Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

Page 101-129

المؤتمر العلمي الثاني لكلية
علوم الحاسوب والرياضيات
16. 18/اذار/2010

Solvability and Admissibility of Linear Perturbed Unbounded Differential Operator Equations with Unbounded Control input Via Composite perturbation Semigroup Approach

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Abstract

The perturbed linear dynamical system with unbounded perturbed control operator generating by unbounded infinitesimal generator have been adapted. The solvability of this class with admissibility as wheel as some dynamical system properties are also given and proved .The solution is guaranteed via composite perturbation semigroup approach.

1- Introduction

Balachandran in [4] Presented the semigroup theory and optimal control theory in Hilbert space, [9] studied the semigroup formulation of Boundary input problems for systems governed by parabolic partial differential equations and the results were uses to examine the time optimal boundary control problem, [3] searched for sufficient conditions for boundary controllability of integro-differential systems of Banach spaces and the result was obtained by using the strongly continuous semigroup theory and Banach contraction principle,[14] concerned with approximating infinite dimensional optimal control problems with non normal system operators and in proving convergence of the infinitesimal generators. Solvability and controllability of semilinear initial value control problem via semigroup approach were studied by [20,23] and the development of approximation technique for infinite dimensional linear-quadratic optimal control problem via semigroup approach searched by [2].The work of [1] focused on the controllability of the mild solution of some semilinear initial and boundary control problems in arbitrary Banach spaces with their optimal controls solution.

The unbounded control operator appears naturally for example, when we model boundary or point control from system described by linear PDE's. There is extensive literature dealing with systems having unbounded control operators. Among those are [7],[8],.[17],[19],[22],[24].

The concept of a composite semigroup was used to drive a necessary and sufficient condition for an unbounded control operator to be admissible in the sense of Weiss [27] that which given a necessary condition for admissibility of unbounded control operators on a Banach space X. It also showed that the conditions are sufficient conditions in the case of invertible semigroups. One can find these conditions, more details, explanations and examples in ([10], [11], [12] ,[16] [25],[26],[28]).

Our argument totally relies on the concept of a composite perturbation semigroup. This is a very promising technique with a potentially wide range of applications in the infinite-dimensional control system theory.

The linear dynamical control system in the presence of unbounded control operators as well as the perturbation for the generators are one of the main interest and themes of this work. The solvability of such system and the study of some of its dynamical properties, up to our knowledge and research are still a challenge for many researchers. So, the main aim of the following work is to define such a dynamical properties as well as the solvability using the concept of composite semigroup generated by some unbounded linear generators. Some preliminaries are then needed to understand the present approac

The following problem have been presented an disused in this paper.

$$\frac{d}{dt}Z(t) = (A_1 + \Delta A_1)Z(t) + Z(t)(A_2 + \Delta A_2) + (B + \Delta B)u(t)$$

$$Z(0) = Z_0$$

where $B : U \longrightarrow L(H)$ is a linear unbounded perturbed control operator, and $\Delta B \in L(U, L(H))$where $u \in L^2_{loc}([0,\infty), U)$ is the control function. Let A_1:$D(A_1) \subset L(H)$ and $A_2 \subset L(H)$ are unbounded linear operators $\Delta A_1 \in L(H)$ and $\Delta A_2 \in L(H)$ that $D(A_1) \subset D(\Delta A_1)$and $D(A_2) \subset D(\Delta A_2)$. $A_1 + \Delta A_1$, $A_2 + \Delta A_2$ are defined as linear unbounded operators with $D(A_1 + \Delta A_1)$ and $D(A_2 + \Delta A_2)$ contained in H.

2. Some mathematical concepts

In this section , some necessary mathematical concepts for usual semigroup ,petutbation and compsite semigroups theorys.

Definition(2.1) [5]

A family $\{T(t)\}_{t\geq 0}$ of bounded linear operators on a Banach space X is called a (one-parameter) semigroup on X if it satisfies the following conditions:

$T(t+s) = T(t)T(s),\ \forall\ t, s \geq 0$

$T(0) = I$, where I is identity operator.

Definition(2.2), [18]

The linear operator A defined on the domain:

$$D(A) = \{x \in X : \lim_{t \downarrow 0} \frac{T(t)x - x}{t} \text{ exists}\} \text{ and} \qquad (6)$$

$$Ax = \lim_{t \downarrow 0} \frac{T(t)x - x}{t} = \left.\frac{d^+T(t)}{dt}\right|_{t=0} \quad \text{for } x \in D(A)$$

is the infinitesimal generator of the semigroup T(t), D(A) is the domain of A.

Definition(2.3) [3]

A semigroup $\{T(t)\}_{t\geq 0}$ on a Banach space X is called strongly continuous semigroup of a bounded linear operators or (C_0-semigroup) if the map $T: t \longrightarrow T(t) \in L(X)$, $t \in \mathbb{R}^+$ satisfies the following conditions:

1. $T(t+s) = T(t)T(s),\ \forall\ t, s \in \mathbb{R}^+$.
2. $T(0) = I$, where I identity operator.
3. $\lim_{t \downarrow 0} \|T(t)x - x\|_X = 0$, for every $x \in X$.

Lemma(2.4), [4],[21]

A linear (unbounded)operator A is the infinitesimel generator of a C_o-semigroup satisfying the following conditions:

(i) A is closed and $\overline{D(A)} = X$.

(ii) For $x \in D(A)$, $T(t)x \in D(A)$ and

$$\frac{d}{dt} T(t)x = AT(t)x = T(t)Ax, \text{ for all } t \geq 0$$

(iii)The resolvent set $\rho(A)$ of A contains $\mathbb{R}^+$ and for every $\lambda > 0$

$\|R(\lambda; A)\| \leq \frac{1}{\lambda}$. Then $\lim_{\lambda \to \infty} \lambda R(\lambda; A)x = x$, for $x \in X$.

Definition(2.5) ,[6]

The weakest topology on L(X, Y), such that $E_x : L(X, Y) \longrightarrow Y$ given by:

$E_x(T) = Tx$ are continuous for all $x \in X$ is called the strong operator topology.

Remark(2.6), [5]

A semigroup $\{\{T(t)\}_{t\geq 0}$ is called a continuous in the uniform operator topology, if:

(1) $\|T(t+\Delta)x - T(t)x\|_X \longrightarrow 0$, as $\Delta \longrightarrow 0$, $\forall\, x \in X$.

(2) $\|T(t)x - T(t-\Delta)x\|_X \longrightarrow 0$, as $\Delta \longrightarrow 0$, $\forall\, x \in X$

Lemma(2.7), [6]

If A_1 and A_2 are operators, such that $\sigma(A_1) \cap \sigma(A_2) = \varnothing$, then the equation

$A_1X - XA_2 = C$ has a unique solution X for every operator C.

Theorem(2.8), [21]

Let X be a real Banach space and let A be the unbounded linear infinitesimal generator of a C_o-semigroup T(t) on X, satisfying:

$\|T(t)\|_{L(X)} \leq Me^{wt}$.

If ΔA is a bounded linear operator on X, then $A+\Delta A$ with $D(A+\Delta A)=D(A)$ is the infinitesimal generator of a C_o-perturbation semigroup S(t) on X, satisfying:

$\|S(t)\|_{L(X)} \leq Me^{(w+M\|\Delta A\|_{\mathcal{L}(X)})t}$

for any $t \geq 0$, $w \geq 0$ and $M \geq 1$.

Definition(2.9)

Let L(H) be a Banach space, a one-parameter family

$\{\mathbb{S}(t)\}_{t\geq 0} \subset L(L((H))$, $t\in[0,\infty)$ of bounded linear operators defined by:

$$\mathbb{S}(t) = S_1(t)ZS_2(t)\ , \qquad (7)$$

for generator $\mathbb{A} + \Delta\mathbb{A}$, for any $Z \in L(H)$ and $t \in [0,\infty)$ is called composite perturbation semigroup, where $S_1(t), S_2(t)$ are two perturbation semigroups defined from H into H for $(A_1+\Delta A_1)$ and $(A_2 + \Delta A_2)$ respectively.

Definition(2.10)

The infinitesimal generator $\mathbb{A} + \Delta\mathbb{A}$ of $\mathbb{S}(t)$ of problem formulation on a uniform operator topology defined as the limit:

$$(\mathbb{A} + \Delta\mathbb{A})Z = \tau \text{ - } \lim_{t\downarrow 0}\left\{\frac{\mathbb{S}(t)Zh - Z}{t}\right\} \in D(\mathbb{A} + \Delta\mathbb{A}),$$

where $D(\mathbb{A} + \Delta\mathbb{A}) \subset L(H)$ is the domain of $\mathbb{A}+\Delta\mathbb{A}$ defined as follows:

$$D(\mathbb{A} + \Delta\mathbb{A}) = \left\{ Z \in L(H) : \tau\text{- } \lim_{t\downarrow 0}\left\{\frac{\mathbb{S}(t)Zh - Z}{t}\right\} \text{ exist in } \{L(H),\tau\} \right\}.$$

where $\{L(H), \tau\}$ stands for L(H) equipped with the strong operator topology τ, i.e., topology induced by family of seminorms $\rho = \{\rho_h\}_{h\in H}$, where siminorms $\rho_h(Z) = \|Zh\|_H$, $Z \in L(H)$

Remarks(2.11)

1. $\{L(H), \tau\}$ stands for L(H) equipped with the strong operator topology τ, i.e., topology induced by family of seminorms $\rho = \{\rho_h\}_{h\in H}$, where siminorms $\rho_h(Z) = \|Zh\|_H$, $Z \in L(H)$.

