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Feebly Generalized Closed Mappings

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Dalal Ibraheem Resan
Al-Mustansiriyah University
College of Education
Department of Mathematics

Abstract

The main purpose of this paper is to study feebly generalized closed mappings , we proved several results about that by using some concepts of topological as fg-closed sets,gf-closed sets, feebly closed sets , feebly open sets , g-closed sets, feebly regular and feebly normal .

Keywords

feebly generalized closed set, generalized feebly closed set , feebly closed set , feebly open set , g-closed set, feebly regular and feebly normal .

1- Introduction

During the last few years the study of generalized closed and feebly closed mappings has found considerable interest among general topologists . One reason in these objects are natural generalizations of closed mappings . More importantly feebly closed and generalized closed mappings suggest some new concepts which have been to be very useful in study of a topology . In fact , S. N. Maheshwari and P. C. Jain [1] introduced the concept of feebly open , in a topological space (X,T) , a set A in X is said to be feebly open if there exists an open set U such that $U\subset A\subset scl\ (U)$ .T . Neubrunn [2] gave an example , let X and Y be the set of real numbers with the usual topology . Let the mapping f : X → Y be defined as follows $f(x) = x$, if $x \neq o$ and $x \neq 1$; $f(o) = 1$, $f(1) = o$. Then f is feebly open . Dalal [3] proved , let (X,T) be a topological space and $A\subset X$ then A is fg-closed set iff (fclA-A) non contain non empty feebly closed set . Recently , Dalal [4] proved , if go f : (X,T) → (Z,W) is open and g : (Y,V) → (Z,W) is feebly continuous and injective , then f : (X,T) → (Y,V) is feebly open . In this paper we shall study feebly generalized closed mapping we prove several results about that by using another concepts. Throughout this paper , if f : (X,T) → (Y,V) be a mapping,
$f^{-1}(H)$ is the inverse image of a subset H of (X,T) , clH and f /H denote the closure of H and restriction of f to H respectively .

2- Preliminaries

A subset A of a topological space (X,T) is said to be semi open [5] if there exists an open set U of X such that $U\subset A\subset clU$ and is said semi closed if there exists closed set U such that $U^{\circ} \subset A\subset U$ .The complement of semi open set is said to be semi closed . The semi – closure [6] and [7] of a subset A of (X,T) , denoted by $scl_x(A)$ briefly scl(A) , is defined to be the intersection of all semi closed sets containing A . scl(A) is a semi closed set [6]and[7] . The semi interior [7] of A denoted by sint (A) is defined to be union of all semi open sets contained in A. In a topological space (X,T) , A is called feebly open [8] if $A \subset scl\ int\ A$ and accustomed A is said to be feebly closed if $sint\ clA \subset A$. In [9] proved the complement of feebly open set is feebly closed set . Intersection of all feebly closed sets containing A is feebly – closure [10] of A and is denoted by fcl A , it is accustomed union of all feebly open sets contained in the set A is the feebly – interior of A and is denoted by fint A . In [10] a function f : (X,T) → (Y,V) is called feebly closed if the image of each closed set in X is feebly closed set in Y . In [4] a function f : (X,T) → (Y,V) is called feebly open if the image of each open set in X is feebly open in Y . A set A in a topological space (X,T) is called feebly generalized closed (briefly , fg- closed) [3] if $fclA \subset U$ whenever $A \subset U$ and U is feebly open set in X and a set A in a topological space (X,T) is called feebly generalized open (briefly , fg-open) if $U \subset$ fint A whenever $U \subset A$ and U is feebly closed in X and the complement of fg-open set is fg- closed . A subset A of a space (X,T) is called a generalized closed (briefly g- closed)[11] if $clA \subset U$ whenever $A \subset U$ and U is open .Asubset A of a topological space(X,T) is called generalized feebly closed (briefly , gf- closed) [12] if $fclA \subset U$ whenever $A \subset U$ and U is open set in X. In [13] a topological space (X,T) is called feebly regular (briefly , f- regular) if for all $x \in X$ and for all open set A containing x there exists feebly open set H such that $x \in H \subset fclH \subset A$ and a topological space (X,T) is called feebly normal (briefly , f-normal) if for all disjoint closed sets H_1,H_2in X , there exist feebly open sets U_1,U_2 in X such that $H_1 \subset U_1$, $H_2 \subset U_2$ and $U_1 \cap U_2 = \phi$.

Definition 2.1

A map f:(X,T) → (Y,V) is called a feebly generalized closed map (written as fg-closed map) if for each closed set F of X , f(F) is a fg-closed set of Y .

Example 2.2

Let T ={ ϕ,X,{b,c}} be a topology on X = {a,b,c } and let V = {ϕ,Y,{3,4}} be a topology on Y = {1,2,3,4}.

Define f: (X,T) → (Y,V) as follows :

f(a) = 1 , f(b) = 2 , f(c) =4. Then f is fg-closed map.

