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Topological Approximation Spaces Via Finite Groups

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A R T I C L E I N F O

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A B S T R A C T

In this paper introduced, investigates the relation between finite group and topological space.We presented a new operator topology approximation space via finite groups $(U, * R, \tau *)$ and new class of approximations using the equivalence relation and interior, closure sets. We reached some theories and results related to the new approximation sets and Our research extends to the measure of membership function in topological approximation space.

MSC..

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1. Introduction

A topology is a branch of mathematics that was founded at the beginning of the twentieth century and began to develop . It can be said for the sake of simplicity that this science has mathematical properties that are exploited when developing from one placeto another. Nowadays, the general topology Arkhangl and Adamson(1966)[3] Its principles are among the most influential concepts across various scientific fields. For instance, in structural analysis. Z. Pawlak introduced theory of approximation sets in(1982)[6], as a mathematical technique to deal with the problems of ambiguity and uncertainty .In 1991,Pawlak presented the theoretical aspects of inference about data.Pawlak and Skowron (1994)[10] also presented some important aspects of rough set research and several applications of it .In 1996[11], researchers also introduced rough sets resulting from the equivalence relation, coverings, subsystems, arbitrary binary relations, the relationship between rough set theory and discrete dynamical systems, and the analysis of formal concepts In chemistry. In 1998, some scientists presented constructive and algebraic methods for approximate groups, and many studies were conducted on this subject, which made the connection between Boolean algebra and rough groups an important matter in mathematical operations and in practical applications. As an effective mathematical tool for approximating modeling issues in data analysis, computer science, and medicine. El-Bably (2008), and Mitra (2010), James (2014)[15], Al-hawez (2015)[16],(2014, 2016), Abo Khadra and Zhao (2016) and Zhaowen and Azzam et al.. El- Bably and Fleifel (2017), Tripathy,.Next, Pawlak introduced the idea of rough equality, which is a method for defining rough sets and can be thought of as a way to deal with vagueness.Pomykala introduced intersection and union operations on Pawlak's rough sets in 1988. Researchers looked at topological spaces on rough sets.Additionally, he showed both upper and lower

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approximations.It allowed for the combination of topology and algebra with rough sets in mathematics.Pawlak has suggested other classes of granular computing blocks formed from the intersection (union) of the right or left neighbors in order to broaden the applicability of this approach. This is a revolution in the theory of rough sets with respect to any bilateral relationship in terms of any binary relationship and allows for the investigation of numerous varieties of identical neighborhood systems based on the equivalency relation. In this research , an approximation space generated from finite groups is presented.by converting the binary operation into an equivalence relationship. and deducing and studying the properties of this space and deducing special theories for it , we also showed the concepts of border regions and the accuracy scale through upper and lower approximation.

2. PRELIMINARIES

2.1 DEFINITION OF GROUP AND SOME FACTS

A **group** (G,∗) is defined as a set G equipped with a binary operation (∗) that Satisfies the following four requirements:

- $a * b \in G$, for all $a, b \in G$. (closed)
- $(a * b) * c = a * (b * c)$ for all $a, b, c \in G$. (associative)
- $\exists\ e \in G$ then $e * a = a * e = a$ for all $a \in G$. (identity)
- for all $a \in G, \exists\ a^{-1} \in G \Rightarrow a * a^{-1} = a^{-1} * a = e$. (inverse)

If G is finite set then $(G,*)$ is finite group.

<u>Definition</u> 2.1.1 [13][17]

Cummutative group: Let $(G,*)$ is finite group is called cummutative group Iff $x * y = y * x$ for all $x, y \in G$

<u>Definition</u> 2.1.2 [13]

Let $(U,*)$ be a group and $x \in U$ the syclic group of U produced by the x is represented by $< x >$ such that

$< x >= x^k : k \in Z$, $u =< x >$ is called cyclic group.

<u>Definition</u> 2.1.3 [13]

The center of a group (G, ∗) is defined as:center

$G = \{a \in G : a * x = x * a$ f or all $x \in G\}$ the group center is represented by the symbol C(G)

<u>Definition</u> 2.1.4 [13]

(dihedral group) The dihedral group is the group of symmetries of the regular

$o(D_n) = 2n$, n-sided polygon, Which includes both rotations and reflections.The Dihedral group D_n cousist of 2n elements.

<u>Example 2.1.1</u>

Let D_3 dihedral group, $D_3 = \{f_1, f_2, f_3, f_4, f_5, f_6\}$. $O(D_3) = (2)(3) = 6$ element.