2. Let $D(A_1) \subseteq D(\Delta A_1)$, $D(A_2) \subseteq D(\Delta A_2)$ and $D(\mathbb{A}) \subseteq D(\Delta\mathbb{A})$.Therefore the following are concluded

a-The different between the usual strongly continuous semigroups of problem formulation and the composite perturbation semigroup (7) follows from the fact that in general for $Z \in L(H)$, the function $[0,\infty) \supseteq t \mapsto \mathbb{S}(t)Z \in L(H)$ is continuous in $\{L(H), \tau\}$, and which cannot be continuous in $\{L(H), \|.\|\}$ unless the semigroups $\{\mathbb{S}_1(t)\}_{t\geq 0}$, $\{\mathbb{S}_2(t)\}_{t\geq 0} \subset L(H)$ are uniformly continuous. However, this takes place case only if their generators $A_1 + \Delta A_1$, $A_2 + \Delta A_2$ are bounded operators on H.

b-The generator $\mathbb{A} + \Delta\mathbb{A}$ is densely defined only in $\{L(H), \tau\}$ and does not in $\{L(H), \|.\|\}$. This implies that $D(\overline{A+\Delta A})$ in L(H) is only a proper set and not the whole L(H).

Lemma(2.12)

Let $\mathbb{S}(t)= S_1(t)ZS_2(t)$, $t \geq 0$ be a composite perturbation semigroup defined on L(L(H)); $S_1(t)$ and $S_2(t)$ are, perturbation semigroups defined on L(H) then

a- The family $\{\mathbb{S}(t)\}_{t\geq 0} \subseteq L(H)$, $t \geq 0$ is a semigroup, i.e.,

1. $\mathbb{S}(0)Z = Z$, $\forall\ Z \in L(H)$

2. $\mathbb{S}(t + s)Z = \mathbb{S}(t)(\mathbb{S}(s)z)= \mathbb{S}(s)(\mathbb{S}(t))Z$ for $Z \in L(H)$, and $t, s \in [0, \infty)$.

b- $\|\mathbb{S}(t)\|_{L(H)} \leq M_1M_2 e^{t(w_1+w_2)+M_1\|\Delta A_1\|_{L(H)}+M_2\|\Delta A_2\|_{L(H)}}$, for $t \in [0, \infty)$.

c- $\mathbb{S}(t) \in L(L(H))$ is a strong-operator and continuous at the origin, i.e.,

$$\tau\text{-}\lim_{t\downarrow 0}\|(\mathbb{S}(t)Z)h - (\mathbb{S}(0)Z)h\|_H = 0,\ h \in H,\ Z \in L(H).$$

Lemma(2.13)

The operator $\mathbb{A} + \Delta\mathbb{A}$ of problem formulation is infinitesimal generator for $\mathbb{S}(t)$ defined on its domain $D(\mathbb{A}+ \Delta\mathbb{A})$ satisfying the following properties:

(a) $D(\mathbb{A} + \Delta\mathbb{A})$ is strong-operator dense in L(H).

(b) $\mathbb{A} + \Delta\mathbb{A}$ is uniform-operator closed on L(H).

(c) For $Z \in L(H)$:

$$\int_0^t (S(r)Z)\, dr \in D(\mathbb{A} + \Delta\mathbb{A}),\ \text{and}$$

$$(\mathbb{A} + \Delta\mathbb{A})\left(\int_0^t \mathbb{S}(r)Z\, dr\right)= \mathbb{S}(t)Z - Z.$$

(d) For $Z \in D(\mathbb{A})$:

$\mathbb{S}(t)Z \in D(\mathbb{A} + \Delta\mathbb{A})$, the function $[0,\infty) \ni t \mapsto \mathbb{S}(t)Z\in L(H)$

is continuously differentiable in $\{L(H),\tau\}$ and

$$\frac{d}{dt}(\mathbb{S}(t)Z) = (\mathbb{A} + \Delta\mathbb{A})\,(\mathbb{S}(t)Z)= \mathbb{S}(t)((\mathbb{A} + \Delta\mathbb{A})Z)$$

(e) For $Z \in D(\mathbb{A} + \Delta\mathbb{A})$ and $h \in D(A_1 + \Delta A_1)$

$$((\mathbb{A} + \Delta\mathbb{A})Z)h = (A_1 + \Delta A_1)Zh + Z(A_2 + \Delta A_2)h\ .$$

The problem of admissibility for unbounded linear operator and the necessary and sufficient conditions (assumptions) for the solvability of problem formulation is then intrduced and as follows.

3.Problem Formulation

Let H and U be a pair of real Hilbert spaces with norm $\|.\|_H$ and $\|.\|_U$ respectively. Consider the initial value linear perturbed control problem in infinite dimensional state space:

$$\frac{d}{dt}Z(t) = (A_1 + \Delta A_1)Z(t) + Z(t)(A_2 + \Delta A_2) + (B + \Delta B)u(t)$$

(8)

$$Z(0) = Z_0$$

Where B: U$\longrightarrow$ L(H) is a linear unbounded perturbed control operator, and $\Delta B \in L(U, L(H))$ is a perturbation control operator. where u $\in L^2_{loc}([0,\infty), U)$ is the control function. Let A_1:D(A_1) $\subset$ H$\longrightarrow$H and A_2:D(A_2) $\subset$ H$\longrightarrow$H are two unbounded infinitesimal generators. also $\Delta A_1 \in$L(H) and $\Delta A_2 \in L(H)$ that D(A_1) $\subset$ D(ΔA_1) and D(A_2) $\subset$ D(ΔA_2) defined from H into H are linear unbounded operators with D($A_1 + \Delta A_1$) and D(A + ΔA_2) contained in H.

It should be noticed that the existence and uniqueness of strong solution of (8) have been developed by assuming the following assumptions:

1. H_0 = L(H) is a Banach space with the norm

$$\| Z \|_{H_0} = \|\mathbb{R}(\lambda; (\mathbb{A} + \Delta\mathbb{A}))Z\|_{H1} \text{ for } Z \in H_0 \text{ and } \lambda \in \rho(\mathbb{A} + \Delta\mathbb{A}),$$

we define a family of seminorms $P_* = \{P_*h\}_{h\in H}$, where $P_*h(Z) = \|\mathbb{R}(\lambda; (\mathbb{A} + \Delta\mathbb{A}))Zh\|$, for Z $\in$ H_0, and H_0 with the strong operator topology τ_* induced by p_* is denoted by {H_0, τ_*}. It is clear that $\| Z \|_{H_0} = \|\mathbb{R}(\lambda; (\mathbb{A} + \Delta\mathbb{A}))Z\|_{D(A+\Delta A)} = \sup \frac{P_*h(Z)}{\| h \|_H}$, for Z $\in$ H_0 and $\lambda \in \rho(\mathbb{A} + \Delta\mathbb{A})$ (H_0 does not depend upon λ).A_1, A_2 are linear unbounded operators (infinitesimal generators satisfying the conditions of Hille-Yoside theorem on H generating C_o-semigroup $\{T_1(t)\}_{t\geq 0} \subset H_0$ and $\{T_2(t)\}_{t\geq 0} \subset H_0$, respectively.

2. $A_1 + \Delta A_1$ with domain D($A_1 + \Delta A_1$) = D(A_1) $\subseteq$ H and $A_2 + \Delta A_2$ with the domain D($A_2 + \Delta A_2$) = D(A_2) $\subset$ H are linear perturbation unbounded linear operator on H generating C_o-perturbation semigroups $\{S_1(t)\}_{t\geq 0} \subset H_0$ and $\{S_2(t)\}_{t\geq 0} \subset H_0$, respectively.

3. $\mathbb{A} + \Delta\mathbb{A}$ is infinitesimal generator of a C_o-composite perturbation semigroup $\{\mathbb{S}(t)\}_{t\geq 0}$ and with domain D($\mathbb{A} + \Delta\mathbb{A}$) = D($\mathbb{A}$) $\subseteq$ H_0.

4. ($\mathbb{A} + \Delta\mathbb{A}$)Zh = $\mathbb{A}$Zh + $\Delta\mathbb{A}$Zh= (A_1Zh + ZA_2h) + (ΔA_1Zh + ZΔA_2h)= (A_1 + ΔA_1)Zh +

Z($A_2 + \Delta A_2$)h,for any Z $\in$ H_0 and h $\in$ H, where $\mathbb{A}$ is standing for A_1. +.A_2 and $\Delta\mathbb{A}$

standing for $(\Delta A_1 .+. \Delta A_2)$.

