Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

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Dirac function and its applications in solving some problems in mathematics

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Abstract

The paper presents various ways of defining and introducing Dirac delta function, its application in solving some problems and show the possibility of using delta-function in mathematics and physics.

Introduction

The development of science requires for its theoretical basis more and more "high mathematics", one of the achievements which are generalized functions, in particular the Dirac function. The theory of generalized functions is relevant in physics and mathematics, as have of remarkable properties that extend the classical mathematical analysis extends the range of tasks and, moreover, leads to significant simplifications in the calculations, automating basic operations.

The objectives of this work

1) study the concept of Dirac function;
2) to consider the physical and mathematical approaches to its definition;
3) show the application to the determination of derivatives of discontinuous functions.

1.1. Basic concepts.

In various issues of mathematical analysis, the term "function" has to understand, with varying degrees of generality. Sometimes considered continuous but not differentiable functions, other issues have to assume that we are talking about functions, differentiable once or several times, etc. However, in some cases, the classical notion of function, even interpreted in the broadest sense,that is as an arbitrary rule, which relates each value of x in the domain of this function, a number of

$y = f(x)$, is insufficient.

That's an important example: using the apparatus of mathematical analysis to some problems, we are faced with a situation in which certain operations analysis are impractical, for example, a function that has no derivative (in some points or even everywhere), it is impossible to differentiate if derivatives are understood as an elementary function. Difficulties of this type could be avoided by limiting the consideration only of analytic functions. However, such a narrowing of the stock of admissible functions in many cases is very desirable.

The need to further expand the concept of function has become particularly acute.

In 1930, for the solution of problems of theoretical physics largest British theoretical physicist, Dirac, one of the founders of quantum mechanics, did not have the apparatus of classical mathematics, and he introduced a new object, called "delta function", which goes far beyond the classic definition of the function .

P. Dirac in his book The Principles of Quantum Mechanics "[5] to determine the delta function δ (x) as follows:

$$\delta(x) = \begin{cases} 0, x \neq 0 \\ \infty, x = 0 \end{cases}$$

Also given condition:

$$\int_{-\infty}^{\infty} \delta(x)dx = 1.$$

Intuitively, you can submit a schedule function, similar to the δ(x), as shown in Figure 1. The more narrow stripes make between the left and right branch, the greater must be this strip, to strip the area (that is the integral) to maintain its preset value of 1. If you decrease the strip we get closer to the condition δ (x) = 0 for x ≠ 0, the function approaches the delta function.

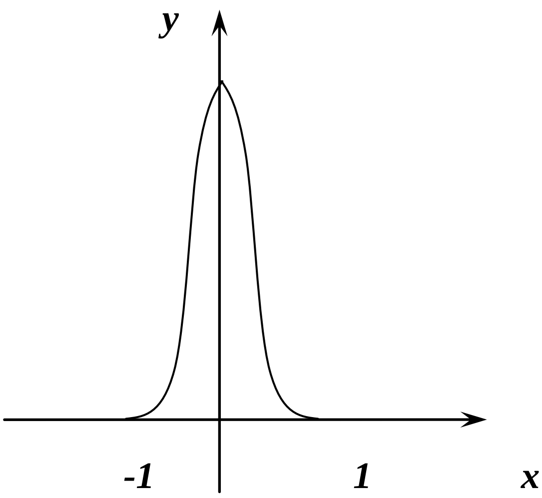

Figure 1

This view is generally accepted in physics.

It should be emphasized that δ (x) is not a function in the usual sense, as follows from this definition incompatible conditions in terms of the classical definition of the function and the integral: $\delta(x) = 0$ when $x \neq 0$ and $\int_{-\infty}^{+\infty} \delta(x)dx = 1.$

In classical analysis there is no function that has the properties prescribed by Dirac. Only a few years later, in the works of SL Sobolev and L. Schwartz delta function derives its mathematical design, but not as usual, but as a generalized function.

Before proceeding to consider the Dirac function, we introduce the basic definitions and theorems that we will need:

Definition 1.

The image function f (t) or L - the image of a given function f (t) is a function of complex variable *p*, defined by the equation:

$$F(p) = \int_{0}^{+\infty} e^{-pt} f(t)dt$$

In this case, we assume that when t <0 f (t) = 0 and t> 0 the inequality $|f(t)| < Me^{at}$, where M and a - some positive constants.

Definition 2.