Proposition 2.3[12 : Theorem 3.2]

Let (X,T) be a topological space and $A \subset X$, then A is feebly closed if and only if fclA = A .

Proposition 2.4 [12 : Corollary 3.4]

Let (X,T) be a topological space , A and B$\subset$X if A$\subset$B then fclA$\subset$fclB.

Proposition 2.5[12 : Theorem 1.3]

Let (X,T) be a topological space ,A and B be feebly closed subsets of X , then A$\cap$B is feebly closed in X .

Proposition 2.6[14:Theorem 4.1]

If f : X$\rightarrow$ Y is gf-closed and A is a closed set of X then f/A: A$\rightarrow$ Y is gf-closed .

Proposition 2.7[3: Proposition 3.10]

If A is feebly open and fg- closed set of a topological space (X,T) , then A is feebly closed.

Proposition 2.8[3: Proposition 3.13]

Every fg-closed set in a topological space (X,T)be gf-closed set.

3- The Main Results

Remark 3.1

Every closed mapping is fg- closed mapping , but the converse is not necessarily true the following example illustrates that .

Example 3.2

Let T = {ф,X,{b,c}}be a topology on X = {a,b,c} and let V = {ф,Y,{c,d}} be a topology on Y = {a,b,c,d} .

Define a map f : (X,T) $\rightarrow$ (Y,V) as follows :

f(a) = a , f(b) = b and f(c) = d . Then f is fg- closed mapping but it is not closed.

Proposition 3.3

Every feebly closed mapping is fg- closed .

Proof :

Let f : (X,T) $\rightarrow$ (Y,V) is feebly closed , let F be a closed in X and let f(F)$\subset$U , where U is feebly open, to prove fcl(f(F))$\subset$U, since f is feebly closed then f(F) is feebly closed in Y , then by Proposition 2.3 , f(F) = fcl(f(F)), thus fcl(f(F))$\subset$U, hence f is fg-closed mapping .

Remark 3.4

The converse of Proposition 3.3 , is not necessarily true, the Example 3.2 , illustrates that .

Proposition 3.5

If $f : (X,T) \rightarrow (Y,V)$ is fg- closed mapping such that every fg- closed subset of Y is feebly open in Y , then f is feebly closed .

Proof

Let H be a closed set in X , since f is fg- closed , then f(H) is fg-closed set in Y and by hypothesis f(H) is also feebly open , then by Proposition 2.7, f(H) is feebly closed in Y , therefore , f is feebly closed .

Proposition 3.6

Every fg-closed map is gf-closed map.

The proof follows from Proposition 2.8,but the converse is not true always the following example illustrates that.

Example 3.7

Let $T = \{\phi, X, \{2,3\}\}$be a topology on $X = \{1,2,3\}$ and let $V = \{\phi, Y, \{3,4\}\}$ be a topology on $Y = \{1,2,3,4\}$.

Define a map $f : (X,T) \rightarrow (Y,V)$ as follows :

$f(1) = 2$, $f(2) = 3$ and $f(3) = 4$. Then the image $f(X)= \{2,3,4\}$ is not fg-closed.Hence f is not fg-closed map.However ,f is a gf-closed map.

Theorem 3.8

A map $f : (X,T) \rightarrow (Y,V)$ is fg- closed if and only if for each set H of Y and for each open set U containing $f^{-1}(H)$, there exists a fg- open set G of Y containing H and $f^{-1}(G)\subset U$.

Proof

We have f is fg- closed , let $H\subset Y$ and U is open set in X such that $f^{-1}(H)\subset U$, let $G = Y-f(X-U)$, since U is open in X , then $X - U$ is closed set in X , since f is fg- closed , then $f(X-U)$ is fg- closed in Y , thus $G = Y-f(X-U)$ is fg-open in Y . Now to prove $H\subset G$, let $y \notin G$, we will prove $y\notin H$, then $y\in f(X-U)$, thus there exists $x\in(X-U)$ such that $f(x) = y$, then $x\notin U$ since $f^{-1}(H)\subset U$, then $x\notin f^{-1}(H)$, therefore , $y\notin H$, thus $H\subset G$. Now we will prove $f^{-1}(G)\subset U$, since $G = Y-f(X-U)$, then

$f^{-1}(G) = f^{-1}[Y-f(X-U)] = f^{-1}(Y) - f^{-1}[f(X-U)] \subset X-(X-U) = U$, thus $f^{-1}(G)\subset U$.

Conversely , let A is closed set in X , then $U = X-A$ is open set in X and let $H = Y-f(A)$ is any set in Y, then $f^{-1}(H) = f^{-1}[Y-f(A)] = f^{-1}(Y) - f^{-1}[f(A)]\subset X-A = U$ and by hypothesis , there exists fg- open set G in Y such that $f^{-1}(G)\subset U$ and $H\subset G$, i.e, $Y - f(A)\subset G$ and $f^{-1}(G)\subset X-A$ we get $Y-f(A)\subset G$ and $G\subset f(X-A)$, thus $Y-f(A)\subset G$ and $G\subset Y-f(A)$, then $G = Y-f(A)$, since G is fg- open in Y , then $Y-f(A)$is fg-open in Y, then f(A)is fg-closed in Y , thus f is fg- closed .