4. $D(A_1) \subset D(\Delta A_1)$ is a Hilbert space with the norm $\|.\|_{D(A1)} = \|(\lambda I - (A_1 + \Delta A_1).\|_H$, where $\lambda \in \rho(A_1 + \Delta A_1)$ and $D(A_2) \subset D(\Delta A_2)$ is a Hilbert space with the norm $\|.\|_{D(A2)} = \|(\mu I - (A_2 + \Delta A_2).\|_H$, where $\mu \in \rho(A_2 + \Delta A_2)$. $\{S_1(t)\}_{t\geq 0}$ restricts to a C_o-perturbation semigroup $\{\underline{S_1}(t)\}_{t\geq 0} \subset L(D(A_1))$ with generator $\underline{A_1 + \Delta A_1}$ having the restriction of $A_1 + \Delta A_1$ and $\{S_2(t)\}_{t\geq 0}$ restricts to C_o-perturbation semigroup $\{\underline{S_2}(t)\}_{t\geq 0} \subset L(D(A_2))$ with generator $A_2 + \Delta A_2$ having the restriction $\underline{A_2 + \Delta A_2}$.

5. $H_{-1}(A_2 + \Delta A_2)$ is a Hilbert space defined as the completion of H in the norm $\|.\|_{H_{-1}(A_1+\Delta A_1)} = \|R(\lambda;\ A_1 + \Delta A_1).\|_H$ and $H_{-1}(A_2 + \Delta A_2)$ is a Hilbert space defined as a completion of H in the norm $\|.\|_{H_{-1}(A_2+\Delta A_2)} = \|R(\lambda;\ A_2 + \Delta A_2).\|_H$. $\{S_1(t)\}_{t\geq 0}$ extends to C_o semigroup $\{\overline{S_1}(t)\}_{t>0} \subseteq L(H_{-1}(A_1 + \Delta A_1))$ with generator $\overline{A_1 + \Delta A_1}$ being continuously extension of $A_1 + \Delta A_1$. Analogously, $\{S_2(t)\}_{t\geq 0}$ extends to a C_o-semigroup $\{\overline{S_2}(t)\}_{t>0} \subseteq L(H_{-1}(A_2 + \Delta A_2))$ with generator $\overline{A_2 + \Delta A_2}$ being continuously extension of $A_2 + \Delta A_2$.

6. $H_1 = D(\mathbb{A}) \subset D(\Delta\mathbb{A})$ is a Banach space with the norm $\|Z\|_{H_1} = \|(\lambda I - (\mathbb{A} + \Delta\mathbb{A}))Z\|_{H_0}$, for $Z \in H$ and $\lambda \in \rho(\mathbb{A} + \Delta\mathbb{A})$.

   Moreover, on $D(\mathbb{A} + \Delta\mathbb{A})$ we define a family of seminorms $p_1 = \{p_1 h\}_{h\in H}$, where $p_1h(Z) = \|(\lambda I - (\mathbb{A} + \Delta\mathbb{A}))Zh\|_H$, for $Z \in D(\mathbb{A} + \Delta\mathbb{A})$ H_1 with the topology induced by the uniform norm is denoted by $\{H_1, \|.\|_1\}$ and with strong operator topology τ_1 induced by p_1 is denoted by $\{H_1, \tau_1\}$. It is clear that $\|Z\|_{H_1} = \sup_{\substack{h\in H \\ h\neq 0}} \frac{p_1 h(Z)}{\|h\|_H}$, for $Z \in D(\mathbb{A} + \Delta\mathbb{A})$.

7. The family $\{\mathbb{S}_{-1}(t)\}_{t\geq 0} \subset L(H_{-1})$ is a semigroup continuous in $\{H_{-1}, \tau_{-1}\}$ and infinitesimal generator coincides with $((\mathbb{A} + \Delta\mathbb{A})_{-1},\ D(\mathbb{A} + \Delta\mathbb{A})_{-1})$, where $D((\mathbb{A} + \Delta\mathbb{A})_{-1}) = H_0$.

8. H_{-1} is the set of all equivalence classes of norm bounded Cauchy sequences in $\{H_0, \tau_*\}$ and an element of H_{-1} is denoted by $\tilde{Z} = [\{Z_n\}]$, where $[\{Z_n\}]$ denotes the equivalence class containing $\{Z_n\}_{n\in N}$. We define a family of seminorms $p_{-1} = \{p_{-1}h\}_{h\in H}$, where $p_{-1}h(\tilde{Z}) = p_{-1}([\{Z_n\}]) = \lim_{n\to\infty} p_* h(Z_n)$, for $\tilde{Z} = [\{Z_n\}] \in H_{-1}$ and a norm $\|.\|_{H_{-1}}$ as follows:

$$\|\tilde{Z}\|_{H_{-1}} = \sup_{\substack{h\neq 0 \\ h\in H}} \frac{p_{-1}([Z_n])}{\|h\|} = \tau_*\text{-}\lim_{n\to\infty} \sup_{\substack{h\neq 0 \\ h\in H}} \frac{p_* h(Z_n)}{\|h\|_H} = \lim_{n\to\infty} \sup_{\substack{h\neq 0 \\ h\in H}} \frac{\|\mathbb{R}(\lambda;\mathbb{A}+\Delta\mathbb{A})Z_n h\|_H}{\|h\|_H}$$

$=\tau_*\text{-}\lim_{n\to\infty} \|\mathbb{R}(\lambda;\mathbb{A}+\Delta\mathbb{A})Z_n\|_{H_1} = \|\mathbb{R}(\lambda;\mathbb{A}+\Delta\mathbb{A})Z\|_{H_0}$, for $\lambda \in \rho(\mathbb{A}+\Delta\mathbb{A})$ and $\{Z_n\}\in H_0$, where $Z\in H_{-1}$.

9. The family $\{\mathbb{S}_{-1}(t)\}_{t\geq 0} \subset L(H_{-1})$ is composite perturbation semigroup and continuous extension in the time in $\{H_{-1}, \tau_{-1}\}$ and its infinitesimal generator $((\mathbb{A}+\Delta\mathbb{A})_{-1}, D((\mathbb{A}+\Delta\mathbb{A})_{-1}))$, where $D((\mathbb{A}+\Delta\mathbb{A})_{-1}) = H_0$.
10. The family $\{\mathbb{S}_1(t)\}_{t\geq 0} \subset L(H_1)$ is a composite perturbation semigroup and continuous restriction in time in $\{H_1, \tau_1\}$ and its infinitesimal generator $((\mathbb{A}+\Delta\mathbb{A})_1, D((\mathbb{A}+\Delta\mathbb{A})_1))$, where $D((\mathbb{A}+\Delta\mathbb{A})_1) = D((\mathbb{A}+\Delta\mathbb{A})^2)$ is dense in $D(\mathbb{A}+\Delta\mathbb{A})$ [see theorem 2.1].

By condition (5), $\mathbb{S}_{-1}(t)$ is the C_0-semigrops generated by the linear operators $(\mathbb{A}+\Delta\mathbb{A})_{-1}$, and let $Z(.) \in L(H)$ be the solution of (8) .Then by lemma(2.4)(ii),we have $\mathbb{S}_{-1}(t\text{-}s)Z(s)$ is differentiable, that implies the H_{-1}-valued function

$$H(s)= \mathbb{S}_{-1}(t\text{-}s)Z(s) \text{ for } 0< s < t , \qquad (9)$$

has a derivative as follows:

$$\frac{d}{ds}H(s) = \mathbb{S}_{-1}(t-s)\frac{d}{ds}Z(s) + \frac{d}{ds}\mathbb{S}_{-1}(t-s)Z(s)$$

$$= \mathbb{S}_{-1}(t-s)((\mathbb{A}+\Delta\mathbb{A})_{-1} Z(s) - (\mathbb{A}+\Delta\mathbb{A})_{-1}\mathbb{S}_{-1}(t\text{-}s) + \mathbb{S}_{-1}(t-s)(B+\Delta B)u(s).$$
(10)

From (10),we have that

$$\frac{d}{ds}H(s)= \mathbb{S}_{-1}(t-s)(B+\Delta B)u(s).$$
(11)

Integrating (11) from 0 to t, yields:

$$\int_0^t \frac{d}{ds}H(s)\, ds = \int_0^t \mathbb{S}_{-1}(t-s)\,(B+\Delta B)u(s)\, ds$$

From (9), we have:

$$Z(t)=\mathbb{S}_{-1}(t)Z_0 \quad \int_0^t \mathbb{S}_{-1}(t-s) \quad (B+\Delta B)u(s) \quad ds \quad .$$
(12)

Depending on the above problem formulation with suggested assumption the following definitions, remarks, theoretical results as well as some useful properties are developed.

Definition(3.1)

A continuous function $Z(.)\in L(H)$, given by:

$$Z(t) = \mathbb{S}_{-1}(t)\, Z_0 + \int_0^t \mathbb{S}_{-1}(t-s)\,(B+\Delta B)u(s)\, ds .$$

for any $Z_0 \in L(H)$ and $t \geq 0$,which is strong operator differentiable in L(H) is called a strong solution to the linear perturbation initial value problem (8).