Definition 2.1.5 [13]

quaternion group is a symmetric group Q8. generated by the two element a, b, such that $ba = a^3b$, $a^2 = b^2$. $O(a) = 4$

Example 2.1.2 [13]

Let Q8 quaternin group $Q = \{1, -1, i, -i, j, -j, k, -k\}$, $i^2 = -1$, $j^2 = -1$, $k^2 = -1$ syclic group non commutative group.

2.2. BASIC OF OPEN SET, CLOSED SET AND TOPOLOGICAL SPACE

Definition 2.2.1[2] [11][14]

Let U be a nonempty set of objects, and let τ be a family of subsets of U. The pair (U,τ) is called a topological space if the following conditions are satisfied:

1. The empty set and the universal set are in τ**:** $\emptyset, U \in \tau$.
2. Closure under arbitrary unions**:** The union of any collection of sets in τ is also in τ.
3. Closure under finite intersections:
4. The intersection of any finite number of sets in τ is also in τ.

A subset A of U that belongs to τ is called an open set in (U,τ). The complement of A, denoted A^c, is called a closed set.
As an example of a topological space, consider the topology, $\tau = \{\emptyset, R\} \cup \{(x,\infty) : x \in R\}$ defined on the set of real numbers R. Note that the interval $(0,1) \sqsubseteq R$ not an open set in this topology, while $(0,\infty)$ **is** an open set.

Remark 2.2.1 [8]

1) The topological space (U, τ) is sometime called the space on U
2) the elements of U are called points of the space.
3) Write τ is meaning topology , and $(U, \tau))$ is meaning topological space

Definition 2.2.2 [2] [4][7][11]

Let $(U, \tau))$ is a topological space,and the letter X° is called the interior of

X is defined by: $X^\circ = \cup \{H \subseteq U, H \text{ is open set and } H \subseteq X\}$

Definition 2.2.3 [2][7][11]

Let $(U, \tau))$ *is a topological space,*

and the letter X° is called the interior of X is defined by:

$\overline{X} = \cap \{B_i : B_i \ is\ closed\ setfor\ all\ X \subseteq B_i\}$

Remark 2.2.2 [5][16]

For a topological space (U, τ) and $X \subseteq U$ is called

1) X is an open iff $X = X^{\circ}$

2) X is an close iff $X = \overline{X}$

3)X is open and close (clopen)

4)X is open and not close

5)X is not open and close

6)X is close and not open

Definition 2.2.4 [5]

Let (X, τ) be a topological space, and let BB be a subfamily of τ.

We say that B is a **basis for τ** if every element in τ can be expressed as a union

of (possibly infinitely many) elements of B

Definition 2.2.5 [15]

Let X and Y are two sets and P = X × Y is the Cartesian product of X and Y. Any subset R C P is called a relation from X to Y . If X ≡ Y , then any subset of X × X is called a binary relation on X. If a pair (a, b) ∈ R, then we can write aRb or a ∼ b. The relation R is called equvlinc relation if the folwing conditions are met :

1. reflexive iff for each a ∈ X, a Ra.
2. symmetric iff for any pair a, b ∈ X, in case aRb then b Ra.
3. transitive iff for any triple a, b, c ∈ X, in case a R b and b R c then a R c

For each a∈X, the **equivalence class** of a is defined as

$[a]$={y ∈ X :y R a . To avoid ambiguity, the notation

$[a]R$ is sometimes used instead of $[a]$.

2.3 KEY ELEMENT OF APPROXIMATION SPACE

Definition 2.3.1[6]

"Let U nonempty set R is binary relation the pair (U, R) is called approximation spacep. such that , $R \subseteq U \times U$.

For any subset X of U, the pair lower and upper approximation. $(\underline{R}(X), \overline{R}(X))$ are defined by:

$\underline{R}(X) = \{x \in X: [x]R \subseteq X\}$

Lower approximation

$\overline{R}(X) = \{x \in X: [x]R \cap X \neq \emptyset\}$

Upper approximation

$BN = \overline{R}(X) - \underline{R}(X)$

Bounder region

$BOS(X) = \underline{R}(X)$

Positive region

$NAG(X) = U - \overline{R}(X)$

Negative region

The accuracy measure is given by the following formula:

$\alpha_{R(X)} = \frac{|\underline{R}(X)|}{|\overline{R}(X)|}$, where $0 \leq \alpha_{R(X)} \leq 1$ and |.| denotes the cardinally of X."

