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New Certain Continuous Mappings via Neutrosophic Topological Spaces

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ABSTRACT

In this article, we present the new class of neutrosophic continuous mappings called; $\psi\beta$-continuous map (WN $\psi\beta$-CM) strongly neutrosophic $\psi\beta$ -continuous map (SN $\psi\beta$-CM), neutrosophic $\psi\beta$ -weakly continuous map (N $\psi\beta$ W-CM) and neutrosophic $\psi\beta$ -weakly continuous map (N$\psi\beta$S-CM) includes an examination of the most notable traits and attributes associated with it. Moreover, we introduced the notions of $\psi\beta$-irresolute map (WN $\psi\beta$-CIM) strongly neutrosophic $\psi\beta$ - irresolute map (SN $\psi\beta$-CIM), neutrosophic $\psi\beta$-weakly irresolute map (N$\psi\beta$W-CIM) and neutrosophic $\psi\beta$ -weakly irresolute map (N$\psi\beta$S-CIM) in neutrosophic topological spaces (NTS) were investigated, coupled with a few characteristics that are connected to them. Finally, we studied the relationship among continuous and irresolute mappings for these kinds that have been presented

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1. Introduction

In 1965, L. A. Zadeh [1] established the first successful attempt to incorporate non-probabilistic uncertainty—that is, uncertainty not caused by the improbability of an event—into mathematical modeling with his significant theory on fuzzy sets (FST).

Each component of the universe includes a part of a fuzzy set, but the membership value of each component is a quantity (or level of belongingness) that ranges from 0 to 1. This gradation concept works well for tasks that include ambiguous input, such language processing, artificially intelligent handwriting, speech recognition, etc.

A further development of this fuzzy set, known as fuzzy sets having intuitive characteristics (IFS), was introduced by Atanassov [2] in 1986. Furthermore, to its membership value, each component in IFS has a not being a member value. Furthermore, it is necessary for the sum of these two values to equal or fall below unity. In IFS, belongingness

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and non-belongingness are not independent; the former determines the latter. Fuzzy set theory may be seen as a particular instance of an IFS where the degree of non-belongingness of an element is exactly equal to one belongingness. Both whole and partial ambiguous data can be handled by IFS. Fuzzy sets built around intuition are very useful in scenarios where the level of non-belongingness is as important as the level of belongingness, including database fusion, systems for expertise, and systems of beliefs.

Neogothic logic was introduced by Smarandache [3] in 1995. This logic assigns a grade of falsehood (F), a level of truth (T), and a level of indeterminacy (I) to each proposition. The unusual unit interval $[\mathbf{0},\mathbf{1}]^*$ is a neutrophilic set, meaning that each element of our universe has a certain amount of truth, ambiguity, and deceit, respectively.

Smarandache's neutrosophic concept has several real-time applications in the fields of [4,5,6,7,8,9, and 10]. information systems, practical mathematics, intelligent machines, computer science, and decision-making. Knowledge of administration, medicine, technology and electricity, mechanics, etc.

Salama and Alblowi [11] introduced the new term for neutrosophic topological space in 2012. A neutrosophic closed class with neutrosophic continuous functions was introduced by Salama et al. in 2014 [12]. Arokiarani et al. [13] were the first to present the neutrosophic α-closed framework for neutrosophic topological spaces.

In 2018, enlarged Neutrosophic closed sets were introduced by Dhavaseelan and Jafari [14]. Mani Parimala et al.5 introduced neutrosophic αψ-closed sets in 2018. In 2019, extended closed sets utilizing neutrophilic topological spaces were introduced by Pushpalatha et al. [15]. Renu Thomas et al. [16] introduced and investigated semi-pre-open, or β-open, sets in neutrosophic topological spaces. Neutrosophic Topological spaces $\boldsymbol{N\beta}^*$-closed structures were very recently created and studied in 2022. Furthermore, Subasree and Basari Kodi [17–18] introduced an additional type of categories within neutrosophic topological space are $\boldsymbol{N\psi\beta}$ and $\boldsymbol{N\beta\psi}$.

lately Nandhini and Vigneshwaran [20–21] developed the concept of $\boldsymbol{\mathcal{N}}_{\mathbf{\alpha g^{\#}\psi}}$-closed sets in neutrosophic spaces for topology and studied some of its characteristics. Furthermore, in neutrosophic topological space, $\boldsymbol{\mathcal{N}}_{\mathbf{\alpha g^{\#}\psi}}$-continuous as well as $\boldsymbol{\mathcal{N}}_{\mathbf{\alpha g^{\#}\psi}}$-irresolute function [20] was initiated and studied.

