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Adjacency and Distance Characteristic Soft Topology on Soft Graphs: Properties and Fundamental Results

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ABSTRACT

In this paper, we study the Adjacency and Distance Characteristic Soft Topology on soft graphs. This topology is constructed by combining adjacency and distance relations between vertices within the framework of soft sets. We reformulate the notions of closure and interior in a manner consistent with classical topology and investigate their behavior in the soft graph setting. It is shown that both operators are completely determined by adjacency relations. Several fundamental properties are established and supported by illustrative examples. In addition, the connectedness of the proposed topology is examined, highlighting the relationship between graph structure and topological properties.

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1. Introduction

Graph theory is one of the fundamental areas of mathematics that studies structures consisting of vertices and edges and plays an important role in modeling relationships in various applications. Since the pioneering work of Euler on the Königsberg bridge problem, graph theory has developed into a rich and widely used field [1].

Soft set theory, introduced by Molodtsov in 1999 [2], provides a flexible mathematical framework for dealing with uncertainty through parameterization. This theory has been further developed through various operations and extensions [3], and it has been successfully applied in different fields, particularly in decision-making problems [4,5]. The concept of soft graphs was introduced to extend classical graph theory within the framework of soft sets,

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allowing the study of parameterized graph structures [6]. Further developments in soft graph theory, including operations on soft graphs and connected soft graphs, have been investigated in [7,8].

In parallel, soft topological spaces have been studied extensively as a natural extension of classical topology to the soft set framework [9,10]. Several important properties such as separation axioms, continuity, and connectedness have been explored in soft topology [11–13].

Moreover, the interaction between graph theory and topology has led to the development of new types of topological structures based on graphs, as discussed in [14]. These studies highlight the importance of combining graph relations with topological concepts. Motivated by these developments, the aim of this paper is to construct and study a new soft topological structure based on adjacency and distance relations in soft graphs.

In particular, we introduce the Adjacency and Distance Characteristic Soft Topology and investigate its fundamental properties. The notions of closure and interior are reformulated in a manner consistent with classical topology while preserving their interpretation in terms of adjacency relations. Several properties are established, showing that both closure and interior operators are determined by adjacency relations in the underlying soft graph. In addition, illustrative examples are provided to clarify the results. The basic construction of the proposed topology is adapted from our Ph.D. thesis [15].

The paper is organized as follows. Section 2 presents the necessary preliminaries on soft sets, soft graphs, and soft topology. Section 3 introduces the proposed topology and studies its main properties. Finally, concluding remarks and possible directions for future work are presented.

2. Preliminaries

We recall in this section the basic concepts from graph theory and soft set theory that are required for the development of our results. All definitions are presented in a simplified and consistent manner for clarity.

Definition 2.1. [1] A graph $\mathbb{G}_g = (\mathbb{V}_g, \mathbb{E}_g)$ consists of a set of vertices \mathbb{V}_g and a set of edges \mathbb{E}_g , where each edge connects two vertices in \mathbb{V}_g . Two vertices u^g and v^g are said to be *adjacent* if $\{u^g, v^g\} \in \mathbb{E}_g$.

Definition 2.2. [2] Let \mathcal{X} be an initial universe, and let Σ be a universal set of parameters. Suppose that $\mathbb{A} \subseteq \Sigma$ is a set of parameters, where $\mathfrak{a} \in \mathbb{A}$ denotes a parameter. A soft set $\mathcal{F}_{\mathbb{A}}$ over \mathcal{X} is defined as a mapping $\mathcal{F}: \mathbb{A} \rightarrow \mathcal{P}(\mathcal{X})$, where $\mathcal{P}(\mathcal{X})$ denotes the power set of \mathcal{X} . For each parameter $\mathfrak{a} \in \mathbb{A}$, the set $\mathcal{F}(\mathfrak{a}) \subseteq \mathcal{X}$ represents the elements of \mathcal{X} corresponding to the parameter \mathfrak{a} . Thus, a soft set $\mathcal{F}_{\mathbb{A}}$ can be viewed as a parameterized family of subsets \mathcal{X} .