Proposition 2.3.1 **[9][10]**

"Let (X, R) be an approximation space and let S, G ⊂ X. Then, the approximation operators satisfy the following properties :

1) $\underline{R}(S) \subseteq S \subseteq \bar{R}(S)$

2) $\underline{R}(\phi) = \bar{R}\,\phi = \phi$

3) $(\underline{R}(S)) = \bar{R}\,(\bar{R}\,(S^c))^c$

4) $(R(S)) = (\underline{R}(S^c))^c$

5) $\bar{R}\,(S \cap G) \subseteq \bar{R}\,(S) \cap \bar{R}\,(G)$

6) $\underline{R}(S \cap G) = \underline{R}(S) \cap \underline{R}(G)$

7) $\underline{R}(S) \cup \underline{R}(G) \subseteq \underline{R}(S \cup G$

8) $\bar{R}\,(S) \cup \bar{R}\,G) = \bar{R}\,(S \cup G)$

9) $S \subseteq G$ if only if $\bar{R}\,(S) \subseteq \bar{R}\,(G)$

10) $S \subseteq G$ if only if $\underline{R}(S) \subseteq \underline{R}(G)$

11) $(\bar{R}\,(S))^c = \underline{R}(S^c)$

12) $\underline{R}(X) = \bar{R}(X) = X$"

Definition 2.3.2 [12][9]

Let R denote an equivalence relation on the universal set U. The pair (U,R) is referred to as an **approximation space**.

The set $X \subseteq U$ can be defined, if $\underline{R}(X) \neq \overline{R}(X)$ then X is a rough set, otherwise exact set.

2.4 ROUGH MEMBERSHIP FUNCTION

Definition 2.4.1[10]

Let (U,R) is an approximation space and $X \subseteq U$ with R equivalence relation, and $[x]_R$ we denote an equivalence class of relation R by the object x. the rough membership function is defined by

$\mu_X^R: U \to [0,1]$:

$$\mu_X^R = \frac{|[x] \cap X|}{|[x]|}$$

Proposition 2.4.1.[10]

" Let produced the approximation space (U, R) by equivalence relation R and $X \subseteq U$, for all $x \in U$,

Then the membership function satisfy The\ following properties.

$If\ \mu_X^R = 1,\ then\ x \in \underline{R}(X)$

$If\ \mu_X^R > 0,\ then\ x \in \bar{R}(X)$

$If\ \mu_X^R = 0,\ then\ x \in U - \bar{R}(X)$

$If\ 0 < \mu_X^R < 1,\ then\ x \in BN$"

3. APPROXIMATION SPACE INDUCE BY DIHEDRAL GROUP

Definition 3.2

Let U be a finite set equipped with (*) a binary operation on U making a group $G = (U,*)$.

The subset $*R \subseteq U \times U$ defined a binary relation on U as follow,

$*R = \{(u_1, u_2) \in U \times U: u_1 * g = g * u_2 \text{ for some } g \in G\}$. In the relation symbol $*R$ indicate

to the binary operation of the group G. Then, $(G,*R)$ called approximation space induce by

finite group.

Theorem 3.1

If (G,∗)be afinite group, and $*\,R = \{(g_1, g_2) \in G \times G : \exists x \in G \ni g_1x = xg_2$(Conjugacy relation)$\}$,

Then $*\,R$ is an equivalence relation on G.

Proof :

1) Let $g \in G$, it is clear that $\exists e \in G$, $(g, g) \in * R$

since $ge = eg$ so , the relation is reflexive.

2)Let $g_1, g_2 \in G$ and $(g_1, g_2) \in * R$, so there is an element $x \in G$ such that

$g_1x = xg_2$.

To show that $(g_2, g_1) \in * R$

Let $y = x^{-1}$

Since $g_1x = xg_2 \to x^{-1}g_1x = g_2$

$\to x^{-1}g_1 = g_2x^{-1} \to yg_1 = g_2y \to g_2y = yg_1 \to (g_2, g_1) \in * R$.

Therefor *R is symmetric.

3) Let $g_1, g_2, g_3 \in G$

Such that $(g_1, g_2), (g_2, g_3) \in * R$

To proof $(g_1, g_3) \in * R$

Since:$(g_1, g_2) \in * R$, then there is one element $x \in G$ such that

$g_1x = xg_2$—(1)

and $(g_2, g_3) \in * R$, then there is one element $y \in G$ such that

$g_2y = yg_3$ —(2)

let $z = xy \in G$

From (1) we geat

$g_1xy = xg_2y \to g_1z = xg_2y$ —(3)

From(2) we get

$xg_2y = xyg_3 \rightarrow xg_2y = zg_3$—-(4)

From (3)and(4) we get

$g_1z = zg_3 \rightarrow (g_1, g_3) \in * R$. and all of ther prove $* R$ is equivalence relation.