This article presented a new class of mappings in NTS: weakly neutrosophic $\boldsymbol{\psi\beta}$-continuous mapping (WN $\boldsymbol{\psi\beta}$-CM) strongly neutrosophic $\boldsymbol{\psi\beta}$ - continuous mapping (SN $\boldsymbol{\psi\beta}$-CM), neutrosophic $\boldsymbol{\psi\beta}$-weakly continuous mapping (N$\boldsymbol{\psi\beta}$W-CM) and neutrosophic $\boldsymbol{\psi\beta}$ -weakly continuous mapping (N$\boldsymbol{\psi\beta}$S-CM). Furthermore, we introduced weakly neutrosophic $\boldsymbol{\psi\beta}$-Irresolute mapping (WN $\boldsymbol{\psi\beta}$-IM) strongly neutrosophic $\boldsymbol{\psi\beta}$ - Irresolute mapping (SN $\boldsymbol{\psi\beta}$-IM), neutrosophic $\boldsymbol{\psi\beta}$-weakly Irresolute mapping (N$\boldsymbol{\psi\beta}$W-IM) and neutrosophic $\boldsymbol{\psi\beta}$ -weakly Irresolute mapping (N$\boldsymbol{\psi\beta}$S-IM) and some of its features are investigated. Finally, we studied the composition for these mappings in neutrosophic topological spaces.

2. Basic Concepts

In this section, we will give some definitions of the neutrosophic topological space that we need in our work.

Definition 2.1.[22] "A neutrosophic set (NS, for short) $\wp$ represents a structure of the form $\mathfrak{H} = \{\langle s, \mathcal{U}_{\mathfrak{H}}(s), \mathcal{V}_{\mathfrak{H}}(s), \mathcal{W}_{\mathfrak{H}}(s): s\epsilon\wp\rangle\}$, where $\mathcal{U}_{\mathfrak{H}}(s), \mathcal{V}_{\mathfrak{H}}(s)$, and $\mathcal{W}_{\mathfrak{H}}(s)$ respectively represent the degree for membership, the degree of indeterminacy, and the degree of non-membership for every member $s\epsilon\wp$ to the set $\mathfrak{H}$".

Definition 2.2. [22] "Assume that $\mathfrak{H}$ and $\mathcal{K}$ are NSs of the form $\mathfrak{H} = \{\langle s, \mathcal{U}_{\mathfrak{H}}(s), \mathcal{V}_{\mathfrak{H}}(s), \mathcal{W}_{\mathfrak{H}}(s): s\epsilon\wp\rangle\}$ and $\mathcal{K} = \{\langle s, \mathcal{U}_{\mathcal{K}}(s), \mathcal{V}_{\mathcal{K}}(s), \mathcal{W}_{\mathcal{K}}(s): s\epsilon\wp\rangle\}$. Next,

(i) $\mathfrak{H} \subseteq \mathcal{K}$ if and only if $\mathcal{U}_{\mathfrak{H}}(s) \leq \mathcal{U}_{\mathcal{K}}(s), \mathcal{V}_{\mathfrak{H}}(s) \leq \mathcal{V}_{\mathcal{K}}(s)$ and $\mathcal{W}_{\mathfrak{H}}(s) \geq \mathcal{W}_{\mathcal{K}}(s)$;

(ii) $\overline{\mathfrak{H}} = \{\langle \mathcal{W}_{\mathfrak{H}}(s), \mathcal{V}_{\mathfrak{H}}(s), \mathcal{U}_{\mathfrak{H}}(s): s\epsilon\wp\rangle\}$;

(iii) $\mathfrak{H} \cap \mathcal{K} = \{\langle s, \mathcal{U}_{\mathfrak{H}}(s) \wedge \mathcal{U}_{\mathcal{K}}(s), \mathcal{V}_{\mathfrak{H}}(s) \vee \mathcal{V}_{\mathcal{K}}(s), \mathcal{W}_{\mathfrak{H}}(s) \vee \mathcal{W}_{\mathcal{K}}(s): s\epsilon\wp\rangle\}$;

(iv) $\mathfrak{H} \cup \mathcal{K} = \{\langle s, \mathcal{U}_{\mathfrak{H}}(s) \vee \mathcal{U}_{\mathcal{K}}(s), \mathcal{V}_{\mathfrak{H}}(s) \wedge \mathcal{V}_{\mathcal{K}}(s), \mathcal{W}_{\mathfrak{H}}(s) \wedge \mathcal{W}_{\mathcal{K}}(s): s\epsilon\wp\rangle\}$".