Definition 2.3. [4] Let \mathcal{F} and \mathcal{K} be two soft sets over \mathcal{X} :

- $\mathcal{F} \subseteq \mathcal{K}$ if $\mathcal{F}(\mathfrak{a}) \subseteq \mathcal{K}(\mathfrak{a})$ for all $\mathfrak{a} \in \mathbb{A}$
- $(\mathcal{F} \cup \mathcal{K})(\mathfrak{a}) = \mathcal{F}(\mathfrak{a}) \cup \mathcal{K}(\mathfrak{a})$
- $(\mathcal{F} \cap \mathcal{K})(\mathfrak{a}) = \mathcal{F}(\mathfrak{a}) \cap \mathcal{K}(\mathfrak{a})$

Definition 2.4. [6] Let $\mathbb{G}_g = (\mathbb{V}_g, \mathbb{E}_g)$ be a graph and \mathbb{A} a parameter set. A soft graph is defined as a mapping $\mathcal{F}: \mathbb{A} \rightarrow \mathcal{P}(\mathbb{V})$ such that each $\mathcal{F}(\mathfrak{a})$ induces a subgraph of \mathbb{G}_g .

Definition 2.5. [9] Let \mathcal{X} be a set and \mathbb{A} a set of parameters. A collection τ of soft sets over \mathcal{X} is called a soft topology if:

1. \emptyset and \mathcal{X} belong to \mathcal{T}
2. The union of any family of soft sets in τ belongs to \mathcal{T}
3. The intersection of any two soft sets in τ belongs to \mathcal{T}

Then $(\mathcal{X}, \mathcal{T}, \mathbb{A})$ is called a soft topological space.

Definition 2.6. [9] Let $\mathcal{F}_{\mathbb{A}}$ be a soft set:

- $Cl(\mathcal{F}_{\mathbb{A}})$: smallest closed soft set containing \mathcal{F}

$$Cl(\mathcal{F}_{\mathbb{A}}) = \bigcap \{ \mathcal{H}_{\mathbb{A}} : \mathcal{H}_{\mathbb{A}} \text{ is closed and } \mathcal{F}_{\mathbb{A}} \subseteq \mathcal{H}_{\mathbb{A}} \}$$

- $Int(\mathcal{F}_{\mathbb{A}})$: largest open soft set contained in \mathcal{F}

$$Int(\mathcal{F}_{\mathbb{A}}) = \bigcup \{ \mathcal{H}_{\mathbb{A}} : \mathcal{H}_{\mathbb{A}} \text{ is open and } \mathcal{H}_{\mathbb{A}} \subseteq \mathcal{F}_{\mathbb{A}} \}$$

Definition 2.7. [10] [15] Let \mathbb{V} be the vertex set of a soft graph, and let Σ be a set of parameters associated with \mathbb{V} . Let $\mathbb{A} \subseteq \Sigma$. A mapping $\mathcal{F} : \mathbb{V} \times \mathbb{A} \rightarrow \mathcal{P}(\mathbb{V})$ is called a soft graphical mapping. Then, the triple $(\mathcal{F}, \mathbb{V}, \mathbb{A})$ denoted by $\mathcal{F}_{\mathbb{V}, \mathbb{A}}$, is called a soft graphical soft set over \mathbb{V} .

That is, $\mathcal{F}_{\mathbb{V}, \mathbb{A}} = \{ ((v, e), \mathcal{F}(v, e)) : \mathcal{F}(v, e) \subseteq \mathbb{V}, e \in \mathbb{A} \}$

This definition assigns to each pair (v, e) a subset of vertices in V_s , representing the relationship between vertices under different parameters.

Definition 2.8. [11] Let $\mathcal{F}_{\mathbb{A}}$ and $\mathcal{H}_{\mathbb{B}}$ be a soft set, then **the union** it over the common universe \mathcal{X} is the soft set $\mathcal{H}_{\mathbb{C}}^f$, where $\mathbb{C} = \mathbb{A} \cup \mathbb{B}, \forall e \in \mathbb{C}$.

$$\mathcal{G}(e) = \begin{cases} \mathcal{F}(e) & \text{if } e^{s\sigma} \in \mathbb{A} - \mathbb{B} \\ \mathcal{H}(e) & \text{if } e^{s\sigma} \in \mathbb{B} - \mathbb{A} \\ \mathcal{F}(e) \cup \mathcal{H}(e) & \text{if } e \in \mathbb{A} \cap \mathbb{B} \end{cases}$$

We write $\mathcal{F}_{\mathbb{A}} \cup \mathcal{H}_{\mathbb{B}} = \mathcal{G}_{\mathbb{C}}$.