Example 3.1

Let $U = D_3$, *be a Dihedral group of order 6.*

$U = \{f_1, f_2, f_{,3}, f_4, f_5, f_6\}$ is the underlying set of D_3.

We will use the following binary relation

$*\mathcal{R} = \{(a, b) \in U \times U \mid a = xbx^{-1} \text{ for some } x \in U\}$

By examining each pair in $U \times U$ we will see that

$*\mathcal{R} = \{(f1, f1), (f2, f2), (f2, f3), (f3, f2), (f3, f3), (f4, f4), (f4, f5),$

$(f4, f6), (f5, f4), (f5, f5), (f5, f6), (f6, f4), (f6, f5), (f6, f6)\}$

$*\mathcal{R}$ equivalence relation induce by theorem (3.1).

$(U, * R)$ is approximation space induce by finite group.

4. T OPOLOGICAL APPROXIMATION SPACE VIA FINITE GROUP

we will build topological space using a finite group. If G=(U,*) is a finite group, then

we can obtain apart ion for U as an equivalence classes

induce by some conjugacy relation.

we can get topology for U by consider as a basis for topology.

The base contains all the equivalence classes via some relation from

The approximation space (U,*R) induce by finite group, denoted by (U,* R, $\tau *$).

Definition 4.1. Let U non empty set G = (U, ∗) is a finite group, produce an approximation space (U, ∗R)by conjugacy relation ∗R. U is a partitioned by ∗R as U/ ∗ R be the family of equivalence classes which is a base of topology τ ∗. The resulting topology is called topological approximation space via finite groups, (U, ∗R, τ ∗).

Proposition 4.1

Any f inite group G=(S,*) with conjugacy relation (a*g=g*b) produced a topology $\tau *$ on S.

Proof:

Let S={Si, i∈I} is a finite set with a binary opration * satisfies the properties of group.

Let $* \mathcal{R} = \{(si, sj) \in S \times S: | si * g = g * sj\}$ is an equivalence relation.

So U is a partianed by *R.

As a family of equivlence classes.

Assum U ={[aj] : j ∈ J}

Then $\tau *$ with be the topology on U.

Proposition 4.2

If a finite group G = (U, ∗) produce a topological space (U, ∗R, τ ∗) by a relation ∗R = {(x, x) : x ∈ U }, then τ ∗ is a discrete topology.

Proof:

U is aprtitioned by ∗R as U/ ∗ R = {[x] : x ∈ U }

Which is a base for topology τ ∗.

Then τ ∗ is a discrete topology since the base includes all the singleton subsets.

Example 4.1

let G = (U, .) be a sup group from quaternion group.

U ={1, −1, i, −i}

∗R={(1, 1), (−1, −1), (−i, −i), (i, i)} identity relation.

A ⊆ U , A = {1, i}, ∗B = {{1}, {−1}, {i}, {−i}} ∪ ϕ

τ ∗ = {ϕ, U.{1}, {−1}, {i}, {−i}, {1, −1}{i, −i}, {1, i}, {1, −i}, {−1, i},
{−1, −i}, {1, −1, i} , {1, −1, −i}, {1, i, −i}, {−1, i, −i}}

$*\overline{R}(A)$ = {1, i}, $*\underline{R}(A)$= {1, i}

Corollary 4.1

Let (G, ∗R, τ ∗) is a topological approximation space induce by finite group (G, ∗). If G = C(G), then τ ∗ is a discrete topology.

Lemma 4.1

Let U *is a non empty set in finite group* $G = (U,*)$*and* $*\ R$ be an equivalence relation, then the base of Topology τ ∗ contain all element of U.

Definition 4.2

(rough set in topological approximation space induce by finite group)

Let (U, ∗R, τ ∗) be a topological approximation space induce by finite groups G = (U, ∗) and x ⊆ U. If upper approximation not equal lower approxima- tion (∗$\overline{R}(X)$≠ ∗$\underline{R}(X)$=), then X is a rough set in τ ∗ otherwise is an exact set.

Corollary 4.2

Let (U, ∗R, τ ∗) be a topological approximation space viafinite group andX ⊆ U. If (U, ∗) is a commutative group, then X is an exact set in τ ∗.

Corollary 4.3. Let (U, ∗R, τ ∗) is a topological approximation space via

finite group, then every element in τ ∗ is clopen.

proof

The basis of τ ∗ is a family of equivalence classes {[xi], i ∈ I} for which

[xi] ∩ [xj] = ϕ for any i, j ∈ I, So the topology τ ∗ will contain subset Ai

and their complement Ai^c

5. ROGH MEMBERSHIP FUNCTION $\tau *$

The **rough membership function** is defined using equivalence classes. Below, we introduce this membership function by utilizing the **base for A topology induced by a finite group**.

$$\mu_X^{\tau*} = \frac{|\cap B_x \cap X|}{|\cap B_x|}$$

where B_x is a small member of $* B$ continuing X.

