + \widetilde{T_1})^{*}$$

$$= \tilde{\mathcal{N}}(\widetilde{T_0} + \widetilde{T_1})^{*}((\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*} + \widetilde{T_1}^{*})^{n})$$

$$= \tilde{\mathcal{N}}(\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*n} + n\left(\widetilde{T_0}^{*n-0}\right)\widetilde{T_1}^{*} + \cdots + \widetilde{T_1}^{*n})(\widetilde{T_0} + \widetilde{T_1})^{*}$$

$$= \tilde{\mathcal{N}}(\widetilde{T_0} + \widetilde{T_1})(\widetilde{T_0}^{*n} + \widetilde{T_1}^{*n})(\widetilde{T_0}^{*} + \widetilde{T_1}^{*})$$

$$= \widetilde{\mathcal{N}}(\widetilde{T_0}\widetilde{T_0}^{*n} + \widetilde{T_1}\widetilde{T_0}^{*n} + \widetilde{T_0}\widetilde{T_1}^{*n} + \widetilde{T_1}\widetilde{T_1}^{*n})(\widetilde{T_0}^{*} + \widetilde{T_1}^{*})$$

$$= \widetilde{\mathcal{N}}\left(\widetilde{T_0}\widetilde{T_0}^{*n}\widetilde{T_0}^{*} + \widetilde{T_1}\widetilde{T_0}^{*n}\widetilde{T_0}^{*} + \widetilde{T_0}\widetilde{T_1}^{*n}\widetilde{T_0}^{*} + \widetilde{T_1}\widetilde{T_1}^{*n}\widetilde{T_0}^{*} + \widetilde{T_0}\widetilde{T_0}^{*n}\widetilde{T_1}^{*} + \widetilde{T_1}\widetilde{T_0}^{*n}\widetilde{T_1}^{*} + \widetilde{T_0}\widetilde{T_1}^{*n}\widetilde{T_1}^{*} + \widetilde{T_1}\widetilde{T_1}^{*n}\widetilde{T_1}^{*}\right)$$

$$= \left(\widetilde{\mathcal{N}}\widetilde{T_0}\widetilde{T_0}^{*n}\widetilde{T_0}^{*} + \widetilde{\mathcal{N}}\widetilde{T_1}\widetilde{T_1}^{*n}\widetilde{T_1}^{*}\right)$$

$$= \widetilde{\mathcal{N}}(\widetilde{T_0}\widetilde{T_0}^{*n})\widetilde{T_0}^{*} + \widetilde{\mathcal{N}}(\widetilde{T_1}\widetilde{T_1}^{*n})\widetilde{T_1}^{*} \ldots.(2)$$

$$= \widetilde{\mathcal{N}}\left((\widetilde{T_0} + \widetilde{T_1})\ \ (\widetilde{T_0} + \widetilde{T_1})^{*n}\right)(\widetilde{T_0} + \widetilde{T_1})^{*n}$$

Therefore; $\widetilde{T_0} + \widetilde{T_1}$ is a fuzzy $(\widetilde{\mathcal{N} - n})^*$-quasi-normal operator.

Theorem 3.5: Let $\widetilde{T_0}, \widetilde{T_1}: \widetilde{H} \to \widetilde{H}$ be fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal operator with satisfy the conditions $\widetilde{T_0}\widetilde{T_1} = \widetilde{T_1}\widetilde{T_0}, \widetilde{T_0}\widetilde{T_1}^{*} = \widetilde{T_1}^{*}\widetilde{T_0}$, and $\widetilde{T_1}$,fuzzy quasi-normal then $\widetilde{T_0}\widetilde{T_1}$ is a fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal.

Proof:

$$(\widetilde{T_0}.\widetilde{T_1})^{*}(\widetilde{T_0}.\widetilde{T_1})\ \ (\widetilde{T_0}.\widetilde{T_1})^{*n} = (\widetilde{T_1}.\widetilde{T_0})^{*}(\ (\widetilde{T_0}.\widetilde{T_1})\ \ (\widetilde{T_1}.\widetilde{T_0})^{*n}$$

$$= (\widetilde{T_0}^{*}\widetilde{T_1}^{*})(\widetilde{T_0}.\widetilde{T_1})(\widetilde{T_0}^{*n}\widetilde{T_1}^{*n})\text{ss}$$

$$= (\widetilde{T_0}^{*}\widetilde{T_1}^{*}\widetilde{T_0})(\widetilde{T_1}\widetilde{T_0}^{*n}\widetilde{T_1}^{*n})$$

$$= (\widetilde{T_0}^{*}\widetilde{T_0}\widetilde{T_1}^{*})(\widetilde{T_0}^{*n}\widetilde{T_1}\widetilde{T_1}^{*n})$$

$$= (\widetilde{T_0}^{*}\widetilde{T_0}\widetilde{T_0}^{*n})(\widetilde{T_1}^{*}\widetilde{T_1}\widetilde{T_1}^{*n})$$