Definition(3,2)

Let U be a Hilbert space and $\Delta B \in L(U, H_0)$, then $B \in L(U, H_{-1})$ is said to be admissible perturbed control operator for $\{\mathbb{S}(t)\}_{t\geq 0}$ for the problem(8) with conditions (1)-(11) if for some $\tau > 0$ and any $u \in L^2([0, \infty], U)$, we have that $\phi_\tau u \in H_0$, and

$$\phi_\tau u = \int_0^t \mathbb{S}_{-1}(\tau - r)(B + \Delta B)u(r)\, dr . \qquad (13)$$

Remarks(3,3)

1. If B is admissible perturbed control operator, then for any $\tau > 0$,

   ϕ_τ defined above is a bounded linear operator from $L^2([0, \infty], U)$ to H_0 (this follows from the closed graph theorem). In the other hand:

   $$\| \phi_\tau u \|_{H_0} \leq k_\tau \| u \|_{L^2([0,\infty),U)} , \ \forall\, u \in L^2(0, \infty), U). \qquad (14)$$

2. The space $\beta(U, H_0, \mathbb{S})$ of all admissible perturbed control operator for $\{\mathbb{S}(t)\}_{t\geq 0}$ with domain U is a subspace of $L(U, H_{-1})$ and is a Banach space with the norm:

   $$\|B + \Delta B\|_t = \sup_{\|u\|} \| \phi_t u \|_{H_0} ,$$

   where the topology of $\beta(U, H_0, \mathbb{S})$ is independent of $\tau > 0$.

Remarks(3.4)

1. $\|.\|_{H_1}$ and $\|.\|_{H_{-1}}$ are equivalence norms do not depend on $\lambda \in \rho(\mathbb{A} + \Delta\mathbb{A})$,[27].

2. $\|.\|_{H_1}$ is equivalent to the graph norm on H_1, so H_{-1} is complete.

3. Let $\mu \in \rho(\mathbb{A} + \Delta\mathbb{A})$ (if H is a real Hilbert space, then $\mu\in\mathbb{R}$).The operator $\mathbb{R}_\mu = (\mu I - (\mathbb{A} + \Delta\mathbb{A}))^{-1}$ has a unique continuous extension to an operator in H_{-1},

which is denoted by the same symbol. $\mathbb{R}_\mu$ is an isomorphic from H_{-1} to H_0 and from H_0 to H_1.

Proposition(3.5)

If $\mathbb{S}(t) \in L(H_0)$ commutes with $\mathbb{A} + \Delta\mathbb{A}$, i.e.,$(\mathbb{S}(t)(\mathbb{A} + \Delta\mathbb{A}))Z = (\mathbb{A} + \Delta\mathbb{A})\,\mathbb{S}(t)Z$, for all $Z \in H_1$), then the restriction $\mathbb{S}(t)$ to H_1 belong to $L(H_1)$ and is the image of $\mathbb{S}(t)$ via any of the isomorphisim $\mathbb{R}_\mu$. Further $\mathbb{S}(t)$ has a unique continuous extension in $L(H_{-1})$, which is the image of $\mathbb{S}(t)$via any of the isomorphism in $\mathbb{R}_\mu^{-1}$.

Proof

From remark (3.4), it follows that $\mathbb{R}_\mu$ is isomorphism from H_{-1} to H and from H to H_1.

$\mathbb{S}(t)Z = \mathbb{R}_\mu\, \mathbb{S}(t)\, \mathbb{R}_\mu^{-1}Z$, for all $Z \in H_1$

$\mathbb{S}(t)Z = \mathbb{R}_\mu^{-1}\mathbb{S}(t)\mathbb{R}_\mu Z$, for all $Z \in H_{-1}$.

Definition(3.6)

An operator $Y \in L(H_1(A_2 + \Delta A_2), H_{-1}(A_1 + \Delta A_1))$ is said to be admissible for $\mathbb{S}(t) \in L(H_0)$ if for every $t > 0$ the following inequality holds:

$$\left| < \int_0^t (\mathbb{S}_{-1}(t-r)Y)h\, dr, g >_{H_{-1}(A_1+\Delta A_1),(H_1(A_1+\Delta A_1))^*} \right| \leq m(t)k\|h\|_H\|g\|_H \qquad (15)$$

for $h \in H_1(A_2 + \Delta A_2)$, $g \in H_1^*(A_1 + \Delta A_1)$ and $k > 0$.

It is clear that in this case (15) extends to all h, g $\in$ H, which can equivalently written as:

$$\int_0^t (\mathbb{S}_{-1}(t-r)Y)\, dr \in L(H_1(A_1 + \Delta A_1), H_{-1}(A_1 + \Delta A_1)) \cap H_0\,.$$

Definition(3.7)

Let $\{\mathbb{S}(t)\}_{t\geq 0}$ be a composite perturbation semigroup in H_0 of problem formulation with generator $\mathbb{A} + \Delta\mathbb{A} \in H_0$ and let $C \in L(H_1, U)$. The operator C is called an admissible observation operator for the composite perturbation semigroup $\{\mathbb{S}(t)\}_{t\geq 0}$ if for any $\tau > 0$, there exists:

$$\int_0^\tau \|C\mathbb{S}(t)Z\|_U^2 \le C_\tau \|Z\|_{H_0}^2, \text{ for } Z \in H_1.$$

Theorem(3.8)

If $C \in L(H_1(A_2 + \Delta A_2), U)$ is an admissible observation operator for $S_2(t)$ and B is an admissible perturbed control operator for $S_1(t)$,such that $(B+ \Delta B) \in L(U, H_{-1}(A_1 + \Delta A_1))$ then $(B+ \Delta B)C \in L(H_1(A_2 + \Delta A_2), H_{-1}(A_1 + \Delta A_1))$ is an admissible perturbed for $\mathbb{S}(t) \subset L(H_0)$, where $\Delta B \in L(U, H_0)$.

Proof

Let $(B+ \Delta B)C \in L(H_1(A_2 + \Delta A_2), H_{-1}(A_1 + \Delta A_1))$,$h \in H_1(A_2 + \Delta A_2)$, and $g \in H_1^*(A_1 + \Delta A_1)$. Using definition(3.6),we have that

$$\left|< \int_0^\tau \mathbb{S}_{-1}(\tau - r)(B + \Delta B)C.h.\,dr, g >_{H_{-1}(A_1+\Delta A_1),\,(H_1(A_1+\Delta A_1))^*}\right|$$

$$\le \int_0^\tau |< \bar{S}_1(\tau - r)(B + \Delta B)C.\bar{S}_2(\tau - r)dr, g >_{H_{-1}(A_1+\Delta A_1),(H_1(A_1+\Delta A_1))^*}$$

$$\le \int_0^\tau ||C.\bar{S}_2(\tau - r)||_U^2 dr\ ||(B + \Delta B)^*.\ \bar{S}_1(\tau - r)g||_U .$$

from definition (3.6), we get

$$\le \int_0^\tau ||C.\bar{S}_2(\tau - r)||_U^2 dr\ ||B^*.\ \bar{S}_1(\tau - r)g||_U||\ \Delta^*B.\ \bar{S}_1(\tau - r)g||_U \le C(t)||h||_H b(t)||g||_H K .$$

Set $m(t) = C(t)b(t)$, thus:

$$\left|< \int_0^\tau \mathbb{S}_{-1}(\tau - r)(B + \Delta B)Chdr, g >_{H_{-1}(A_1+\Delta A_1),\,D((A_1+\Delta A_1)^*)}\right| \le m(t)k||h||_H||g||_H .$$

Then from definition (3.6), we obtain $(B + \Delta B)C$ is admissible for $\{\mathbb{S}(t)\}_{t\ge 0}$.

Corollary(3.9)

Let $\mathbb{S}(t) = S(t)ZS^*(t)$ be a composite perturbation semigroup generated by $((A + \Delta A).\ +.(A + \Delta A)^*)$, for all $Z \in L(H_0)$. B is an admissible perturbed control operator such that $(B + \Delta B) \in L(U, H_{-1}(A + \Delta A))$ of a perturbation composite semigroup $\{\mathbb{S}(t)\}_{t\ge 0}$ if and only if:

$$\|(B^* + \Delta^*B)\ \mathbb{S}_{-1}(\tau - r)h\|_{L^2(0,\tau,U)} \le b(\tau)\|h\|_H^2. \qquad (16)$$

Proof

Let $(B + \Delta B) \in L(U, H_{-1}(A + \Delta A))$. Using definition(3.6),we have that

$$\left| < \int_0^{\tau} \mathbb{S}_{-1}(\tau - r)(B + \Delta B)(B + \Delta B)^* dr h, g >_{H_{-1}(A+\Delta A),(H_1(A+\Delta A))^*} \right| =$$

$$\int_0^{\tau} | < \bar{S} (\tau - r)(B + \Delta B)(B + \Delta B)^* \bar{S}^* (\tau - r) dr h, g > | =$$

$$\int_0^{\tau} | < (B + \Delta B)^* \bar{S}^* (\tau - r) dr h, (B + \Delta B)^* \bar{S}^* (\tau - r) g >_U |$$