Definition 2.3. [11] "If an extended family $\mathcal{T}$ of a NS in $\mathcal{X}$ satisfies the following axioms, then $\mathcal{X}$ has a neutrosophic topology (NT):

(i) $0_N, 1_N \in \mathcal{T}$;

(ii) For any $\mathcal{M}, \mathcal{N} \in \mathcal{T}$ then $\mathcal{M} \cap \mathcal{N} \in \mathcal{T}$.

(iii) For each random family $\{\mathcal{M}_i\ i \in K\} \subseteq \mathcal{T}$, then $\cup\, \mathcal{M}_i \in \mathcal{T}$".

Definition 2.4.[11] "Suppose $\mathcal{Q}$ be a NS of a neutrosophic topological space $\mathcal{X}$. It follows that:

i. $\mathcal{N}int(\mathcal{M}) = \cup \{\mathcal{W}: \mathcal{W}$ represents NOS in $\mathcal{X}$, and $\mathcal{W} \subseteq \mathcal{M}\}$ is referred to as a neutrosophic interior for $\mathcal{M}$.

ii. $\mathcal{N}cl(\mathcal{M}) = \cap \{\mathcal{G}: \mathcal{G}$ represents NCS in $\mathcal{X}$, and $\mathcal{G} \supseteq \mathcal{M}\}$ is referred to as a neutrosophic closure of $\mathcal{M}$".

Definition 2.5.[17] "If $\mathcal{M}$ is a NOS in (X, τ) and $Ncl(\mathcal{M}) \subseteq U$ whenever $\mathcal{M} \subseteq U$, then a subset $\mathcal{M}$ of a NTS (X, τ) is a neutrosophic generalized closed (NgC) set".

Definition 2.6.[17]. "Let (X, τ) be a NTS and let $\mathcal{M}$ be a subset of (X, τ). Then $\mathcal{M}$ is:

1. A neutrosophic semi generalized –closed (NsgC) sets if $Nscl(\mathcal{M}) \subseteq U$ whenever $\mathcal{M} \subseteq U$ and U is a NsO-set in (X, τ).
2. A neutrosophic ψ–closed (NψC) sets if $Nscl(\mathcal{M}) \subseteq U$ whenever $\mathcal{M} \subseteq U$ and U is a NsgO-set in (X, τ)".

Definition 2.7.[17] "If $Nscl(\mathcal{M}) \subseteq U$ whenever $\mathcal{M} \subseteq U$ and U is a neutrosophic ψ-open set in (X, τ), then a subset $\mathcal{M}$ of a NTS (X, τ) is referred to as a neutrosophic $\beta\psi$-closed ($N\beta\psi CS$)".

Definition 2.8.[23] "If $\mathcal{M}$ is a subset of NTS (X, τ) then $\mathcal{M}$ is called:

i. WN $\psi\beta$-CS if it meets the requirements listed below:

1. $\mathcal{M} = N\psi cl(\mathcal{M})$;
2. If $\mathcal{M} \subseteq U$, then for each U is $N\beta\psi OS$ in X:

$N\psi int(U) \cup \mathcal{M} \neq \emptyset$.

ii. SN $\psi\beta$-CS if it meets the requirements listed below:

1. $\mathcal{M} = N\psi cl(\mathcal{M})$;
2. If $\mathcal{M} \subseteq U$, then for each U is $N\beta\psi OS$ in X:

$N\psi int(U) \cap \mathcal{M} \neq \emptyset$.

iii. N$\psi\beta$W-CS if $\mathcal{M} \subseteq N\psi cl(U)$ whenever $\mathcal{M} \subseteq U$, and U is a $N\beta OS$ such that

$$N\psi int(U) \cup N\psi cl(\mathcal{M}) \neq \emptyset.$$

iv. N$\psi\beta$S-CS if $\mathcal{M} \subseteq N\psi cl(U)$ whenever $\mathcal{M} \subseteq U$, and U is a $N\beta OS$ such that

$$N\psi int(U) \cap N\psi cl(\mathcal{M}) \neq \emptyset".$$

Definition 2.9.[23] "let $(X, \tau)$and (Y, σ) be a NTS and let h be a mapping from $X\ to\ Y$ then h is neutrosophic continuous mapping, if the preimage $h^{-1}(\mathcal{M})$ is NOS in (X, τ) for any NOS $V \in (Y, \sigma)$".

Definition 2.9.[24] "let (X, τ) and (Y, σ) be a NTS and let h be a mapping from $X\ to\ Y$ then h is neutrosophic irresolute mapping, if the preimage $h^{-1}(\mathcal{M})$ the NSOS in (X, τ) for any NSOS $V \in (Y, \sigma)$".