Definition 2.9. [11] Let $\mathcal{F}_{\mathbb{A}}$ and $\mathcal{H}_{\mathbb{B}}$ represent a soft set so **the intersection** of them over a common universe \mathcal{X} is defined as $\mathbb{C} = \mathbb{A} \cap \mathbb{B}, \mathcal{G}(e) = \mathcal{F}(e) \cap \mathcal{H}(e)$ and $\forall e \in \mathbb{C}$. And it symbolizes by $\mathcal{F}_{\mathbb{A}} \cap \mathcal{H}_{\mathbb{B}} = \mathcal{G}_{\mathbb{C}}$.

3. Adjacency and Distance Characteristic Soft Topology on Soft Graphs

This section presents the construction of the proposed topology and its fundamental results.

Definition 3.1. [15] Let $\mathbb{G}_s = (\mathbb{G}_g, \mathcal{F}, \mathbb{A})$ be a soft graph, where: $\mathbb{G}_g = (\mathbb{V}_g, \mathbb{E}_g)$ is a simple graph and \mathbb{A} is a set of parameters.

The **Adjacency and Distance Characteristic Soft Topology on Soft Graph**, denoted by $\mathcal{AD} - \mathcal{ST}_g$ and written as $\mathcal{ST}_g(\mathbb{G}_s)$, is defined as a soft topology generated by a sub-basis \mathcal{SS}_g .

whose elements are soft sets of the form $\mathcal{F}_{\mathbb{V}, \mathbb{A}_i}$, where $\mathbb{A}_i \in \{\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{D}\}$.

The parameter sets are defined as follows:

$$\begin{aligned} \mathbb{A} &= \{e_1 : \text{adjacency}, e_2 : \text{path of length 3}\} \\ \mathbb{B} &= \{e_1 : \text{non-adjacency}, e_2 : \text{path of length 2}\} \\ \mathbb{C} &= \{e_1 : d(v, u) \leq n - 1, e_2 : \text{adjacency}\} \\ \mathbb{D} &= \{e_1 : \text{adjacency}, e_2 : d(v, u) \leq n - 1\} \text{ for all } v, u \in \mathbb{V}(\mathbb{G}_s). \end{aligned}$$

The sub-basis \mathcal{SS}_g consists of all such soft sets generated by these parameter families. Finite intersections of elements of \mathcal{SS}_g form a basis, and arbitrary unions generate the topology $\mathcal{ST}_g(\mathbb{G}_s)$.

This construction shows that the proposed soft topology is generated from graph relations, particularly adjacency and distance, through parameterized soft sets. These relations play a fundamental role in determining the structure of the topology.

Construction of the Topology 3.2. The sub-basis $s\mathcal{S}_g$ generates a basis through finite intersections, and the topology $\mathcal{ST}_g(\mathbb{G}_s)$ is obtained by arbitrary unions of these basis elements.

Example 3.3. Let \mathbb{G}_s be a cyclic soft graph $s\mathbb{C}_3$ with vertex set $\mathbb{V}(\mathbb{G}_s) = \{v_1, v_2, v_3\}$, as shown in the Fig (1): -

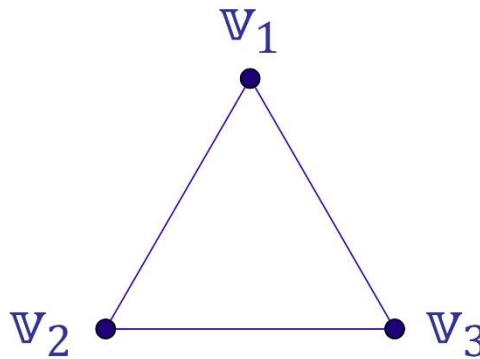


Figure 1. cyclic soft graph $s\mathbb{C}_3$

Let \mathbb{A} be a set of parameters representing adjacency and distance relations.

The mapping \mathcal{F} assigns to each pair (v, e) a subset of vertices depending on the relation.