By setting h = g, we obtain:

$$= \int_0^{\tau} \|(B + \Delta B)^* \bar{S}^* (\tau - r) h\|_U^2 \, dr = \|(B + \Delta B)^* \bar{S}^* (\tau - r) h\|^2_{L^2(0,\tau:U)} \leq b(\tau) \| h \|_H^2 .$$

On the other hand, by using Cauchy Shwarz's inequality we gets

$$\left| < (B + \Delta B)^* \bar{S}^* (\tau - r) h, (B + \Delta B)^* \bar{S}^* (\tau - r) g >_{L^2(0,\tau:U)} \right|$$

$$\leq \left\|(B + \Delta B)^* \bar{S}^* (\tau - r) h\right\|_{L^2(0,\tau:U)} \left\|(B + \Delta B)^* \bar{S}^* (\tau - r) g\right\|_{L^2(0,\tau:U)} .$$

By setting g = h, we obtain:

$$\left| < (B + \Delta B)^* \bar{S}^* (\tau - r) h, (B + \Delta B)^* \bar{S}^* (\tau - r) h > \right| \leq \left\|(B + \Delta B)^* \bar{S}^* (\tau - r) h\right\|^2 \leq b(\tau) \| h \|_H^2$$

$$\Rightarrow \left| < \int_0^{\tau} \mathbb{S}_{-1}(\tau - r)(B + \Delta B)(B + \Delta B)^* dr h, g >_{H_{-1}(A+\Delta A),(H_1(A+\Delta A))^*} \right|$$

$\leq b(\tau)\|h\|_H$ [see definition (3.6)].

Lemma(3.10) , [18]

$Y \in L(D(A^*), H_{-1}(A))$ is an element of H_{-1} if and only if the following equation has a unique solution $Z \in H_0$, such that:

$$Y = \lambda Z - (AZ - ZA^*)$$

Theorem(3.11)

Let $BB^* \in L(H^*_1(A), H_{-1}(A)) \cap H_{-1}$ and suppose that

$\sigma(\Delta A) \cap \sigma((\Delta A)^*) = \varnothing$, and $(\lambda I - (\mathbb{A} + \Delta\mathbb{A}))Z = (B + \Delta B)(B+\Delta B)^*$

for any $Z \in D(\mathbb{A} + \Delta\mathbb{A})$, then $(B + \Delta B)(B + \Delta B)^* \in L((H_1(A + \Delta A))^*, H_{-1}(A + \Delta A)) \cap H_{-1}$.

Proof

Let $(\lambda I - (\mathbb{A} + \Delta\mathbb{A}))Z = (B + \Delta B)(B+\Delta B)^*$,

$\lambda Z - ((A+\Delta A)Z + Z(A+\Delta A)^* = BB^* + B(\Delta B)^* + \Delta B(\Delta B)^* + \Delta BB^*$,

Since $BB^* \in L(H^*_1(A), H_{-1}(A)) \cap H_{-1}$ .Then from lemma (3.10), we get:

$\lambda I - AZ - ZA^* = BB^*$, for unique $Z \in H_0$,

and since $\sigma(\Delta A) \cap \sigma((\Delta A)^*) = \varnothing$, then from lemma (2.7), the following equation:

$\Delta AZ - Z(\Delta A)^* = B(\Delta B)^* + \Delta B(\Delta B)^* + (\Delta B)B^*$,

Has a unique solution $Z \in H_0$.

So again from lemma (3.10), we get:

$(B + \Delta B)(B + \Delta B)^* \in L(H^*_1(A + \Delta A), H_{-1}(A + \Delta A)) \cap H_{-1}$.

Concluding Remarks(3.12)

The following remakes are generalized results of [11],[12].

1. H_0 is a subspace of $L(H^*_1(A + \Delta A), H_{-1}(A + \Delta A))$, which is dense in the strong operator topology of $L(H^*_1(A + \Delta A), H_{-1}(A + \Delta A))$. This follows from:

   $p_\lambda = \lambda(\lambda I - (A + \Delta A))^{-1} p_\lambda(\lambda I - (A + \Delta A)^*)^{-1} \in H_0$,

   which is an approximation for any $p \in L((H_1{*}(A + \Delta A), H_{-1}(A + \Delta A))$ satisfies:

   $\lim_{\lambda\to\infty} \|p_\lambda h - ph\|_{H_{-1}(A+\Delta A)} = 0$, $h \in (H_1^*(A + \Delta A)$.

2. $\{\mathbb{S}(t)\}_{t\geq 0}$ is a composite perturbation semigroup, such that $\mathbb{S}(t) \in L(H_0)$ can be extended to a composite perturbation smeigroup $\mathbb{S}(t) \in L((H^*_1(A + \Delta A), H_{-1}(A + \Delta A))$.

3. The resolvent operator $\mathbb{R}(\lambda;\mathbb{A} + \Delta\mathbb{A}) \in L(H_0)$ can be extended by continuity to $\mathbb{R}(\lambda; \mathbb{A} + \Delta\mathbb{A})$: $H_{-1} \longrightarrow H_0$, such that $\operatorname{im} \mathbb{R}(\lambda; \mathbb{A} + \Delta\mathbb{A}) = H_0$ and every element $Y \in H_{-1}$ can be uniquely expressed in the form:

   $Y = (\lambda I - (\mathbb{A} + \Delta\mathbb{A}))Z$, where $Z \in H_0$ [since the resolvent

   operator $\mathbb{R}(\lambda; \mathbb{A} + \Delta\mathbb{A})$ is invertible].

Theorem(3.13)

$(B + \Delta B) \in L(U, H_{-1}(A + \Delta A))$ is an admissible perturbed control operator for $\{\mathbb{S}(t)\}_{t\geq 0}$ if and only if $(B + \Delta B)(B + \Delta B)^{*} \in L(D((A + \Delta A)^{*})_1, H_{-1}(A + \Delta A)) \cap H_{-1}$.

Proof

From corollary (3.9), one gets

$$\int_0^{\tau} \mathbb{S}_{-1}(\tau - r)(B + \Delta B)(B + \Delta B)^{*}\, dr \in L(H_1^{*}(A + \Delta A), H_{-1}(A + \Delta A)) \cap H_0 \quad (17)$$

is equivalent to (16).

Now, we have to prove that (17) holds if and only if $(B + \Delta B)(B + \Delta B)^{*} \in L(H_1^{*}(A + \Delta A), H_{-1}(A + \Delta A)) \cap H_{-1}$.

Let $(B + \Delta B)(B + \Delta B)^{*} \in L(H_1^{*}(A + \Delta A), H_{-1}(A + \Delta A)) \cap H_{-1}$.

Hence from theorem (3.11), one can get

$$(\lambda\mathbb{I} - \mathbb{A} + \Delta\mathbb{A})Z = (B + \Delta B)(B + \Delta B)^{*},$$

For a unique $Z \in H_0$ and $\lambda \in \rho(\mathbb{A} + \Delta\mathbb{A})$.

Thus:

$$\int_0^{\tau} \mathbb{S}_{-1}(\tau - r)(B + \Delta B)(B + \Delta B)^{*}\, dr = \int_0^{\tau} \mathbb{S}(\tau - r)(\lambda\,\mathbb{I} - (B + \Delta B))Z\, dr$$

$$= \lambda\int_0^{\tau} \mathbb{S}_{-1}(\tau - r)\, Z\, dr - \int_0^{\tau} \mathbb{S}(\tau - r)\,(B + \Delta B)\, Z\, dr, \quad (18)$$

for any $Z\in H_0$.

The integrals in the right sides of (18)are in H_0, which implies that:

$$\int_0^{\tau} \mathbb{S}_{-1}(\tau-r)(B+\Delta B)(B+\Delta B)^{*}\, dr \in H_0 \cap L(H_1^{*}(A+\Delta A), H_{-1}(A+\Delta A))$$

Set $P_{\lambda} = \lambda(\lambda I-(A+\Delta A))^{-1}(B+\Delta B)(B+\Delta B)^{*}\lambda(\lambda I- (A+\Delta A)^{*})^{-1} \in H_0$.