Function f (t), defined as follows:

$$f(t) = \begin{cases} 1, t \geq 0 \\ 0, t < 0 \end{cases}$$

is the unit Heaviside function, denoted by $\sigma_0(t)$. This function is shown in Fig.2

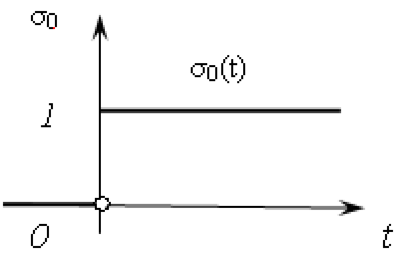

Fig.2

We find L - image Heaviside function:

$$L\{\sigma_0(t)\} = \int_{0}^{+\infty} e^{-pt} dt = \lim_{A \to +\infty} \int_{0}^{A} e^{-pt} dt =$$

$$= \lim_{A \to +\infty} (-\frac{e^{-pA}}{p} + \frac{e^{-p*0}}{p}) = \frac{1}{p}$$

So

$$L\{\sigma_0(t)\} = \frac{1}{p} \qquad (1)$$

Let the function f (t) for t <0 is identically equal to zero (Fig. 3). Then the function $f(t-t_0)$ is identically equal to zero for t $<t_0$ (Fig. 4).

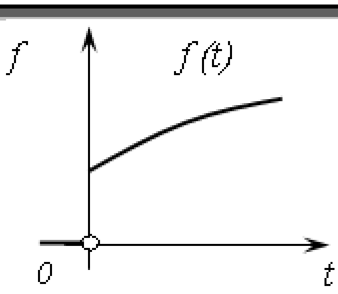

Fig. 3

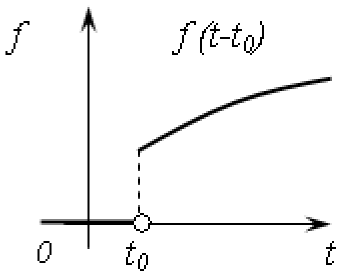

Fig. 4

To find the image δ (x) with an auxiliary function, consider the theorem of delay:

Theorem 1

If F (p) is the image function f (t), then $e^{-pt_0}F(p)$ *is the image function f (t-t₀) that is , if L (f (t)) = F (p), then* $L\{f(t-t_0)\}=e^{-pt_0}F(p)$.

Proof.

By definition images have

$$L\{f(t-t_0)\}=\int_0^{\infty}e^{-pt}f(t-t_0)dt=\int_0^{t_0}e^{-pt}f(t-t_0)dt+\int_{t_0}^{\infty}e^{-pt}f(t-t_0)dt$$

The first integral is equal to zero, because f $(t-t_0) = 0$ for $t < t_0$. In the last integral, make the change of variable $t-t_0 = z$:

$$L\{f(t-t_0)\}=\int_0^{\infty}e^{-p(z+t_0)}f(z)dz=e^{-pt_0}\int_0^{\infty}e^{-pz}f(z)dz=e^{-pt_0}F(p)$$

Thus $L\{f(t-t_0)\}=e^{-pt_0}F(p)$.

For the Heaviside unit step function, it was found that $L\{\sigma_0(t)\}=\frac{1}{p}$. Based on this theorem, it follows that the function $\sigma_0(t-h)$, L-image will be $\frac{1}{p}e^{-ph}$, that is

$$L\{\sigma_0(t-h)\}=\frac{1}{p}e^{-ph} \qquad (2)$$

Definition 3

Continuous or piecewise continuous function δ (t, λ) of the argument t, depending on the parameter λ, called *acicular*, if:

1) $\delta(t,\lambda)=0$ at $|t|>\lambda$;

2) $\delta(t,\lambda)\geq 0$ when $|t|<\lambda$;

3) $\int_{-\infty}^{+\infty} \delta(t,\lambda)dt = \int_{-\lambda}^{\lambda}\delta(t,\lambda)dt = 1$

Definition 4

Numerical function *f*, defined on some linear space L, is called functional. Define a set of those functions, which will act functional. As this set we consider the set K of all real functions φ (x), each of which has continuous derivatives of all orders and finite, that is zero outside a limited area (for each of its functions φ (x)). These functions will be called the core, and their entire collection of K - the main space.

Definition 5

Generalized function is called every continuous linear functional defined on the main space K.
Decode the definition of generalized functions:
1) the generalized function f is a functional on the basic functions of φ, that is, each φ matches (complex) number (f, φ);
2) linear functional f, that is $(f,\lambda_1\varphi_1+\lambda_2\varphi_2)=\lambda_1(f,\varphi_1)+\lambda_2(f,\varphi_2)$, for any complex numbers λ_1 and λ_2, and all the main functions φ1 and φ_2;
3) the functional f is continuous, that is $(f,\varphi_k)\to(f,\varphi), k\to\infty$, if $\varphi_k\to\varphi$.