For example:

$$\mathcal{F}(v_1, e_1) = \{v_2, v_3\}, \mathcal{F}(v_1, e_2) = \{v_1\}.$$

Similarly, soft sets are defined for other vertices.

The collection

$$s\mathcal{S}_g = \{\mathcal{F}_{v,\mathbb{A}}, \mathcal{F}_{v,\mathbb{B}}, \mathcal{F}_{v,\mathbb{C}}, \mathcal{F}_{v,\mathbb{D}}\}$$
 forms the sub-basis of the topology.

Finite intersections of elements of $s\mathcal{S}_g$ generate a basis, and arbitrary unions generate the soft topology $\mathcal{ST}_g(\mathbb{G}_s)$.

This example illustrates how the proposed topology is constructed from adjacency and distance relations.

Definition 3.4. [10] Let $(\mathbb{V}, \mathcal{ST}_g(\mathbb{G}_s))$ be the adjacency and distance characteristic soft topological space, and let \mathcal{F} be a soft set over \mathbb{V} .

The **closure** of $\mathcal{F}_\mathbb{A}$, denoted by $Cl(\mathcal{F}_\mathbb{A})$, is defined as:

$$Cl(\mathcal{F}_\mathbb{A}) = \bigcap \{ \mathcal{H}_\mathbb{A} : \mathcal{H}_\mathbb{A} \text{ is closed and } \mathcal{F}_\mathbb{A} \subseteq \mathcal{H}_\mathbb{A} \}$$

The closure of a soft set is the smallest closed soft set containing it.

Definition 3.5. [10] Let $\mathcal{F}_\mathbb{A}$ be a soft set in $(\mathbb{V}, \mathcal{ST}_g(\mathbb{G}_s))$.

The **interior** of $\mathcal{F}_\mathbb{A}$, denoted by $Int(\mathcal{F}_\mathbb{A})$, is defined as:

$$Int(\mathcal{F}_\mathbb{A}) = \bigcup \{ \mathcal{H}_\mathbb{A} : \mathcal{H}_\mathbb{A} \text{ is open and } \mathcal{H}_\mathbb{A} \subseteq \mathcal{F}_\mathbb{A} \}$$

The interior of a soft set is the largest open soft set contained in it.

Remark 3.6. The definitions of closure and interior are consistent with the classical topological notions, adapted to the framework of soft sets on soft graphs.

The following results are derived within the framework of the proposed topology.

Proposition 3.7. In the topology $\mathcal{ST}_g(\mathbb{G}_s)$, the closure and interior of a soft set \mathcal{F}_A can be characterized in terms of adjacency relations as follows:

- A soft vertex $v \in \mathbb{V}$ belongs to the closure $Cl(\mathcal{F}_A)$ if and only if it is adjacent to at least one vertex in \mathcal{F}_A .
- A soft vertex $v \in \mathbb{V}$ belongs to the interior $Int(\mathcal{F}_A)$ if and only if its adjacency neighborhood is completely contained in \mathcal{F}_A .

Proof. The topology $\mathcal{ST}_g(\mathbb{G}_s)$ is generated from adjacency-based soft sets.

Thus, a vertex belongs to the closure if it is related by adjacency to elements of the set, and it belongs to the interior if its entire adjacency neighborhood is contained in the set.

Hence, the result follows directly from the definitions of closure and interior. ■

Proposition 3.8. For any soft set \mathcal{F}_A in $\mathcal{ST}_g(\mathbb{G}_s)$: $Int(\mathcal{F}_A) \subseteq \mathcal{F}_A \subseteq Cl(\mathcal{F}_A)$.

Proof. The result follows directly from the definitions of interior and closure. ■

Remark 3.9. This proposition shows that, although closure and interior are defined in the classical topological sense, they are completely determined by adjacency relations in the underlying soft graph.

Theorem 3.10. The closure and interior operators in $\mathcal{ST}_g(\mathbb{G}_s)$ are completely determined by the adjacency relations of the soft graph.

Example 3.11. Let $\mathbb{G}_s = (\mathbb{V}, \mathbb{E})$ be a soft graph with vertex set $\mathbb{V}(\mathbb{G}_s) = \{v_1, v_2, v_3, v_4, v_5\}$, as shown in Fig(2).