One can prove the following identity in H_0

$$P_{\lambda} = \int_0^{1} (\mathbb{S}_{-1}(1 - r)P_{\lambda})\, dr - (\mathbb{A} + \Delta\mathbb{A})\int_0^{1} (\mathbb{S}(1 - r)P_{\lambda}r)\, dr .$$

Since

$$\int_0^1 (\mathbb{S}(1-r)P_\lambda)\, dr = \int_0^1 S(1-r)P_\lambda S^*(1-r)\, dr$$

$$= \int_0^1 S(1-r)\lambda(\lambda I-(A+\Delta A))^{-1}(B+\Delta B)(B+\Delta B)^*\lambda(\lambda I-(A+\Delta A)^*)^{-1}S^*(1-r)\, dr$$

$$= \lambda(\lambda I-(A+\Delta A))^{-1}\int_0^1 S(1-r)(B+\Delta B)(B+\Delta B)^*S^*(1-r)\,\lambda(\lambda I-(A+\Delta A)^*)^{-1}\, dr \qquad \backslash \qquad (19)$$

and from (19), we have

$$\int_0^1 (\mathbb{S}_{-1}(1-r)P_\lambda r)\, dr = \lambda(\lambda I-(A+\Delta A))^{-1}\int_0^1 (\mathbb{S}_{-1}(1-r)(B+\Delta B)(B+\Delta B)^*\, dr\, \lambda(\lambda I-(A+\Delta A)^*)^{-1}, \qquad (20)$$

then (15) is surely satisfied. Since the integrals on the right sides of (19) and (20) are in H_0. then we have:

$$\tau_{-1}\text{-}\lim_{\lambda\to\infty}\int_0^1 \mathbb{S}_{-1}(1-r)P_\lambda\, dr = \lim_{\lambda\to\infty}\lambda(\lambda I-(A+\Delta A))^{-1}\int_0^1 S(1-r)(B+\Delta B)(B+\Delta B)^*S^*(1-r)\, dr \lim_{\lambda\to\infty}\lambda(\lambda I-(A+\Delta A)^*)^{-1}, \qquad (21)$$

and also:

$$\tau_{-1}\text{-}\lim_{\lambda\to\infty}\int_0^1 \mathbb{S}_{-1}(1-r)P_\lambda\, dr = \lim_{\lambda\to\infty}\lambda(\lambda I-(A+\Delta A)^*)^{-1}\int_0^1 S(1-r)(B+\Delta B)(B+\Delta B)^*rS^*(1-r)\, dr \lim_{\lambda\to\infty}\lambda(\lambda I-(A+\Delta A)^*)^{-1}.$$

Hence by[lemma(2.4)], we obtain

$$\tau_{-1}\text{-}\lim_{\lambda\to\infty}\int_0^1 \mathbb{S}_{-1}(1-r)P_\lambda\, dr = \int_0^1 \mathbb{S}_{-1}(1-r)(B+\Delta B)(B+\Delta B)^*\, dr,$$

and

$$\tau_{-1}\text{-}\lim_{\lambda\to\infty}\int_0^1 \mathbb{S}_{-1}(1-r)P_\lambda\, dr = \int_0^1 \mathbb{S}_{-1}(1-r)(B+\Delta B)(B+\Delta B)^* r\, dr .$$

Thus: $(B + \Delta B)(B + \Delta B)^* = \int_0^1 \mathbb{S}_{-1}(1 - r)(B + \Delta B)(B + \Delta B)^*\, dr - (\mathbb{A} + \Delta\mathbb{A})\int_0^1 \mathbb{S}_{-1}(1 - r).$

$$(B + \Delta B)(B + \Delta B)^*\, dr$$

$$(B+\Delta B)(B + \Delta B)^* = (I-\mathbb{A} + \Delta\mathbb{A})\int_0^1 \mathbb{S}_{-1}(1-r)(B + \Delta B)(B+\Delta B)^* dr. \qquad (22)$$

From concluding remark(3.17), the integral is in H_0, which implies that the right hand side of (2.52) is in H_{-1} as well as the left side.

Definition(3.14) , [28]

Let u, v $\in \Omega$ and let $\tau \geq 0$. Then τ-concatenation of u and v, $u \underset{\tau}{<>} v \in \Omega$ is given by:

$$(u \underset{\tau}{<>} v)(t) = \begin{cases} u(t), & t \in [0,\tau) \\ u(t-\tau), & t \geq \tau, \end{cases}$$

where $\Omega = L^2([0, \infty), U)$ and U is a Hilbert space.

Recall that use work with Ω because we want to define our system as receiving U-valued locally integrable input function, and any segment of such an input function can be thought of as the restriction to a bounded interval of an element of Ω.

Definition(3.15)

Let U and H be Hilbert spaces, $\Omega = L^2([0, \infty), U)$. An abstract linear control system on H_0 and Ω is a pair $\Sigma = (\mathbb{S}, \phi)$, where $\{\mathbb{S}(t)\}_{t\geq 0}$ is a strongly continuous perturbation semigroup on H, and $\phi = \{\phi_t\}_{t\geq 0}$ is a family of bounded operators from Ω to H_0, such that:

$$\phi_{\tau+t}(u \underset{\tau}{<>} v) = \mathbb{S}(t)\phi_\tau u + \phi_t v , \qquad (23)$$

for any u, v $\in \Omega$ and any $\tau, t \geq 0$.

The functional equation (23) is called composition property. The operators ϕ_t are called input maps.

Remark(3.16)

Taking $\tau = t = 0$ in (23), we get $\phi_0 = \mathbb{S}(0)\phi_0 u + \phi_0 v$, whence, taking now only $t = 0$ in (23) we get that:

For any $\tau \geq 0$

$$\phi_{\tau+0}(u<>0) = \mathbb{S}(0)\phi_\tau u + \phi_0 v \quad (24)$$

$$= \phi_\tau u .$$

Then, one can define, generally:

$$\phi_\tau u = \phi_\tau P_\tau u , \quad (25)$$

Where P_τ is the projection of Ω onto $L^p([0, \tau],U)$ defined by $P_\tau u = u \underset{\tau}{<>} 0$, [52].

Proposition(3.17)

Let H and Ω be as in definition (3.15) and let $\Sigma = (\mathbb{S}, \phi)$ be an abstract linear control system on H_0 and Ω, then the function $\varphi(t, u) = \phi_t u$ is continuous on the product $[0, \infty)\times\Omega$, in particular $\phi = \{\phi_t\}_{t\geq 0}$ is strongly continuous family of operators.

Proof

Taking in (23) $u = 0$ and taking the supremum for $\|v\| = 1$ and denoting $T = t + \tau$ we get:

$$\phi_{t+\tau}(0 \underset{\tau}{<>} v) = \phi_T(0<>v) = \phi_t v ,$$

$$\|\phi_t v\| = \|\phi_T(0<>v)\| \leq \|\phi_\tau\| \, \|(0 \underset{\tau}{<>} v)\| \leq \| \phi_T \|_{H_0} \| v \|_{L^2([0,\infty),U)} .$$

By using $\| v \|_{L^2([0,\infty),U)} = 1$, we obtain

$$\| \phi_t \|_{H_0} \leq \| \phi_T \|_{H_0} , \text{ for } t \leq T . \quad (26)$$

Let us first prove the continuity of $\phi(t, u)$ with respect to the time t, so for the time being, let $u \in \Omega$ be fixed and let:

$$f(t) = \phi_t u ,$$

$$\|f(t)\| = \|\phi_t u\| = \|\phi_t p_t u\| \leq \|\phi_1\| \, \|p_t u\|, \text{ for } t \in [0, 1] ,$$

$$\lim_{t\to 0} \|f(t)\| \leq \lim_{t\to 0} \|\phi_1\| \, \|p_t u\| .$$

since $\|p_t u\| = \int_0^t \|u(t)\|^2 \, dt$, then $\lim_{t\to 0} \|p_t u\| = 0$.

Hence $\lim_{t\to 0} \|f(t)\| = 0$, which implies that $\lim_{t\to 0} f(t) = 0$.

To prove the right continuity of f(t).We proceed as follows.

From (23), we have:

$$f(\tau + h) = \phi_{\tau+h}u = \mathbb{S}(h)\phi_\tau u = \mathbb{S}(h)\phi_\tau u - \phi_\tau u .$$

Hence:

$$\lim_{h\to 0} \|f(\tau + h) - f(h)\|_{H_0} = \lim_{h\to 0} \| \mathbb{S}(h)\phi_\tau u - \phi_\tau u\|_{H_0} = \| \lim_{h\to 0} \mathbb{S}(h)\phi_\tau u - \phi_\tau u\|_{H_0}$$

$$= \| \mathbb{S}(0)\phi_\tau u - \phi_\tau u\|_{H_0} .$$

Then from definition of C_o composite perturbation semigroup, thus: $\lim_{h\to 0} f(\tau + h) = f(\tau)$.

To prove the left continuity of f in $\tau > 0$, we take a sequence $\{\varepsilon_n\}$ with $\varepsilon_n \in [0, \tau]$ and $\varepsilon_n \longrightarrow 0$ and $u_n(t) = u(\varepsilon_n + t)$ so $u_n \in \Omega$ and:

$$\lim_{n\to\infty} u_n = \lim_{\varepsilon_n \to 0} \int_0^{t+\varepsilon_n} \|u(t)\|^2 \, dt = \int_0^t \|u(t)\|^2 \, dt = u .$$

Hence $u_n \longrightarrow 0$ as $\varepsilon_n \longrightarrow 0$, we have $u = u \underset{\varepsilon_n}{<>} u_n$,

so according to (23)

$$\phi_{\varepsilon_n+(\tau-\varepsilon_n)}(u \underset{\varepsilon_n}{<>} u_n) = \mathbb{S}(\tau - \varepsilon_n)\phi_{\varepsilon_n} u + \phi_{\tau-\varepsilon_n} u_n$$

$$\|\phi_\tau u - \phi_{\tau-\varepsilon_n} u\| \le \| \mathbb{S}(\tau - \varepsilon_n)\| \, \|\phi_{\varepsilon_n} u\| + \|\phi_{\tau-\varepsilon_n} u_n\| - \|\phi_{\tau-\varepsilon_n} u\| .$$

From lemma (2.12)(b), we get:

$$\|\phi_\tau u - \phi_{\tau-\varepsilon_n} u\| \le M_1M_2 e^{(t-\varepsilon_n)((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)} \|f(\varepsilon_n)\| + \|\phi_{\tau-\varepsilon_n} u_n\| - \|\phi_{\tau-\varepsilon_n} u\| .$$

Since $\|\phi_{\tau-\varepsilon_n}\| \, \|u_n - u\| \, \varepsilon_n \longrightarrow 0$ as $n \longrightarrow \infty$, then: $\|\phi_\tau u - \phi_{\tau-\varepsilon_n} u\| \le 0$,

so $\phi_\tau u$ is left continuous.The joint continuity of f follows easily now from the decomposition:

$$\phi_t v - \phi_\tau u = \phi_t(v - u) + (\phi_t - \phi_\tau)u$$

where $(t, v) \longrightarrow (\tau, u)$.