Definition 6

Impulse - single, short-term jump in electrical current or voltage [2, p. 482].

Definition 7

Average density - the ratio of body mass m to its volume V, ie [2, p. 134].

Theorem 2. (Generalized mean value theorem).

If $f(t)$ - continuous, and $\varphi(t)$ - an integrable function on [a; b], where $\varphi(t)$ in this interval does not change sign, then $\int_a^b f(t)\varphi(t)dt = f(\tau)\int_a^b \varphi(t)dt$, *where* $\tau\in(a;b)$ [1, p. 228].

Theorem 3

Let the function f (x), is bounded on [a, b] and has only a finite number of points of discontinuity. Then the function is a primitive for the function f (x) on the interval [a, b] and for any antiderivative F (x) the formula [1, p. 220].

Definition 8

The set of all continuous linear functional defined on some linear space E forms a linear space. It is called the dual space of E and denoted by E *.

Definition 9

Linear space E, which is given a certain rate, called a normed space.

Definition 10

The sequence is weakly convergent to, if for each the relation.
Theorem 4. If (xn) - weakly convergent sequence in a normed space, then there exists a constant C that [10, p. 187].

1.2 problems, carrying into definition of Dirac delta function.

From a physical point of view, the Dirac delta function, used in mathematical physics for solving problems, which are concentrated at a single point value (load, charge, etc.) is presented as a simple generic function that allows to record the spatial density of a physical quantity (mass, charge, the intensity of the heat source, power, etc.), concentrated and applied at the point of a space Rn. It describes, for example, the density of the mass distribution, which at one point focused unit mass, and any interval not containing this point, free from the masses.

1.2.1. problem on impulse.
Consider the function

$$\sigma_1(t,h) = \frac{1}{h}\left[\sigma_0(t) - \sigma_0(t-h)\right] = \begin{cases} 0, & t < 0 \\ \frac{1}{h}, & 0 \le t \le h \\ 0, & h < t \end{cases}$$

shown in Figure 5.

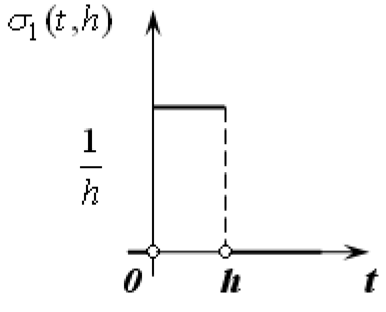

Figure 5

If this feature interpreted as a force acting during the time interval from 0 to h, and the rest of the time equal to zero, the momentum of this force, calculated by the formula is equal to unity.

On the basis of formulas (1) and (2) image of this function will

$$L\{\sigma_1(t,h)\} = \frac{1}{h}\left(\frac{1}{p} - \frac{1}{p}e^{-ph}\right)$$

$$L\{\sigma_1(t,h)\} = \frac{1}{p}\left(\frac{1-e^{-ph}}{h}\right)$$

In mechanics it is convenient to consider the forces acting very short period of time, as the forces acting instantaneously, but with a finite momentum. Therefore, we introduce the function δ (t) as the limit function $\sigma_1(t,h)$ at $h \to 0$:

$$\delta(t) = \lim_{h\to 0} \sigma_1(t,h)$$

This is called the unit impulse function or a delta function, and $\int_{-\infty}^{+\infty} \delta(t)dt = 1$, as the momentum of force equal to unity.

1.2.2. The problem of the density of the material point.

Try to determine the density, created a material point of mass 1.
We assume that this point is the origin. To determine the density, the unit mass distribute uniformly inside a ball of radius ε centered at 0. The result is the average density $f_\varepsilon(x)$, equal

$$f_\varepsilon(x) = \begin{cases} \dfrac{1}{\frac{4}{3}\pi\varepsilon^3}, & |x| < \varepsilon, \\ 0, & |x| > \varepsilon. \end{cases}$$

But we are interested in density $\varepsilon \to 0+$ (ie, ε tends to 0 on the right). Let us first as the desired density δ (x) the limit of the mean densities $f_\varepsilon(x)$ if $\varepsilon \to 0+$ that is the function

$$\delta(x) = \lim_{\varepsilon\to 0} f_\varepsilon(x) \begin{cases} +\infty, & x = 0, \\ 0, & x \neq 0. \end{cases}$$

(3)

From the density δ is natural to require that its integral over all space would give the total mass of substance, ie

$$\int_{-\infty}^{+\infty} \delta(x)dx = 1 \qquad (4)$$

But for the function δ (x), defined by (3), $\int_{-\infty}^{+\infty}\delta(x)dx = 0$. This means that the function does not restore a lot (does not meet the requirement (4)) and therefore can not be taken as the desired density. Thus, the limit of the average densities fε (x) is not suitable for our purposes, ie can not be taken as the density δ (x).

