Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

On Smarandache M-Semigroup

Page 71-76

Hassan Rashed Yassein
University of Al-Qadisiya
College of Education
Department of Mathematics

Mazen Omran Karim
University of Al-Qadisiya
College of Education
Department of Mathematics

Abstract

In this research, we defined the term a smarandache M – semigroup (S-M-semigroup) and studied some basic properties.

Also defined smarandache fuzzy M-semigroup and some elementary properties about this concepts are discussed .

Introduction

In 1965 Zadeh introduced the concept of fuzzy set, in 1971 Rosenfeld formulated the term of fuzzy subgroup.In 1994 W.X.Gu , S.Y.Li and D.G.Chen studied fuzzy groups and gave some new concepts as M- fuzzy groups . In 1999 W.B.Vasantha introduced the concepts of smarandache semigroups . Smarandache fuzzy semigroups are studied in 2003 by W.B.Vasantha .

In this research , the concept of Smarandache M- fuzzy semigroup are given and its some elementary properties are discussed

1- Preliminaries

Definition(1.1)

A fuzzy set μ of a group G is called a fuzzy subgroup if
$\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$ for every $x,y \in G$. [2]

Definition(1.2)

A fuzzy subgroup μ of a group G is called a fuzzy normal subgroup if
$\mu(xyx^{-1}) \geq \mu(y)$ for every $x,y \in G$.[2]

Definition(1.3)

A group with operators is an algebraic system consisting of a group G , set M and a function defined in the product M×G and having value in G such that , if ma denotes the elements in G determined by the element m of M , then m(ab)=(ma)(mb) hold for all a,b in G ,m in M . [4]

We shall usually use the phrase "G is an M-group " to a group with operators .

Definition (1.4)

If μ is a fuzzy set of G and $t \in [0,1]$ then $\mu_t = \{ x \in G \mid \mu(x) \geq t \}$ is called a t- level set μ . [1]

Definition (1.5)

Let G and G' both be M – groups , f be a homomorphism from G onto G' , if f(mx)= mf(x) for every $m \in M$, $x \in X$, then f is called a M – homomorphism . [4]

Definition (1.6)

Let G be M – group and μ be a fuzzy subgroup of G if $\mu(mx) \geq \mu(x)$ for every $x \in G$, $m \in M$,then μ is said to be a fuzzy subgroup with operators of G , we use the pharse μ is an M – fuzzy subgroup of G instead of a fuzzy subgroup with operators of G . [4]

Proposition (1.7)

If μ is an M – fuzzy subgroup of G , then the following statements hold for every $x,y \in G$, $m \in M$: [4]

1- $\mu(m(xy)) \geq \mu(mx) \wedge \mu(my)$
2- $\mu(mx^{-1})) \geq \mu(x)$

Proposition (1.8)

Let G and G' both M – groups and f an M- homomorphism from G onto G' , if μ' is an M – fuzzy subgroup of G' then $f^{-1}(\mu')$ is an M-fuzzy subgroup of G. [4]

Proposition (1.9)

Let G and G' both M – groups and f an M- homomorphism from G onto G' if μ is an M – fuzzy subgroup of G then f(μ) is an M-fuzzy subgroup of G' . [1]

Definition (1.10)

Let S be a semigroup , S is said to be a smarandache semigroup (S –semigroup) if S has a proper subset P such that P is a group under the operation of G . [2]

Definition (1.11)

Let S be an S-semigroup . A fuzzy subset $\mu : S \rightarrow [0,1]$ is said to be smarandache fuzzy semigroup (S- fuzzy semigroup) if μ restricted to at least one subset P of S which is a subgroup is a fuzzy subgroup . [3]
that is for all $x,y \in P \subset S$, $\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$.
this S- fuzzy semigroup is denoted by $\mu_p : P \rightarrow [0,1]$ is fuzzy group .

Definition (1.12)

A semigroup H with operators is an algebraic system consisting of a semigroup H , set M , and a function defined in the product $M \times H$ and having values in H such that , if ma denotes the element in H determined by the element a in H and the element m in M , then m(ab)=(ma)(mb) , $a,b \in H$ and $m \in M$ then H is M – semigroup . [4]

Definition (1.13)

Let f be a function from a set X to a set Y while μ is fuzzy set of X then the image f(μ) of μ is the fuzzy set $f(\mu) : Y \to [0,1]$ defined by : [1]

$$f(\mu(y)) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi \end{cases}$$

Definition (1.14)

Let f be a function from a set X to a set Y while μ is fuzzy set of Y then the inverse image $f^{-1}(\mu)$ of μ under f is the fuzzy set $f^{-1}(\mu) : X \to [0,1]$ defined by $f^{-1}(\mu)(x) = \mu(f(x))$. [1]

2-The Main Results

In this section we shall define smarandache M – semigroup and smarandache fuzzy M-semigroup and given some its results .

Definition (2.1)

Let H be M- semigroup . H is said to be a smarandache M – semigroup (S-M-semigroup) if H has a proper subset K such that K is M- group under the operation of H .

Definition (2.2)

Let H be a S - M –semigroup. A fuzzy subset $\mu : H \to [0,1]$ is said be smarandache fuzzy M-semigroup if μ restricted to at least one subset K of H which is subgroup is fuzzy subgroup **.**

Definition (2.3)

Let H be a S-M- semigroup . A fuzzy subset $\mu : H \to [0,1]$ is said to be smarandache fuzzy M-semi group if restricted to at least one subset K of H which is M- subgroup is fuzzy M- subgroup

Definition (2.4)

Let S and S' be any two S- semigroup . A map φ from S to S' is said to be S- semigroup homomorphism if φ restricted to a subgroup $A \subset S \to A' \subset S'$ is a group homomorphism .