Now one can give an estimate for the growth rate of ϕ.

Proposition(3.18)

Let H and Ω be as in definition (3.15) ad let $(\mathbb{S}, \phi)$ be an abstract linear control system on H_0 and Ω of problem formulation. If $M_1M_2 \geq 1$ and $W_1, W_2 \geq 0$ are such that:

$$\| \mathbb{S}(t)\| \leq M_1M_2\, e^{t((W_1+W_2)+\|\Delta A_1\|+\|\Delta A_2\|)} \quad \forall\ t \geq 0\,,$$

then there is some $L \geq 0$, such that:

$$\|\phi_t\| \leq L\, e^{(W_1+W_2)+\|\Delta A_1\|+\|\Delta A_2\|},\ \forall\ t \geq 0\,.$$

Proof

From (23), and the definition(3.14),one gets

$$\phi_n(\ldots(n_1 \underset{1}{<>} n_2) \underset{2}{<>} \ldots \underset{n-1}{<>} u_n) = \mathbb{S}(n\text{-}1)\phi_1u_1 + \mathbb{S}(n\text{-}2)\phi_1u_2 + \ldots + \phi_1u_n\,,$$

for $t \in (n-1, n]$. By using (26), we have that $\|\phi_t\| \leq \|\phi_n\|$.

Now, let $K_n = \mathbb{S}(n\text{-}1)\phi_1 + \mathbb{S}(n\text{-}2)\phi_1 + \ldots + \phi_1$, thus

$$\|K_n\|\left(e^{(W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1\right) \leq M_1M_2 \left(e^{n((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}-1\right) \ \ldots\ +\ e^{(W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}\|\phi_1\| - M_1M_2\left(e^{(n-1)((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}+\ldots+1\right)\|\phi_1\|\,.$$

Hence

$$\|K_n\|\left(e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1\right)\|\phi_1\| \leq \left(e^{n((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}-1\right)$$

$$\|Kn\| < \frac{\left(M_1M_2e^{n((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}-1\right)}{e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1}\|\phi_1\|, (e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1) \neq 0$$

which implies:

$$\|\phi_t\| \leq \frac{\left(M_1M_2e^{n((W_1+W_2)+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}-1\right)}{e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1}\|\phi_1\|\,.$$

Since $t \in (n-1, n)$, i.e $t \geq n-1$ and $t + 1 \geq n$, so the following is well defined:

$$\|\phi_t\| \le M_1M_2 \frac{e^{(t+1)(W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)}}{e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1},$$

and

$$\|\phi_t\| \le L\, e^{t(W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|)},\ \forall\, t \ge 0,$$

where

$$L = \frac{e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}}{e^{W_1+W_2+M_1\|\Delta A_1\|+M_2\|\Delta A_2\|}-1}.$$

Definition(3.19)

Let H_0 be a Banach space, let $\mathbb{S}$ be a semigroup on H_0 with generator $\mathbb{A}+\Delta\mathbb{A}$ and let $f \in L^1_{loc}([0,\infty), \{H_{-1}, \tau_{-1}\})$. Then we say that the operator $Z(.) \in L^1_{loc}([0,\infty), \{H_0, \tau\})$ is a strong solution of the differential equation:

$$\dot{Z}(t) = (\mathbb{A}+\Delta\mathbb{A})Z(t) + f(t), \qquad (27)$$

if for any $t \ge 0$,

$$Z(t) - Z(0) = \int_0^t [(\mathbb{A}+\Delta\mathbb{A})Z(s) + f(s)]\, ds .$$

Remarks(3.20)

1. Any strong solution of (27) as an $\{H_{-1}, \tau_{-1}\}$-valued function, is absolutely continuous and almost everywhere differentiable, and as an H-valued function it is only defined almost everywhere.
2. Let $\tilde{\Omega} = L^p_{loc}([0,\infty), U)$ is a Frechet space with the family of seminorms $p_n(u) = \|p_n u\|$, $n \in \mathbb{N}$.
3. If $f=(B+\Delta B)u$ then definition(3.19) is very good for control perturbed

$$\dot{Z}(t) = (\mathbb{A}+\Delta\mathbb{A})Z(t) + (B+\Delta B)u$$

4. Any family of independent maps ϕ defined on Ω can be extended to $\tilde{\Omega}$ by continuity.

Theorem(3.21)

Consider problem formulation,let U and H be a Hilbert spaces and let

$\tilde{\Omega} = L^2_{loc}([0, \infty), U)$. Let $(\mathbb{S}_{-1}, \phi)$ be an abstract linear control system on $\{H_{-1}, \tau\}$ and $\tilde{\Omega}$. Then there is a unique operator $B + \Delta B \in L(U, \{H_{-1}, \tau_{-1}\})$ such that for any $u \in \tilde{\Omega}$ and $t \geq 0$

$$\phi_t u = \int_0^t \mathbb{S}_{-1}(t-\sigma)(B+\Delta B)u(\sigma)\, d\sigma . \tag{28}$$

Moreover for any $Z_0 \in H_0$ and $u \in \Omega$ the function Z of $t \geq 0$

$$Z(t) = \mathbb{S}_{-1}(t)Z_0 + \phi_t u , \tag{29}$$

is the (unique) continuous state strong solution of the differential equation:

$$\dot{Z}(t) = (\mathbb{A} + \Delta\mathbb{A})Z(t) + (B + \Delta B)u(t),\ Z(0) = Z_0 . \tag{30}$$

Proof

Let $u \in \tilde{\Omega}$ (see Remark (3.20)) and so that

$\phi_u(t) = \phi_t p_t u$ is a well defined function from $[0, \infty)$ to $\{H_0, \tau\}$.

By using proposition (3.17) and equation (25) we have $\phi_t u$ is continuous.

Using again equation (25) and

$$\| p_t u \|_{L^2([0,t],U)} = \left(\int_0^t \| u(\tau) \|^2\, d\tau \right)^{1/2} \leq \left(\int_0^t d\tau \right)^{1/2} (\| u(\tau) \|^2)^{1/2} d\tau$$

$$= t^{1/2} \| u \|_{L^2([0,\infty),U)} , \tag{31}$$

we get from proposition (3.18) that

$$W_1 + W_2 + M_1 \| \Delta A_1 \|_{H_0} + M_2 \| \Delta A_2 \|_{H_0} > 0$$

$$\| \phi_t u \|_{H_0} \leq L\, e^{((W_1+W_2)+M_1\|\Delta A_1\|_{H_0}+M_2\|\Delta A_2\|_{H_0})t}\, t^{1/2}.$$

It follows that for $s \in \mathbb{C}$ with Re s sufficiently big, the Laplace transform

$\tilde{\phi}_u(t)$ of $\phi_u(t)$ is well defined.

Now, one can define the composition property (23) with input u for any $t, \tau \geq 0$ as follows:

$$\phi_u(t+\tau) = \mathbb{S}_t \phi_u(\tau) + \phi_u(t) . \tag{32}$$

Applying the Laplace transform with respect to t on both sides of(31), we obtain

$e^{s\tau}\tilde{\phi}_u(s) - e^{s\tau}\int_0^{\tau} e^{-st}\phi_u(t)\,dt = (sI-(\mathbb{A}+\Delta\mathbb{A}))^{-1}\phi_u(\tau)+\tilde{\phi}_u(s)$. assuming $\tau > 0$, we have:

$$\frac{e^{s\tau}-1}{\tau}\tilde{\phi}_u(s) = \frac{e^{s\tau}}{\tau}\int_0^{\tau} e^{-st}\phi_u(t)\,dt + (sI-(\mathbb{A}+\Delta\mathbb{A}))^{-1}\frac{\phi_u(\tau)}{\tau} .$$

Taking the limit for $\tau \longrightarrow 0$ (with respect to the norm on $\{H_0, \tau\}$) and using the continuity of ϕ_u and the fact that $\phi_u(0) = 0$ (see Remark (3.16)) and using Lohopital rule, we get:

$$s\tilde{\phi}_u(s) = \lim_{\tau\to 0}(sI-(\mathbb{A}+\Delta\mathbb{A})_{-1})^{-1}\frac{\phi_u(\tau)}{\tau} . \tag{33}$$

From concluding remark (3.16), we have $(sI-(\mathbb{A}+\Delta\mathbb{A}))^{-1}$ is isomorphism from $\{H_{-1}, \tau_{-1}\}$ into $\{H_0, \tau\}$, it follows that:

$$\|sI-(\mathbb{A}+\Delta\mathbb{A})_{-1})^{-1}\lim_{\tau\to 0}\frac{\phi_u(\tau)}{\tau}\|_{H_0} = \|\lim_{\tau\to 0}\frac{\phi_u(\tau)}{\tau}\|_{H_{-1}} .$$

Hence, the limit

$$(B+\Delta B)u = \lim_{\tau\to 0}\frac{\phi_u(\tau)}{\tau} , \tag{34}$$

exists and defined a linear operator from U to H_{-1}.