For any continuous function φ (x) we find the weak limit of a sequence $\int f_{\varepsilon}(x)\varphi(x)dx$ with $\varepsilon \to 0$.
Show that

$$\lim_{\varepsilon\to 0}\int f_{\varepsilon}(x)\varphi(x)dx = \varphi(0) \qquad (5)$$

Indeed, the continuity of φ (x) for any η> 0, there is $\varepsilon_0>0$ such that $|\varphi(x)-\varphi(0)|<\eta$, as soon $|x|<\varepsilon_0$. Hence, for all $\varepsilon \le \varepsilon_0$ we get

$$\left|\int f_{\varepsilon}(x)\varphi(x)dx-\varphi(0)\right| = \frac{3}{4\pi\varepsilon^3}\left|\int_{|x|<\varepsilon}[\varphi(x)-\varphi(0)]dx\right| \le \frac{3}{4\pi\varepsilon^3}\int_{|x|<\varepsilon}|\varphi(x)-\varphi(0)|dx <$$

$$<\eta\frac{3}{4\pi\varepsilon^3}\int_{|x|<\varepsilon}dx=\eta .$$

We will show that $\left|\int f_{\varepsilon}(x)\varphi(x)dx-\varphi(0)\right| = \frac{3}{4\pi\varepsilon^3}\left|\int_{|x|<\varepsilon}[\varphi(x)-\varphi(0)]dx\right|$.

$$\left|\int f_{\varepsilon}(x)\varphi(x)dx-\varphi(0)\right| = \left|\frac{3}{4\pi\varepsilon^3}\int_{|x|<\varepsilon}\varphi(x)dx-\varphi(0)*1\right| =$$

$$=\left|\frac{3}{4\pi\varepsilon^3}\int_{|x|<\varepsilon}\varphi(x)dx-\varphi(0)*\frac{3}{4\pi\varepsilon^3}*\frac{4\pi\varepsilon^3}{3}\right| =$$

Since $\int_{|x|<\varepsilon}dx=\frac{4\pi\varepsilon^3}{3}$ (dx is actually equal to dV), then $\int_{|x|<\varepsilon}dx=\int_{|x|<\varepsilon}dV=V$ - the volume of a sphere of radius ε. Consequently

$$\left|\frac{3}{4\pi\varepsilon^3}\int_{|x|<\varepsilon}\varphi(x)dx-\varphi(0)*\frac{3}{4\pi\varepsilon^3}*\frac{4\pi\varepsilon^3}{3}\right| =$$

$$=\left|\frac{3}{4\pi\varepsilon^3}\left(\int_{|x|<\varepsilon}\varphi(x)dx-\varphi(0)\int_{|x|<\varepsilon}dx\right)\right| = \frac{3}{4\pi\varepsilon^3}\left|\int_{|x|<\varepsilon}[\varphi(x)-\varphi(0)]dx\right| .$$

Equation (5) indicates that the weak limit of a sequence of functions fε (x),

$\varepsilon \to +0$, is the functional φ (0) (and not a function!), Associating each continuous function
φ (x) the number of φ (0) - its value at x = 0. This functional was adopted for determining the density δ (x) - this is the Dirac delta function. So, we can write

$$f_{\varepsilon}(x) \to \delta(x)\ ,\ \varepsilon \to +0$$

meaning there by limiting relation (5). The value of the functional δ function at φ - the number of φ (0) - denoted as follows:

$$(\delta, \varphi) = \varphi(0) \qquad (6)$$

This equation gives the exact meaning of the delta functions, introduced by Dirac, which has the following properties:

δ (x) = 0, x ≠ 0, $\int \delta(x)\varphi(x)dx = \varphi(0)$, $\varphi \in$C.

The role is played by the integral $\int \delta(x)\varphi(x)dx$ value (δ, φ) - the value of the functional δ function on φ.
Thus, the delta-function - a functional that assigns the formula (δ, φ) *= φ (0) for each continuous function φ the number of φ (0) - its value at zero.*
Verify that the functional δ recovers the total mass. Indeed, the integral role $\int_{-\infty}^{+\infty} \delta(x)dx$ played by the quantity $(\delta, 1)$, equal to, by virtue of (6), the value of the function identically equal to 1, at x = 0,that is $(\delta, 1)$ = 1 (0) = 1. Thus, the density corresponding to a point mass distribution can not be described within the classical notion of function, and its description should include the linear (continuous) functional.