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3. New Classes of Continuous Mappings in NTS

In this part, we present a new class of continuous mapping including $WN\psi\beta - CM, S\psi\beta W - CM, N\psi\beta W - CM, N\psi\beta S - CM$ and $WN\psi\beta - IM, S\psi\beta W - IM, N\psi\beta W - IM, N\psi\beta S - IM$ and investigating relationships among themselves.

Definition 3.1. let $(X,\tau),(Y,\sigma)$ be a NTS and let h be a function from X to Y then h is:

i. $N\psi\beta W - CM$ if the preimage $h^{-1}(\mathcal{M})$ WN$\psi\beta$-OS in (X,τ) for any NOS $\mathcal{M} \in (Y,\sigma)$.

ii. $SN\psi\beta - CM$ If the preimage $h^{-1}(\mathcal{M})$ SN$\psi\beta$-OS in (X,τ) for any NOS $\mathcal{M} \in (Y,\sigma)$.

iii. $N\psi\beta W - CM$If the preimage $h^{-1}(\mathcal{M})$ N$\psi\beta$W-OS in (X,τ) for any NOS $\mathcal{M} \in (Y,\sigma)$.

iv. $N\psi\beta S - CM$If the preimage $h^{-1}(\mathcal{M})$ N$\psi\beta S$-OS in (X,τ) for any NOS $\mathcal{M} \in (Y,\sigma)$.

Example 3.2. Let $X = \{\xi, \nu, \mu\}$, $\tau_x = \{0_N, 1_N, A\}$ with $A = \{(\xi, 0.5,0.7,0.4), (\nu, 0.4,0.5,0.3), (\mu, 0.8,0.7,0.9)\}$ and $C = \{(\xi, 0.1,0.7,0.3), (\nu, 0.2,0.2,0), (\mu, 0.8,0.3,0.9)\}$ satisfies $N\psi\beta W - OS$

$Y = \{\alpha, \beta, \gamma\}$, $\tau_y = \{0_N, 1_N, B\}$ with $B = \{(\alpha, 0.4,0.6,0.3), (\beta, 0.3,0.4,0.2), (\gamma, 0.7,0.6,0.8)\}$

and $D = \{(\alpha, 0.2,0.2,0.1), (\beta, 0.6,0.6,0.6), (\gamma, 0.8,0.9,0.9)\}$ satisfies NOS. Then a map $f: X \longrightarrow Y$ defined by $f(\alpha) = \xi$, $f(\beta) = v$, $f(\gamma) = \mu$ represents $N\psi\beta W - CM$, where τ_x, τ_y are $N\psi\beta W - OS$

Example 3.3. Let $X = \{\xi, \nu, \mu\}$, $\tau_y = \{0_N, 1_N, A\}$ *with* $A = \{(\xi, 0.4,0.5,0.3), (\nu, 0.3,0.4,0.1), (\mu, 0.6,0.5,0.8)\}$

$C = \{(\xi, 0.2,0.1,0.7), (\nu, 0,0.2,0.5), (\mu, 0.6,0.1,0.7)\}$ satisfies $WN\psi\beta - OS$

$Y = \{\alpha, \beta, \gamma\}$, $\tau_y = \{0_N, 1_N, B\}$ with $B = \{(\alpha, 0.5,0.6,0.4), (\beta, 0.4,0.5,0.2), (\gamma, 0.7,0.6,0.9)\}$

$D = \{(\alpha, 0.3,0.2,0.8), (\beta, 0.1,0.3,0.6), (\gamma, 0.7,0.2,0.8)\}$ satisfies NOS. Then a map $f: X \longrightarrow Y$ defined by $f(\alpha) = \xi$, $f(\beta) = v$, $f(\gamma) = \mu$ *represents* $WN\psi\beta - CM$, *where* τ_x, τ_y *are* $N\psi\beta W - OS$.

Example 3.4. *Let* $X = \{\xi, \nu, \mu\}$, $\tau_x = \{0_N, 1_N, A, B\}$ with $A = \{(\xi, 0.7,0.8,0.5), (\nu, 0.5,0.6,0.4), (\mu, 0.9,0.9,0.8)\}$,$B = \{(\xi, 0.5,0.8,0.7), (\nu, 0.4,0.6,0.5), (\mu, 0.8,0.9,0.9)\}$ *and* $C = \{(\xi, 0.2,0.4,0.6), (\nu, 0.4,0.5,0.4), (\mu, 0.8,0.9,0.9)\}$ *satisfies* $N\psi\beta S - OS$

$Y = \{\alpha, \beta, \gamma\}$, $\tau_y = \{0_N, 1_N, M, N\}$ *with* $M = \{(\alpha, 0.6,0.7,0.4), (\beta, 0.4,0.5,0.3), (\gamma, 0.8,0.8,0.7)\}$, $N = \{(\alpha, 0.4,0.7,0.6), (\beta, 0.3,0.5,0.4), (\gamma, 0.7,0.8,0.8)\}$ *and* $S = \{(\alpha, 0.1,0.3,0.5), (\beta, 0.3,0.4,0.3), (\gamma, 0.7,0.8,0.8)\}$ satisfies NOS. *Then a map* $f: X \longrightarrow Y$ *defined by* $f(\alpha) = \xi$, $f(\beta) = v$, $f(\gamma) = \mu$ *represents* $N\psi\beta S - CM$, *where* τ_x, τ_y *are* $N\psi\beta W - OS$.