$$= \widetilde{\mathcal{N}_0}((\widetilde{T_0}\widetilde{T_0}^{*n})\widetilde{T_0}^{*})\widetilde{\mathcal{N}_1}((\widetilde{T_1}\widetilde{T_1}^{*n})\widetilde{T_1}^{*})$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[((\widetilde{T_0}\widetilde{T_0}^{*n})\widetilde{T_0}^{*})((\widetilde{T_1}\widetilde{T_1}^{*n})\widetilde{T_1}^{*})]$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[((\widetilde{T_0}\widetilde{T_0}^{*n})\widetilde{T_1}\widetilde{T_0}^{*})((\widetilde{T_1}^{*n})\widetilde{T_1}^{*}$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[((\widetilde{T_0}\widetilde{T_1}\widetilde{T_0}^{*n})\widetilde{T_0}^{*})((\widetilde{T_1}^{*n})\widetilde{T_1}^{*}$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[((\widetilde{T_0}\widetilde{T_1}\widetilde{T_0}^{*n})\widetilde{T_1}^{*n}\widetilde{T_0}^{*}\widetilde{T_1}^{*}$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[(\left(\widetilde{T_0}\widetilde{T_1}\widetilde{T_0}^{*n}\widetilde{T_1}^{*n}\right)(\widetilde{T_0}^{*}\widetilde{T_1}^{*})$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[\left((\widetilde{T_0}\widetilde{T_1})(T_1.\widetilde{T_0})^{*n}\right)(\widetilde{T_1}.\widetilde{T_0})^{*}],$$

$$= \widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1}[\left((\widetilde{T_0}\widetilde{T_1})(\widetilde{T_0}.\widetilde{T_1})^{*n}\right)(\widetilde{T_0}\widetilde{T_1}.)^{*}]$$

By choosing $\widetilde{\mathcal{N}_0}\widetilde{\mathcal{N}_1} = \widetilde{\mathcal{N}}$, then, we have $(\widetilde{T_0}.\widetilde{T_1})^{*}(\ (\widetilde{T_0}.\widetilde{T_1})\ \ (\widetilde{T_0}.\widetilde{T_1})^{*n} = \widetilde{\mathcal{N}}[(\widetilde{T_0}.\widetilde{T_1})\ \ (\widetilde{T_0}.\widetilde{T_1})^{*n}](\widetilde{T_0}.\widetilde{T_1})^{*}$

Therefore; $\widetilde{T_0}\widetilde{T_1}.$ is a fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal.

Theorem 3.6: Let $\widetilde{T}: \widetilde{H} \to \widetilde{H}$ be fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal defined on fuzzy Hilbert space then $\tilde{\lambda}\widetilde{T}$ is a fuzzy $\widetilde{\mathcal{N} - \boldsymbol{n}}$-quasi-normal, for any fuzzy real scalar $\tilde{\lambda}$.

Proof:

$$(\tilde{\lambda}\widetilde{T})^{*}\left(\tilde{\lambda}\widetilde{T}(\tilde{\lambda}\widetilde{T}^{*^n}\right) = \tilde{\lambda}\tilde{\lambda}\ \tilde{\lambda}^{n}(\widetilde{T}^{*}\left(\widetilde{T}\widetilde{T}^{*^n}\right))$$

$$= \tilde{\lambda}\tilde{\lambda}\ \tilde{\lambda}^{n}\widetilde{\mathcal{N}}(\left(\widetilde{T}\widetilde{T}^{*^n}\right)\widetilde{T}^{*})$$

$$= \widetilde{\mathcal{N}}\left(\left(\tilde{\lambda}\tilde{T}\tilde{\lambda}^{n}\tilde{T}^{*^{n}}\right)\tilde{\lambda}\tilde{T}^{*}\right)$$

Therefore; $\tilde{\lambda}\tilde{T}$ is a fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operator

Theorem 3.7: Let $\tilde{T}:\tilde{H}\to\tilde{H}$ be fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal defined on fuzzy Hilbert space then $\tilde{T}/_{\tilde{\omega}}$ is a fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal, where $\tilde{\omega}$ is closed subspace

Proof:

$$\tilde{T}/_{\tilde{\omega}}^{\,*}\left(\tilde{T}/_{\tilde{\omega}}\tilde{T}/_{\tilde{\omega}}^{\,*n}\right) = \tilde{T}^{*}\left(\tilde{T}\tilde{T}^{*^{n}}\right)/_{\tilde{\omega}}$$

$$= \widetilde{\mathcal{N}}\left[\left(\tilde{T}\tilde{T}^{*^{n}}\right)\tilde{T}^{*}\right]/_{\tilde{\omega}}$$

$$= \widetilde{\mathcal{N}}\left(\tilde{T}/_{\tilde{\omega}}\tilde{T}/_{\tilde{\omega}}^{\,*n}\right)\tilde{T}/_{\tilde{\omega}}^{\,*}$$

Therefore; $\tilde{T}/_{\tilde{\omega}}$ is a fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operator.

4. Conclusions

One result is the introduction of particular generalization of a type of fuzzy bounded operator which is fuzzy $\widetilde{\mathcal{N}-\boldsymbol{n}}$-quasi-normal operators with some operations such as addition and multiplication as well as the relation of this with fuzzy self-adjoint operator one can have in this paper.

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