1.3. Matematical definition of Dirac.

The function δ (x) is applied not only in mechanics, and in many areas of mathematics, particularly in solving many problems of mathematical physics equations.
Let f (t) - function, continuous on (a; b), and $\delta(t, \lambda)$ - needle-shaped function.
To further determine the introduction of Dirac delta function the behavior of the integral

$$\int_a^b f(t)\delta(t,\lambda)dt \text{ at } \lambda \to 0$$

Let (a; b), containing the point t = 0, that is a <0 <b and $\lambda \le \min(|a|, b)$. The definition of an acicular function and the generalized mean value theorems we obtain:

$$\int_a^b f(t)\delta(t,\lambda)dt = \int_{-\lambda}^{\lambda} f(t)\delta(t,\lambda)dt = f(\tau)\int_{-\lambda}^{\lambda}\delta(t,\lambda)dt = f(\tau)$$

where $\tau \in (-\lambda;\lambda)$.

If $\lambda \to 0$, then $\tau \to 0$, and in the continuity of the function f (t) and $f(\tau) \to f(0)$. Therefore, when a <0 <b

$$\lim_{\lambda \to 0}\int_a^b f(t)\delta(t,\lambda)dt = f(0) \quad (7)$$

If the numbers a and b of the same characters (a <b <0 or 0 <a <b), ie (a; b) does not contain within itself the point t = 0, then

$$\int_a^b f(t)\delta(t,\lambda)dt = 0$$

for all sufficiently small λ.
If the numbers a and b have the same sign, then when $\lambda \to a$, if a> 0 (Fig. 6), or $\lambda < |b|$ if b <0 (Fig. 7), the interval $(-\lambda;\lambda)$ will not overlap with (a; b), and therefore for all $t \in (a;b)$

$$\delta(t,\lambda) = 0 \quad \text{and} \quad \int_a^b f(t)\delta(t,\lambda)dt = 0 .$$

Consequently

$$\lim_{\lambda \to 0}\int_a^b f(t)\delta(t,\lambda)dt = 0 \quad (8)$$

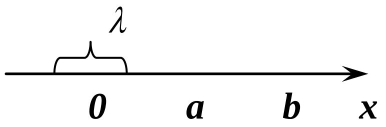

Fig.6

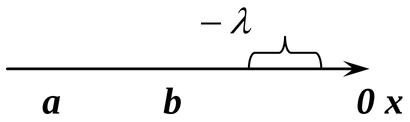

Fig.7

Introduce the notation: $\lim_{\lambda \to 0}\int_a^b f(t)\delta(t,\lambda)dt = \int_a^b f(t)\delta(t)dt \quad (9)$

Thus, δ (t) - a distribution which characterizes the limit behavior of acicular function $\delta(t,\lambda)$ at $\lambda \to 0$ and used in the calculation of integrals

Delta-function can be applied formally, using only read the *main properties* derived from equation (7) - (9) for any continuous function.

$$\int_a^b f(t)\delta(t)dt = \begin{cases} f(0), 0 \in (a;b) \\ 0, 0 \notin (a;b) \end{cases} \quad (10)$$

We introduce the substitution $\int_a^b f(t)\delta(t-t_0)dt = \lim_{\lambda\to 0}\int_a^b f(t)\delta(t-t_0,\lambda)dt$, then

$$\int_a^b f(t)\delta(t-t_0)dt = \begin{cases} f(t_0),\ t_0 \in (a;b) \\ 0,\ t_0 \notin (a;b) \end{cases} \qquad (11)$$

The property, described by relations (10) and (11) is called the *filtering properties* of the delta- function.

If f (t) ≡ 1, (9) - (11) take the form

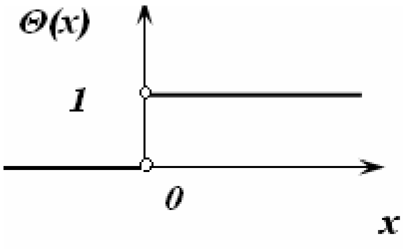

Fig .8

$$\lim_{\lambda\to 0}\int_a^b \delta(t,\lambda)dt = \int_a^b \delta(t)dt$$

$$\int_a^b \delta(t)dt = \begin{cases} 0,\ 0 \notin (a;b) \\ 1,\ 0 \in (a;b) \end{cases}$$

$$\int_a^b \delta(t-t_0)dt = \begin{cases} 0,\ t_0 \notin (a;b) \\ 1,\ t_0 \in (a;b) \end{cases}$$

If the interval (a; b) take the entire axis, then $\int_{-\infty}^{\infty} \delta(t)dt = 1$.