Example 3.5. Let $X = \{\xi, \nu, \mu\}$, $\tau_x = \{0_N, 1_N, A, B\}$ *with* $A = \{(\xi, 0.7,0.8,0.5), (\nu, 0.1,03,0.1), (\mu, 0.8,0.8,0.7)\}$, $B = \{(\xi, 0.3,0.5,0.6), (\nu, 0.5,0.5,0.5), (\mu, 0.6,0.8,0.8)\}$ *and* $C = \{(\xi, 0.1,0.5,0.3), (\nu, 0.6,0.4,0.5), (\mu, 0.6,0.5,0.8)\}$ satisfies $SN\psi\beta - OS$ and

$Y = \{\alpha, \beta, \gamma\}$, $\tau_y = \{0_N, 1_N, M, N\}$ *with* $M = \{(\alpha, 0.7,0.5,0.6), (\beta, 0.5,0.5,0.5), (\gamma, 0.6,0.5,0.5)\}$, $N = \{(\alpha, 0.3,0.4,0.7), (\beta, 0.5,0.5,0.5), (\gamma, 0.6,0.6,0.6)\}$ *and* $S = \{(\alpha, 0.2,0.4,0.3), (\beta, 0.4,0.4,0.4), (\gamma, 0.6,0.6,0.7)\}$ satisfies NOS. *Then a map* $f: X \longrightarrow Y$ *defined by* $f(\alpha) = \xi$, $f(\beta) = v$, $f(\gamma) = \mu$ *represents* $SN\psi\beta - CM$, *where* τ_x, τ_y *are* $N\psi\beta S - OS$.

Theorem 3.6. Every $SN\psi\beta - CM$ is $WN\psi\beta - CM$

Proof: Assume that$(X,\tau),(Y,\sigma)$ be a NTS and $f: X \longrightarrow Y$ *be a mapping. Since* f *represents* $SN\psi\beta - CM$, *this implies* $h^{-1}(\mathcal{M})$ SN$\psi\beta$-OS in X for any NOS $\mathcal{M} \in Y$. Since every SN $\psi\beta$-CS is WN $\psi\beta$-CS. Thus $h^{-1}(\mathcal{M})$ WN$\psi\beta$-OS [23] for any NOS $\mathcal{M} \in Y$.Consequently, a *map* f *represents* $WN\psi\beta - CM$.

The example 3.3. demonstrates that the converse of Theorem 3.6. is not required to be true.

Theorem 3.7. Every $SN\psi\beta - CM$ is $N\psi\beta S - CM$

Proof: In the same way above.

The example 3.4. demonstrates that the converse of Theorem 3.6. is not required to be true.

Theorem 3.8. Every $WN\psi\beta - continuous\ mapping$ is $N\psi\beta W - continuous\ mapping$

Proof: Obvious.

The example 3.2. demonstrates that the converse of Theorem 3.7. is not required to be true.

Theorem 3.9. Every $N\psi\beta S - continuous\ mapping$ is $N\psi\beta W - continuous\ mapping$

Proof: Obvious.

The example 3.2. demonstrates that the converse of Theorem 3.8. is not required to be true.

Remark 3.10. The diagram that follows Figure 3.1 be Show the Relationships across different neutrophilic continuous mapping.