Definition (2.5)

Let H and H' be any two S-M- semigroup . A map φ from H to H' is said to be S-M- semigroup homomorphism if φ restricted to a M- subgroup $A \subset S \to A' \subset S'$ is M- homomorphism .

Proposition (2.6)

If μ is S- fuzzy M-semigroup of S-M- semigroup then :

1) $\mu_K(m(xy)) \geq \min \{ \mu_K(mx) , \mu_k(my) \}$
2) $\mu_k(mx^{-1}) \geq \mu_k(x)$

For all $x \in M$, $x,y \in K$

Proof

μ is S- fuzzy M-semigroup
Then there exist subset K of H which is M- subgroup such μ restricted of K which is fuzzy M- subgroup
i.e . $\mu_K : K \to [0,1]$, M- fuzzy subgroup
for all $x,y \in K$, $m \in M$, it is clear that

1) $\mu_K(m(xy)) \geq \mu_K((mx)(my))$
$\geq \min \{ \mu_K(mx) , \mu_k(my) \}$
2) $\mu_k(mx^{-1}) = \mu_k(mx)^{-1}$
$\geq \mu_K(mx)$
$\geq \mu_K(mx)$ ■

Proposition (2.7)

Let G be S- semigroup , μ fuzzy set of G , then μ is an S- fuzzy M-semigroup
of G iff $\forall t \in [0,1]$, μ_t is an S-M- semigroup $\mu_t \neq \emptyset$.

Proof

It is clear μ_t is semigroup of G while $\mu_t \neq \emptyset$ holds .
for any $x \in \mu_t$, $m \in M$
$\mu(mx) \geq \mu(x) \geq t$
hence mx in μ_t , hence μ_t is an M- semigroup of **G** .

since S-fuzzy M- semigroup $\exists$ K $\subset$G subgroup $\ni$ μ_t :K $\rightarrow$ [0,1] fuzzy M- subgroup.

$\mu_{K_t} = \{ x \in K \mid \mu_K(x) \geq t \}$.

It is clear μ_{K_t} is group .

hence μ_t S-M- semigroup .

Conversely ,

Since μ_t S-M- semigroup then there exists aproper subset K of G such that K is M- subgroup .

If there exists x$\in$K , m$\in$ M such that $\mu_K(mx) < \mu_K(x)$.

let $t=\frac{1}{2}(\mu_K(mx)+\mu_K(x))$

then $\mu_K(x) > t > \mu_K(mx)$

$mx \notin \mu_{K_t}$ so here emerges a contradiction .

$\mu_K(mx) \geq \mu_K(x)$ always holds for any x$\in$K ,m$\in$ M .

μ_K is M- fuzzy subgroup

hence μ is S – fuzzy M- subgroup **.**

Proposition(2.8)

Let H and H' both be S-M- semigroup and f as S-M- semigroup homomorphism from H onto H' . if μ' is an S- fuzzy M- semigroup of H' then $f^{-1}(\mu')$is an S- fuzzy M-semigroup of H.

Proof:

Since f :H $\rightarrow$ H' is as S-M- semigroup homomorphism then f restricted to M-subgroup .

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism ,

$f^{-1}(\mu)_A : A \rightarrow [0,1]$ such that A M-subgroup ,

For any m$\in$M , x$\in$ A

$$f^{-1}(\mu)_A(mx) = \mu'_A(f(mx))$$
$$= \mu'_A\, m(f(x)) \geq \mu'_A(f(x))$$
$$= f^{-1}(\mu')(x)$$

$f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

Proposition(2.9)

Let H and H' both be S-M- semigroups and f as S-M- semigroup homomorphism from H onto H' . if μ is an S- fuzzy M- semigroup of H then f(μ) is an S- fuzzy M-semigroup of H' .

Proof

Since f :H $\rightarrow$ H' is as S-M- semigroup homomorphism then f restricted to M- subgroup .

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism

$f(\mu)_{A'} : A' \to [0,1]$ such that A' M-subgroup ,

For any $m \in M$, $y \in A'$

$$\begin{aligned} f(\mu)(my) &= \sup \mu(x) \quad , x \in f^{-1}(my) \\ &= \sup \mu(x) \quad , f(x)=my \\ &\geq \sup \mu(mx') \, , f(mx')=mx \, , mx' \in H \\ &= \sup \mu(x') \quad , mf(x')=my \, , mx' \in H \\ &\geq \sup \mu(x') \quad , f(x')=y \, , x' \in H \\ &= f(\mu)(y) \end{aligned}$$

hence $f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

References

[1]K.A.Al-Shamari , "An Annalus Approach To Fuzzy Subgroup " , M.Sc. thesis , Saudui Arabia , 1998 .

[2]W.B.Vasantha ,"Smarandache Fuzzy Algebra ", American research press , 2003.

[3]W.B.Vasantha , "Smarandache Semigroups" , American research press ,2002.

[4]W.X.Gu , S.Y.Li and D.G.Chen , "Fuzzy Groups with Operators " , Fuzzy sets and system , 66 (363-371) , 1994 .