2- Application of the Dirac

2.1. Discontinuous functions and their derivatives.

XX - XI century finds a lot of constructive solutions to what seemed impossible in the XIX century. Since the delta function, determine the derivative at the point of discontinuity (in particular, to break the form of a finite jump). Consider the integral of the function δ (x) depending on its upper limit, ie the function

$$\theta(x) = \int_{-\infty}^{x} \delta(x)dx \ . \ (12)$$

This function has the form of "step" (Fig. 8). While x <0, the region of integration in formula (12) is entirely where δ (x) = 0. Consequently, θ (x) = 0 for x <0. If x> 0, then the integration includes the neighborhood of the origin, where $\delta(0)=\infty$. On the other hand, since for x> 0 and δ (x) = 0, then the value of the integral does not change when the upper limit varies from 0,1 to 1, or to 10, or to ∞. Consequently, for x> 0 we have

$$\theta(x) = \int_{-\infty}^{x} \delta(x)dx = \int_{-\infty}^{+\infty} \delta(x)dx = 1$$

as shown in Figure 8.

Thus, with the help of the delta function to construct a simple discontinuous function θ (x) such that x <0, θ (x) = 0, and in x> 0, θ (x) = 1. When x = 0, θ discontinuity from 0 to 1.

Not knowing the delta-function, have to say that the derivative can not find where the function is discontinuous. We've built a discontinuous function of θ (x). By the theorem on the existence of a primitive for a limited function with finite or countable number of discontinuity points, the general rule between the integral and derivative has the form:

$$F(x) = \int_{x_0}^{x} g(x)dx \quad .$$

Then $$g(x) = \frac{dF}{dx}.$$

Apply it to the expression (12), we obtain

$$\frac{d\theta(x)}{dx} = \delta(x) \quad .$$

So, for the derivative of a discontinuous function does not need to make an exception: just at the point of discontinuity is equal to the derivative of "special" function - Dirac delta function.
The derivative of the discontinuous function is defined as follows:

$$f'(x)=\{f'(x)\}+[f_{x0}]\delta(x - x_0)$$

where f_{x0} - the value of the gap at the point x_0, $\{f'(x)\}$ - a derivative everywhere except at the point x_0.
With Dirac delta function derivatives can be found in more complex cases.

2.2. Finding derivatives of discontinuous functions.

Example 1: Find the derivative of

$$y = \begin{cases} x, x < 1 \\ x - 2, x > 1 \end{cases} .$$

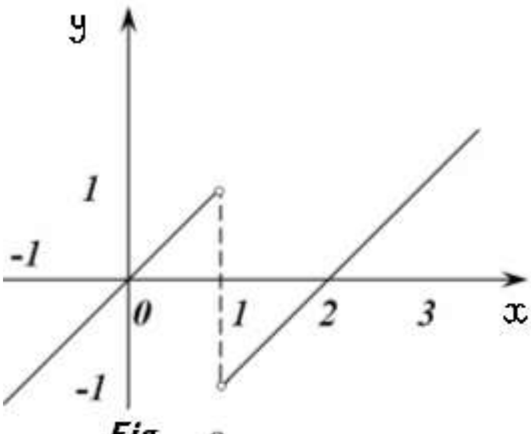

Fig .8

The graph of Figure 8. The gap occurs at x = 1. The value of discontinuity y (1 +0)-y (1-0) = 1-2-1 = -2, where y (1 +0) - is the limit when approaching y at x → 1 on the right (from x> 1), y (1 - 0) - the same on the left. Hence, we find that

$$\frac{dy}{dx} = 1 - 2\delta(x - 1) \qquad (13)$$

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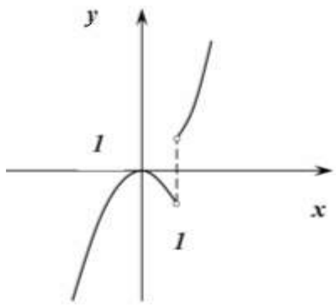
Such a record better than $\frac{dy}{dx} = 1$ the assertion that everywhere except the point x = 1, where the function is discontinuous and has no derivative. In writing (13) is contained in one line and the fact that the gap (once entered δ), and his place (x = 1), and magnitude (factor (- 2) with δ).