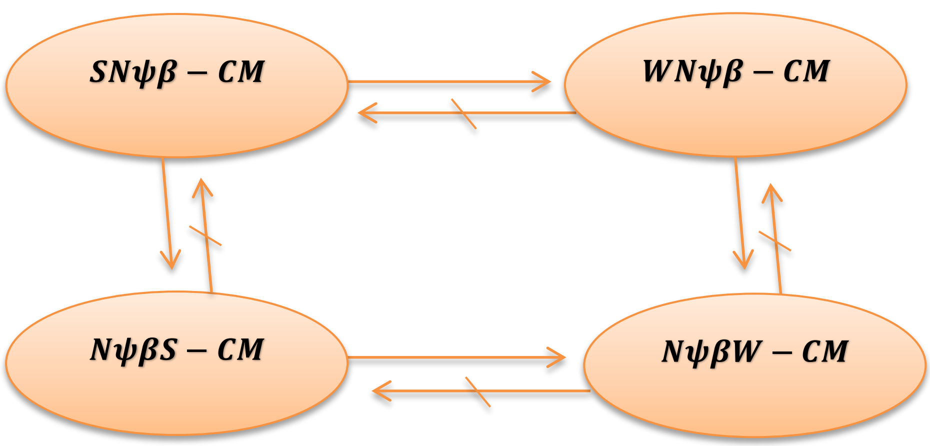

Fig 3.1. The relationship between the new varieties of continuous mappings

Definition 3.12. let $(X,\tau),(Y,\sigma)$ be a two neutrosophic topological space and let f be a mapping from $X\ to\ Y$ then f is neutrosophic irresolute continuous mapping,

i. If the preimage $f^{-1}(\mathcal{M})$ is WN$\psi\beta$-OS in (X,τ) for any WN$\psi\beta$-SOS $\in (Y,\sigma)$.

ii. If the preimage $f^{-1}(\mathcal{M})$ is SN$\psi\beta$-OS in (X,τ) for any SN$\psi\beta$-SOS $\in (Y,\sigma)$.

iii. If the preimage $f^{-1}(\mathcal{M})$ is N$\psi\beta$W-OS in (X,τ) for any N$\psi\beta$W-SOS $\in (Y,\sigma)$.

iv. If the preimage $f^{-1}(\mathcal{M})$ is N$\psi\beta S$-OS in (X,τ) for any N$\psi\beta S$-SOS $\in (Y,\sigma)$.

Example 3.13. Let $X = \{\xi,\nu,\mu\}$, $\tau_x = \{0_N, 1_N, A\}$ with $A = \{(\xi, 0.2,0.1,0.7), (\nu, 0,0.2,0.5), (\mu, 0.6,0.1,0.7)\}$ satisfies $WN\psi\beta - OS$

$Y=\{\alpha,\beta,\mu\},\ \tau_y=\{0_N,1_N,B\}$ with $B=\{(\alpha,0.1,0,0.6),(\beta,0,0.1,0.4),(\gamma,0.5,0,0.6)\}$ satisfies $WN\psi\beta-OS$. *Then a map* $f{:}\,X\longrightarrow Y$ *defined by* $f(\alpha)=\xi,\ f(\beta)=v,\ f(\gamma)=\mu$ *represents* $WN\psi\beta-IM$*, where* τ_x,τ_y *are* $WN\psi\beta-OS$*.*

Example 3.14. Let $X=\{\xi,\nu,\mu\},\ \tau_x=\{0_N,1_N,A\}$ with $A=\{(\xi,0,0.6,0.2),(\nu,0.1,0.1,0),(\mu,0.7,0.2,0.8)\}$ satisfies $N\psi\beta W-OS$

$Y=\{\alpha,\beta,\gamma\},\ \tau_y=\{0_N,1_N,B\}$ with $B=\{(\alpha,0.1,0.1,0),(\beta,0.5,0.5,0.5),(\gamma,0.7,0.6,0.6)\}$ satisfies $N\psi\beta W-OS$. *Then a map* $f{:}\,X\longrightarrow Y$ *defined by* $f(\alpha)=\xi,\ f(\beta)=v,\ f(\gamma)=\mu$ *represents* $N\psi\beta W-IM$*, where* τ_x,τ_y *are* $N\psi\beta W-OS$*.*

Example 3.15. Let $X=\{\xi,\nu,\mu\},\ \tau_x=\{0_N,1_N,A\}$ with $A=\{(\xi,0.1,0.3,0.5),(\nu,0.3,0.4,0.3),(\mu,0.7,0.8,0.8)\}$ this satisfies $N\psi\beta S-OS$

$Y=\{\alpha,\beta,\gamma\},\ \tau_y=\{0_N,1_N,B\}$ with $B=\{(\alpha,0,0.2,0.4),(\beta,0.2,0.3,0.2),(\gamma,0.6,0.7,0.7)\}$ this satisfies $N\psi\beta S-OS$. *Then a map* $f{:}\,X\longrightarrow Y$ *defined by* $f(\alpha)=\xi,\ f(\beta)=v,\ f(\gamma)=\mu$ *represents* $N\psi\beta S-IM$*, where* τ_x,τ_y *are* $N\psi\beta S-OS$*.*

Example 3.16. Let $X=\{\xi,\nu,\mu\},\ \tau_x=\{0_N,1_N,A\}$ with $A=\{(\xi,0.2,0.4,0.6),(\nu,0.4,0.5,0.4),(\mu,0.8,0.9,0.9)\}$ satisfies $SN\psi\beta-OS$