Theorem 3.9

If f : (X,T) → (Y,V) is a continuous , fg-closed surjection from a normal space (X,T) to a space (Y,V) , then (Y,V) is feebly normal .

Proof

Let A and B be disjoint closed sets of Y . Since f is continuous , then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets in X . Since X is normal , there exist disjoint open sets $U_1$and U_2 of X such that $f^{-1}(A)\subset U_1$and $f^{-1}(B)\subset U_2$. By Theorem 3.8, there exist fg- open sets G and H of Y such that $A\subset G$, $B\subset H$ and $f^{-1}(G)\subset U_1$ and $f^{-1}(H)\subset U_2$. Then we have $f^{-1}(G)\cap f^{-1}(H) = \phi$ and hence $G\cap H = \phi$. Since A and B are closed then A,B feebly closed sets,since G is fg-open and A is feebly closed , $A\subset G$ implies $A\subset fint(G)$. Similarly we have $B\subset f\,int(H)$, therefore $fint\,G\cap fint\,H = G\cap H = \phi$ and hence Y is feebly normal .

Theorem 3.10

If f : (X,T) → (Y,V) is a continuous , feebly open and fg-closed surjection from regular space (X,T) to a space (Y,V) , then (Y,V) is feebly regular .

Proof

Let U be an open set containing a point y in Y , let x be a point of X such that y = f(x) . Since X is regular space then there is an open set H in X such that $x\in H\subset clH\subset f^{-1}(U)$, where $f^{-1}(U)$ is open since f is continuous , then $y\in f(H)\subset f(clH)\subset U$, where f(H) is feely open since f is feebly open . Since U is open then U is feebly open, clH is closed in X and by hypothesis f is fg- closed then f(clH) is fg- closed in Y , then $fcl(f(clH))\subset U$, since $H\subset clH$, then $f(H)\subset f(clH)$, then by Proposition 2.4, $fcl\ (f(H))\subset fcl(f(clH))\subset U$, then $fcl(f(H))\subset U$, thus $y\in f(H) \subset fcl\ (f(H)) \subset U$, where f(H) is feebly open in Y , hence Y is feebly regular .

Theorem 3.11

If f : (X,T) → (Y,V) is fg- closed and A is closed set of X , then f/A: $(A,T_A)\rightarrow (Y,V)$ is fg- closed .

Proof

Let H be a closed subset of A , then H is closed in X and f is fg-closed ,then f(H) is fg-closed in Y, but f/A(H) = f(H), thus f/A (H) is fg- closed in Y , hence f/A is fg- closed .

Corollary 3.12

(i) If $f:X\rightarrow Y$ is closed and A is a closed set of X, then $f/A:A\rightarrow Y$ is fg -closed.

(ii) If $f:X\to Y$ is feebly closed and A is a closed in X , then $f/A:A\to Y$ is fg - closed .
(iii) If $f:X\to Y$ is fg-closed and A is a closed in X , then $f/A:A\to Y$ is gf -closed .

Proof
(i) The proof is obvious from Theorem 4.1 and by Proposition3.1.
(ii) The proof is obvious from Theorem 4.1 and by Proposition3.2.
(iii) The proof is obvious from Propositions(3.6 and 2.6).

Theorem 3.13
Let B be feebly open and fg-closed subset of Y . If $f:X\to Y$ is feebly closed then $f/A:A\to Y$ is fg -closed where $A=f^{-1}(B)$.

Proof
Let H be a closed set in A then $H = G\cap A$ where G is closed in X , since $f_A(H)=f(H)$ then $f_A(H)=f(G\cap A)=$
$f(G\cap f^{-1}(B))=f(G)\cap B$, since f is feebly closed and G is closed in X, then f(G) is feebly closed in Y , since B is feebly open and fg-closed then by Proposition 2.7,B is feebly closed then by Proposition2.5, $f(G)\cap B$ is feebly closed , thus $f_A(H)$ is feebly closed then $f_A(H)$ is fg-closed , thus f/A is fg-closed.

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الخلاصة

الغرض الرئيسي من هذا البحث هو دراسة التطبيقات المغلقة الاعم الضئيلة وقد تم برهان عدد من النتائج حول ذلك باستخدام بعض المفاهيم التبولوجية كالمجموعات المغلقة الضئيلة الاعم والمجموعات المغلقة الاعم الضئيلة والمجموعات المغلقة الضئيلة والمجموعات المفتوحة الضئيلة والمجموعات المغلقة – g والمنتظم الضئيل والسوي الضئيل .