Example

$$y = \begin{cases} -x^2, x < 1 \\ x^2, x > 1 \end{cases}$$

$$y' = \begin{cases} -2x, x < 1 \\ 2x, x > 1 \end{cases}$$

The gap at the point x = 1. The value of the gap: y (1 +0)-y (1 - 0) = 2.
Now we can point x = 1 attached to the left pane, and then write

$$y' = \begin{cases} -2x + 2\delta(x-1), x \le 1 \\ 2x, x > 1 \end{cases}.$$

Or another option - you can add x = 1 to the right pane, and then with an equal right to write

$$y' = \begin{cases} -2x, x < 1 \\ 2x + 2\delta(x-1), x \ge 1 \end{cases}.$$

You can also write

$$y' = \varphi(x) + 2\delta(x-1),$$

where $\varphi(x) = \begin{cases} -2x(x<1), \\ 2x(x>1). \end{cases}$

Example 3

Consider the model of current flow along the chain provided in the MN Dubaylovoy "Application of Fourier series for solving problems in electrodynamics" [7].
We find the derivative of the function provided by the plot of the current

strength of the time:

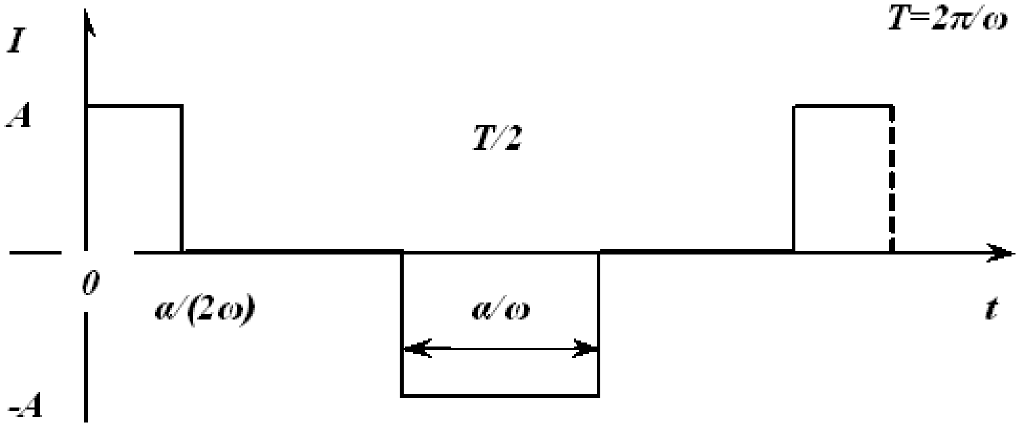


Fig.9

The graph shows that the current strength at the points α / (2ω), 2π-α / (2ω), 2π + α / (2ω), 4π-α / (2ω), ... instantly falls from A to 0 or from 0 to -A, ie, the current instantly reduced to 0, and again appears with a negative value. The disappearance of current in the circuit means that the chain is broken, so the actual process appears again after some time can not talk spontaneously. Such a model of current flow along the chain is controversial.

In fact, current strength does not change instantaneously, but within a short finite period of time. The actual process can be represented by the following graph (Fig. 10).

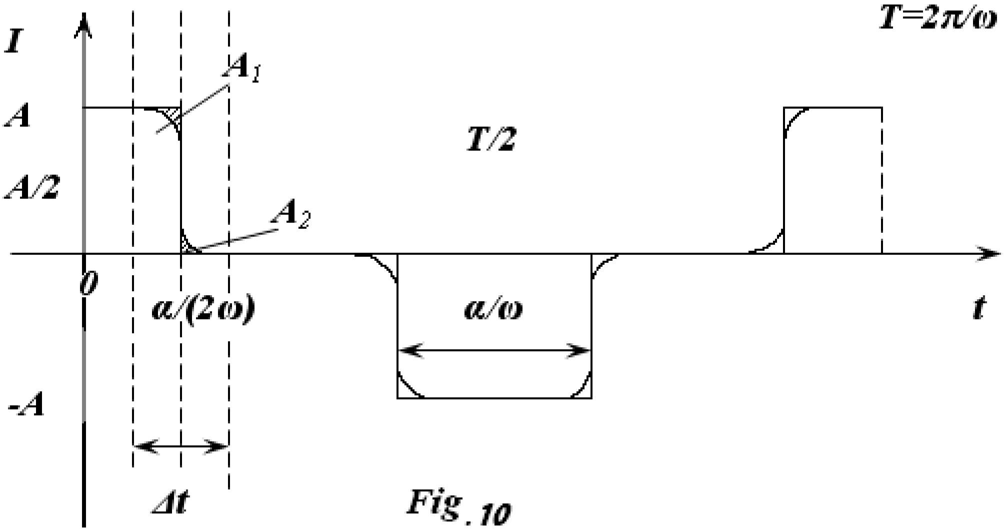


Fig. 10

In physics, a simplified model, the schedule is presented in Figure 9, as well as the work of the current in a short finite time interval Δt is equal to zero ($A=\int Idt=A_1+A_2=A_1+(-A_1)=0$, geometrically the number of A_1 and A_2 express Square shaded figures, see Figure 10).