$Y=\{\alpha,\beta,\gamma\},\ \tau_y=\{0_N,1_N,B\}$ with $B=\{(\alpha,0.1,0.3,0.5),(\beta,0.3,0.4,0.3),(\gamma,0.7,0.8,0.8)\}$ satisfies $SN\psi\beta-OS$. *Then a map* $f{:}\,X\longrightarrow Y$ *defined by* $f(\alpha)=\xi,\ f(\beta)=v,\ f(\gamma)=\mu$ *represents* $SN\psi\beta-CM$*, where* τ_x,τ_y *are* $SN\psi\beta-OS$

Theorem 3.17. Every $SN\psi\beta-IM$ is $WN\psi\beta-IM$

Proof: Assume that$(X,\tau),(Y,\sigma)$ be a NTS and $f{:}\,X\longrightarrow Y$ *be a mapping. Since* f *represents* $SN\psi\beta-IM$*, this implies* $h^{-1}(\mathcal{M})$ SN$\psi\beta$-OS in X for any SN$\psi\beta$-OS $\mathcal{M}\in Y$. Since every SN $\psi\beta$-CS is WN $\psi\beta$-CS. Thus $h^{-1}(\mathcal{M})$ WN$\psi\beta$-OS [23] for any SN$\psi\beta$-OS $\mathcal{M}\in Y$.Consequently, a *map* f *represents* $WN\psi\beta-IM$*.*

The example 3.13. demonstrates that the converse of Theorem 3.17. is not required to be true.

Theorem 3.18. Every $SN\psi\beta-IM$is $N\psi\beta S-IM$

Proof: In the same way above.

The example 3.15. demonstrates that the converse of Theorem 3.18. is not required to be true.

Theorem 3.19. Every $WN\psi\beta-IM$is $N\psi\beta W-IM$

Proof: In the same way above.

The example 3.14. demonstrates that the converse of Theorem 3.19. is not required to be true.

Theorem 3.20. Every $N\psi\beta S-IM$ is $N\psi\beta W-IM$

Proof: In the same way above.

The example 3.14. demonstrates that the converse of Theorem 3.20. is not required to be true.

Remark 3.20. The diagram that follows Figure 3.2 be Show the Relationships across different neutrophilic continuous mapping.

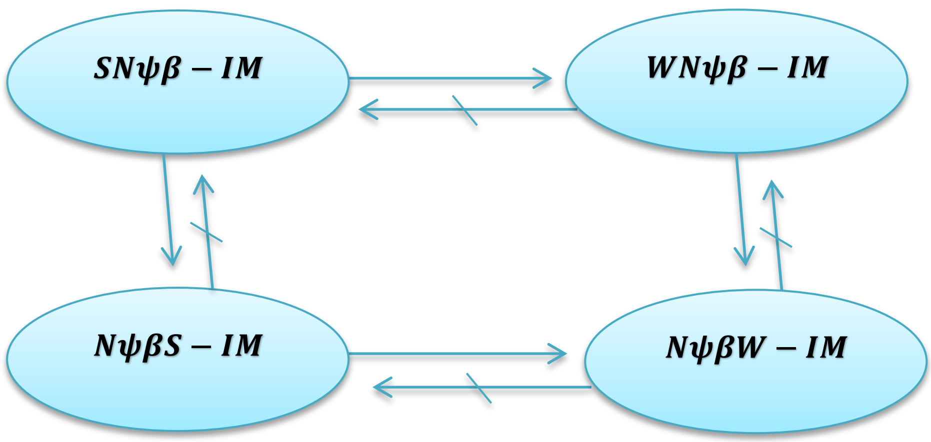

Figure 3.2. The relationship between the new varieties of irresolute mappings

Theorem 3.21. Suppose that $d_1: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then

1. Every $SN\psi\beta - CM$ is $SN\psi\beta - IM$
2. Every $WN\psi\beta - CM$ is $WN\psi\beta - IM$
3. Every $N\psi\beta W - CM$is $N\psi\beta W - IM$
4. Every $N\psi\beta S - CM$is $N\psi\beta S - IM$

Proof: 1. Assume that$(X, \tau), (Y, \sigma)$ be a NTS and *$f: X \longrightarrow Y$ be a $SN\psi\beta - CM$. Since f represents $SN\psi\beta - CM$, this implies* $h^{-1}(\mathcal{M})$ SN$\psi\beta$-OS in X for any NOS $\mathcal{M} \in Y$. Since every NOS is SN $\psi\beta$-CS. Thus $h^{-1}(\mathcal{M})$ WN$\psi\beta$-OS [23] for any SN$\psi\beta$-OS $\mathcal{M} \in Y$.Consequently, a *map f represents $WN\psi\beta - IM$.*

proof of the rest in the same way.