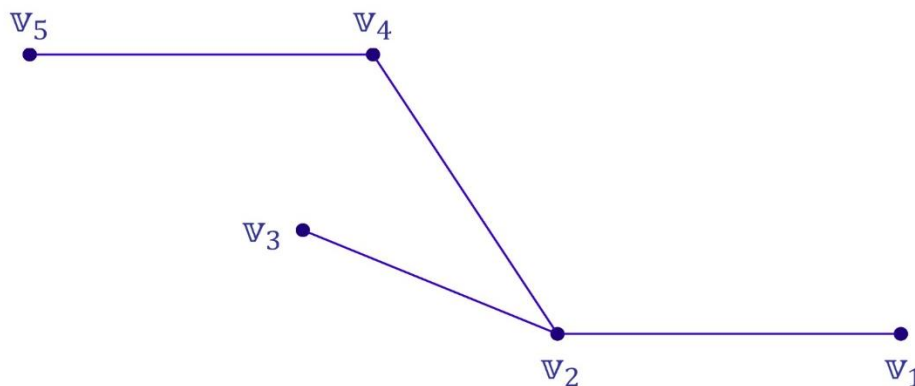


Figure 2. $\mathbb{G}_s = (\mathbb{V}, \mathbb{E})$ soft graph

Let $\mathcal{M} \subseteq \mathbb{V}$ be a soft subset defined by: $\mathbb{V}(\mathcal{M}) = \{\mathbb{v}_2, \mathbb{v}_3, \mathbb{v}_4\}$, as shown in Fig(3).

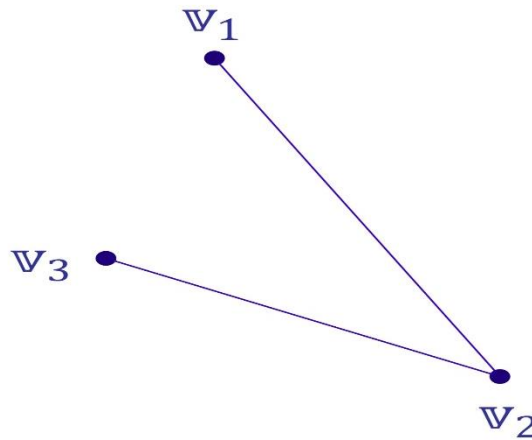


Figure 3. $\mathcal{M} = (\mathbb{V}, \mathbb{E})$ soft subgraph

Interior

The adjacency neighborhoods of \mathbb{v}_2 and \mathbb{v}_3 are contained in \mathcal{M} .

Hence,

$$Int(\mathcal{F}_A) = \{\mathbb{v}_2, \mathbb{v}_3\}$$

Closure

The vertices \mathbb{v}_1 and \mathbb{v}_2 are adjacent to vertices in \mathcal{M} .

Thus,

$$Cl(\mathcal{F}_A) = \{\mathbb{v}_1, \mathbb{v}_2, \mathbb{v}_3, \mathbb{v}_4\}$$

Interpretation

This example shows that:

- the interior depends on full containment of adjacency neighborhoods,
- the closure includes all vertices adjacent to the set.

Hence, both operations are determined by adjacency relations in the soft graph.

Proposition 3.12. The topology $\mathcal{ST}_g(\mathbb{G}_s)$ is connected.

Proof. The topology is generated by adjacency relations, which prevent separation into disjoint nonempty open soft sets. ■

This result highlights the strong connection between graph structure and topological behavior.

4. Conclusion

In this paper, we studied the Adjacency and Distance Characteristic Soft Topology on soft graphs. This topology is constructed based on adjacency and distance relations between vertices within the framework of soft sets.

We reformulated the notions of closure and interior so that they are consistent with the classical topological definitions, while still reflecting the role of adjacency in soft graphs. It was shown that both closure and interior are determined by adjacency relations in the underlying soft graph.

Some fundamental properties of these operators were established and supported by examples. The results indicate that adjacency plays a central role in defining the structure of the proposed topology.

The approach presented in this paper provides a natural way to connect graph structures with soft topological concepts. Further work may consider extending these results to other types of soft topologies and exploring additional applications.

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