In mathematics Fig.9 is not a graph of the function (one value of t corresponds to an infinite set of values of I). Therefore, mathematics is considering a simplified model, abstracted from the real process, disrupting the function graph of this model presented in Figure 11.

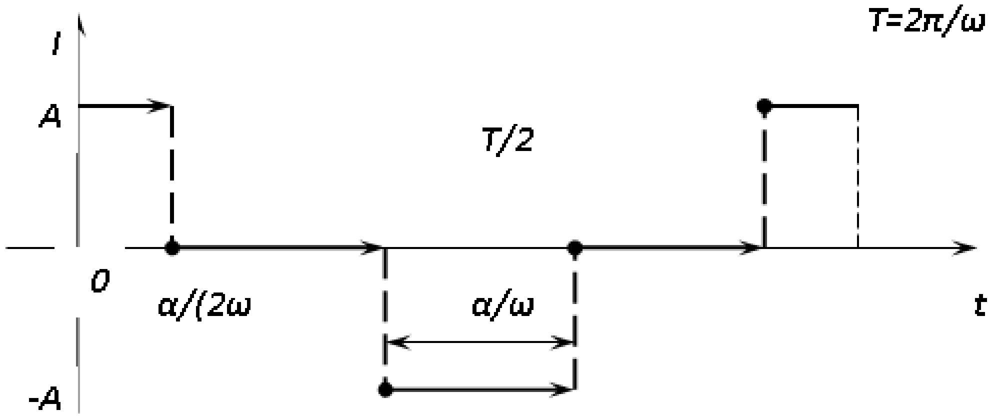


Fig.11

We find the derivative of this function. For this function ask the following:

$$f(t)=\begin{cases} A, t\in[0,\frac{\alpha}{2\omega})\cup[T-\frac{\alpha}{2\omega},T] \\ 0, t\in[\frac{\alpha}{2\omega},\frac{T}{2}-\frac{\alpha}{2\omega})\cup[\frac{T}{2}+\frac{\alpha}{2\omega},T-\frac{\alpha}{2\omega}) \\ -A, t\in[\frac{T}{2}-\frac{\alpha}{2\omega},\frac{T}{2}+\frac{\alpha}{2\omega}) \end{cases}.$$

Gaps occur when $\frac{\alpha}{2\omega},(\frac{T}{2}-\frac{\alpha}{2\omega}),(\frac{T}{2}+\frac{\alpha}{2\omega}),(T-\frac{\alpha}{2\omega})$. Values are gaps-A,-A, A, A, respectively. Hence, we find that

$$f'(t)=\begin{cases} -A\delta(t-\frac{\alpha}{2\omega}), t\in[0,\frac{T}{2}-\frac{\alpha}{2\omega}) \\ -A\delta(t-(\frac{T}{2}-\frac{\alpha}{2\omega})), t\in[\frac{\alpha}{2\omega},\frac{T}{2}+\frac{\alpha}{2\omega}) \\ A\delta(t-(\frac{T}{2}+\frac{\alpha}{2\omega})), t\in[\frac{T}{2}-\frac{\alpha}{2\omega},T-\frac{\alpha}{2\omega}) \\ A\delta(t-(T-\frac{\alpha}{2\omega})), t\in[\frac{T}{2}+\frac{\alpha}{2\omega},T] \end{cases}.$$

Conclusion

In the final qualifying work goals achieved, there were some detail the mathematical and physical approaches to the definition of Dirac, and physical approach to the determination carried through the solution of physical problems of impulse and density of the material point. Application of the Dirac function for finding the derivatives of discontinuous functions was illustrated with the help of mathematical and physical examples, revealed the usefulness of the delta function for finding the derivatives of discontinuous functions. Theoretical material is confirmed by the decision of various examples. Thus, the Dirac delta function - one of the most essential and widely used concepts in physics and in mathematical analysis.

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الخلاصة

هذا البحث يقدم طرق مختلفة لعرض وتحديد دالة دلتا ديراك (Dirac delta function) وتطبيقاتها في حل بعض المشاكل واضهار امكانية استخدام دالة دلتا(delta function) في الرياضيات التحليلية والفيزياء.