Theorem 3.22. Suppose that $d_1: (X, \tau) \rightarrow (Y, \sigma)$ and $d_2: (Y, \sigma) \rightarrow (\mathcal{K}, \mathcal{J})$ are two mappings. Then the following holds:

1. If d_1 and d_2 is $N\psi\beta W-$ CM, then the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ is $N\psi\beta W$- CM.
2. If d_1 and d_2 is $N\psi\beta S-$ CM, then the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ is $N\psi\beta S$- CM.
3. If d_1 and d_2 is $WN\psi\beta-$ CM, then the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ is $WN\psi\beta$- CM.
4. If d_1 and d_2 is $SN\psi\beta-$ CM, then the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ is $SN\psi\beta$- CM.

Proof: 1. Let $\mathcal{M}$ be a NOS in (X, τ). Since d_1 is $N\psi\beta W-$ CM, the image $d_1(\mathcal{M})$ is $N\psi\beta W$-OS in (Y, σ). Since d_2 is $N\psi\beta W-$ CM so, the image $e(d_1(\mathcal{M}))$ is $N\psi\beta W$-OS in $(\mathcal{K}, \mathcal{J})$. Now, consider the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ For any NOS $\mathcal{M}$ in (X, τ), $d_1(\mathcal{M})$ is NOS in (Y, σ), and $e(d_1(\mathcal{M}))$ is $N\psi\beta W$-OS in $(\mathcal{K}, \mathcal{J})$. By the definitions of $N\psi\beta W$-CM, $d_1 \circ d_2(\mathcal{M})$ satisfies the conditions of $N\psi\beta W$-CM. Therefore, the composition $d_1 \circ d_2: (X, \tau) \rightarrow (\mathcal{K}, \mathcal{J})$ is $N\psi\beta W$-CM.

proof of the rest in the same way.

Theorem 3.23. Suppose that $d_1: (X,\tau) \rightarrow (Y,\sigma)$ and $d_2: (Y,\sigma) \rightarrow (\mathcal{K},\mathcal{J})$ are two mappings. Then the following holds:

1. If d_1 and d_2 is $N\psi\beta W-$ IM, then the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ is $N\psi\beta W\ IM$.
2. If d_1 is and d_2 is $N\psi\beta S-$ IM, then the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ is $N\psi\beta S$- IM.
3. If d_1 and d_2 is $WN\psi\beta-$ IM, then the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ is $WN\psi\beta$- IM.
4. If d_1 and d_2 is $SN\psi\beta-$ IM, then the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ is $SN\psi\beta$- IM.

Proof: 1. Let $\mathcal{M}$ be a $N\psi\beta W$ -OS in (X,τ). Since d_1 is $N\psi\beta W-$ IM, the image $d_1(\mathcal{M})$ is $N\psi\beta W$ -OS in (Y,σ). Since d_2 is $N\psi\beta W-$ IM so, the image $e(d_1(\mathcal{M}))$ is $N\psi\beta W$-OS in $(\mathcal{K},\mathcal{J})$. Now, consider the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ For any $N\psi\beta W$-OS $\mathcal{M}$ in (X,τ), $d_1(\mathcal{M})$ is $N\psi\beta W$-OS in (Y,σ), and $e(d_1(\mathcal{M}))$ is $N\psi\beta W$-OS in $(\mathcal{K},\mathcal{J})$. By the definitions of $N\psi\beta W$-IM, $d_1 \circ d_2(\mathcal{M})$ satisfies the conditions of $N\psi\beta W$-IM. Therefore, the composition $d_1 \circ d_2: (X,\tau) \rightarrow (\mathcal{K},\mathcal{J})$ is $N\psi\beta W$-IM.

proof of the rest in the same way.

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4. Conclusions

we presented the new class of neutrosophic continuous mappings called; WN $\psi\beta$-CM, SN $\psi\beta$-CM, N$\psi\beta$W-CM and N$\psi\beta$S-CM and discussed a few of their characteristics. Moreover, we introduced the notions of WN $\psi\beta$-CIM, SN $\psi\beta$-CIM, N$\psi\beta$W-CIM and N$\psi\beta$S-CIM in NTS were examined, along with a few traits that are associated with them. Lastly, we looked at how continuous with irresolute mappings for various types relate to one another. These findings significantly advance our understanding for neutrosophic topological spaces with broaden their applications. Among other areas, connectedness, compactness, topological qualities, and hereditary features are areas where this idea may be developed and improved.

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