We can rewrite (33) as the follows:

$$\tilde{\phi}_s u = \frac{1}{s}(sI-(\mathbb{A}+\Delta\mathbb{A})_{-1})^{-1}(B+\Delta B)u , \tag{35}$$

which shows that $\frac{1}{s}(sI-(\mathbb{A}+\Delta\mathbb{A})_{-1})^{-1}$ being isomporphism so that,

$B+\Delta B \in L(U, H_{-1})$.

The Laplace inverse transform of (35) is

$$\phi_u(t) = \int_0^t \mathbb{S}_{-1}(\sigma)(B+\Delta B)u(\sigma)\,d\sigma . \tag{36}$$

Hence the uniqueness of the operator $B+\Delta B$ for which (28) holds is Obvious from (34).

Let $u = u \underset{\tau}{<>} 0$.We have for $t \geq \tau$ and by using (23), we get:

$$\phi_t u = \mathbb{S}(t-\tau)\phi_\tau u + \phi_{t-\tau}.0 = \int_0^t \mathbb{S}_{-1}(t-\sigma)(B + \Delta B)u(\sigma)\, d\sigma . \qquad (37)$$

Let us prove that Z(.) given by (29) is the continuous state strong solution of (30) with $Z(0) = Z_0$. Now let g(s) be H_{-1} valued function defines as:

$$g(s) = \mathbb{S}_{-1}(t-s)Z(s) ,$$

is differentiable for $0 < s < t$, then:

$$g'(s) = -(\mathbb{A} + \Delta\mathbb{A})\mathbb{S}_{-1}(t-s)Z(s) + \mathbb{S}_{-1}(t-s)\dot{Z}(s)$$

$$= -(\mathbb{A} + \Delta\mathbb{A})\,\mathbb{S}_{-1}(t-s)Z(s) + \mathbb{S}_{-1}(t-s)[(\mathbb{A} + \Delta\mathbb{A})Z(t) + (B + \Delta B)u(t)] . \qquad (38)$$

From (38), we have

$$\int_0^t g'(s)\, ds = -\int_0^t (\mathbb{A} + \Delta\mathbb{A})\mathbb{S}_{-1}(t-s)Z(s)\, ds + \int_0^t \mathbb{S}_{-1}(t-s)[(\mathbb{A} + \Delta\mathbb{A})Z(s) + (B + \Delta B)u(s)]\, ds ,$$

and

$$Z(t) = \mathbb{S}_{-1}(t)Z_0 + \int_0^t \mathbb{S}_{-1}(t-s)(B + \Delta B)u(s)\, ds . \qquad (39)$$

Using:

$$\mathbb{S}_{-1}(t)Z_0 - Z_0 = \int_0^t (\mathbb{A} + \Delta\mathbb{A})\,\mathbb{S}_{-1}(s)Z_0\, ds , \qquad (40)$$

to reduce the following integral equation, $Z \in L(H)$,

$$Z(t) - Z(0) = \int_0^t (\mathbb{A} + \Delta\mathbb{A})\, Z(s)\, ds + \int_0^t (B + \Delta B)u(s)\, ds, \qquad (41)$$

to another integral equation to satisfy the condition in definition (3.19).

$$Z(t) - Z(0) - \int_0^t (\mathbb{A} + \Delta\mathbb{A})\,\mathbb{S}_{-1}(t)Z_0\, ds = \int_0^t (\mathbb{A} + \Delta\mathbb{A})Z(s)\, ds$$

$$+ \int_0^t (B + \Delta B)u(s)\, ds - \int_0^t (\mathbb{A} + \Delta\mathbb{A})\int_0^t \mathbb{S}_{-1}(s)Z_0\, ds .$$

From (40), we have:

$$Z(t) - \mathbb{S}_{-1}(t)Z_0 = \int_0^t (B + \Delta B)u(s)\, ds + \int_0^t (\mathbb{A} + \Delta\mathbb{A})[Z(s) - \mathbb{S}_{-1}(s)Z_0]$$

$$\int_0^t \mathbb{S}_{-1}(t-s)(B + \Delta B)\, u(s)\, ds = \int_0^t (B + \Delta B)u(s)\, ds + \int_0^t (\mathbb{A} + \Delta\mathbb{A}))\int_0^t \mathbb{S}_{-1}(t - s)(B + \Delta B)u(s)\, ds \ .$$

We obtain that:

$$\phi_t u = \int_0^t (\mathbb{A} + \Delta\mathbb{A})\phi_s u\, ds + \int_0^t (B + \Delta B)u(s)\, ds \ .$$

Then from proposition (3.17) it follows that $Z(.) \in C([0, \infty),\{ H_{-1},\tau\})$.

To prove the uniqueness of the strong continuous solution Z(.) in (29), it is clear that any strong continuous solution in H_0 is in H_{-1}.

Let $\{Z_{n,0}\}$ be a sequence in $\{H_0,\tau\}$ converges to Z_0, thus:

$$\{Z_n \mid Z_n = \mathbb{S}_{-1}(t)Z_{n,0} + \int_0^t \mathbb{S}(t - \sigma)(B + \Delta B)u(\sigma)\, d\sigma\} \quad (42)$$

be a sequence in $\{H_{-1},\tau\}$.

Since H_{-2} is a completion space of Banach space $H_{-1.}$ Therefore the sequence in (42) converges to:

$$Z(t) = \mathbb{S}_{-1}(t)Z_0 + \int_0^t \mathbb{S}_{-1}(t - \sigma)(B + \Delta B)u(\sigma)\, d\sigma \quad \text{in } \{H_{-2},\tau_{-2}\},.$$

Hence, from property uniqueness of the limit, we obtain the strong continuous solution is unique.

Corollary(3.24)

Consider problem formulation , let U and H are Hilbert spaces and

$\tilde{\Omega} = L^2_{loc}([0, \infty), U)$.Let B be an admissible perturbed control operator such that $B + \Delta B \in L(U, \{H_{-1}, \tau_{-1}\})$. Then for any $Z_0 \in H_0$ and $u \in \tilde{\Omega}$, the function

$$Z(t) = \mathbb{S}_{-1}(t)Z_0 + \phi_t u \ , \quad (43)$$

is the (unique) continuous state strong solution of the differential equation:

$$\dot{Z}(t) = (\mathbb{A} + \Delta\mathbb{A})Z(t) + (B + \Delta B)u(t),\ Z(0) = Z_0 . \quad (44)$$

The proof is the same as above and is omitted for simplicity.

Proposition(3.25)

Let U, H are Hilbert spaces, $\{\mathbb{S}(t)\}_{t\geq 0}$ is a composite perturbation semigroup on H_0 and $\Omega = L^2_{loc}([0, \infty), U)$. Let $B + \Delta B \in L(U, H_{-1})$ and let ϕ_τ be given by (28). If for some fixed $\tau > 0$ and any $u \in \Omega$, $\phi_\tau u \in H$, then $(B + \Delta B)$ is admissible.

Proof

Let $(B + \Delta B)_0 = R(\lambda: \mathbb{A} + \Delta\mathbb{A})(B + \Delta B)$, for $\lambda \in \rho(\mathbb{A} + \Delta\mathbb{A})$.

Then $(B + \Delta B)_0 \in L(U, H_0)$, and we have:

$$\phi_T u = (\lambda I - (\mathbb{A} + \Delta\mathbb{A}))\int_0^T \mathbb{S}(t - r)(B + \Delta B)_0 u(r)\, dr .$$

By closed graph theorem , ϕ_T is bounded. Then from (26), yield:

$$\|\phi_t\| \leq \|\phi_T\|,\ \forall\ t \leq T ,$$

and from remark (3.3), we get if ϕ_t is bounded for some t, it is bounded for all multiples of t.Thus ϕ_t is bounded for all $t \geq 0$.

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Journal of Al-Qadisiyah for Computer Science and Mathematics
Vol. 3 No.1 Year 2011

قابلية الحل و المقبولية لمعادلات المؤثر الخطى التفاضلي القلق غير المقيد

ذات مؤثر سيطرة غير مقيد باستخدام مفهوم الزمرة المركبة القلق

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الخلاصة

تم أفتراض نضام ديناميكي خطي قلق مع مؤثر سيطرة غير مقيد والمتولد بواسطة مولد غير مقيد .ولقد اعطية وبرهنة قابلية الحل لهذا الصف من المعادلات مع قابلية المقبولية وأيضا بعض الخواص الديناميكية لها.فتم الحصول على الحل تماما باستخدام مفهوم الزمرة المركبة القلق .