Example 5.1

Let U = D4 is a finite U = { r1, r2, r3, r4, v, h, d1, d2}with

∗R̦ = {(a, b) ∈ U × U : a = xbx−1 for some g ∈ U } is a conjugacy relation U produce a base of ∗.

∗ B = {ϕ, {r1}, {r2, r4}, {r3}{h, v}, {d1, d2}} is a base of topology τ ∗,

τ ∗ = {U, ϕ, {r1}, {r2, r4}{r3}, {h, v}, {d1, d2}, {r1, r3}, {r1, h, v}{r1, r2, r4}

{r1, d1, d2}, {r3, d1, d2}, {r2, r4, r3}, {r3, h, v}, {h, v, d1, d2},{r1, h, v, d1, d2},

{r3, h, v, d1, d2}{r1, r3, h, v, d1, d2}, {r1, r2, r3, r4}, {r1, r3, d1, d2}, {r1, r3, h, v, },

{r1, r2, r4, d1, d2}, {r3, r2, r4, d1, d2}, {r2, r4, d1, d2}, {r1, v, d1, d2},

{r1, r2, r4, h, v, d1, d2}{r1, r2, r4, h, v}, {r3, r2, r4, h, v}, {r1, r3, r2, r4, h, v},

{r1, r3, r2, r4, d1, d2}{r1, r3, h, v, d1, d2}, {r2, r3, r4, h, v, d1, d2}}

If X = $\{r3, r4\}$, X ⊆ U, then $\mu_X^{\tau*}(r1) = 0$, $\mu_X^{\tau*}(r2) = 1/2$, $\mu_X^{\tau*}(r3) = 1$, $\mu_X^{\tau*}(r4) = 1/2$, $\mu_X^{\tau*}(h) = 0$,

$\mu_X^{\tau*}(v) = 0$, $\mu_X^{\tau*}(d_1) = 0$, $\mu_X^{\tau*}(d_2) = 0$

Remark 5.1

Let $(U,*R,\tau *)$ *is a Toplogy approximation space induce by finite group .*

If $0 < \mu_X^{\tau *} < 1$ for all $x \in U$, then $*\underline{R}(X) = \phi$

Proposition 5.2

Let $(G,*)$ *is a finite group,* $*B = \{Bi, i \in I\}$ is a Base of some topology on G induced by on equivalence relation $*R$, for every element a in Bi has the value $\mu_B i^{\tau *}(a) = 1$

proof

Let G is a finite group and f is a function defined to produce an equivalence relation

$*R = \{(a,b) \in G \times G : b = f(a)\}$

So the set G, will be a partitianed of equivalence classes, these classes make a base.

$*B = \{[a] : x \in G\}$.

for some topology τ on G, So

$$\mu_{Bi}^{*R}(a) = \frac{|[a] \cap Bi|}{|[a]|} = 1$$

Since $a \in \mu_X^{\tau *}(f5) = 1/3Bi$ and $[a] = Bi$

Example 5.2

Let *U*= D_3 *be the Dihedral group of order 6.*

by example(3.1)we can see. and X⊂ U, X = $\{f2, f3\}$ the $*B = \{\{f_1\}, \{f_2, f_3\} \{f_4, f5, f6\}, \emptyset\}$ is abace for

$$\tau *= \{\phi, U\{f_1\}, \{f_2, f_3\}\{f_4, f5, f6\}, \{f_1, f_2, f_3\}, \{f_2, f_3, f_4, f5, f6\}, \{f_1, f_4, f5, f6\}\}$$

$\mu_X^{\tau *}(f1) = 0$ then $f_1 \notin X$

$\mu_X^{\tau *}(f2) = 1$ then , $f_2 \in X$

$\mu_X^{\tau *}(f3) = 1$ then , $f_3 \in X$

$\mu_X^{\tau *}(f4) = 0$ then , $f_4 \notin X$

$\mu_X^{\tau *}(f5) = 0$ then , $f_5 \notin X$

$\mu_X^{\tau *}(f6) = 0$ then , $f_6 \notin X$

Lemma5.1

$(U,*R,\tau*)$ is *a topological approximation space induce by a finite group and* $X \subseteq U$.

If $0 \le \mu_X^{\tau*} < 1$ for all $x \in U$, then $X \notin \tau*$

Example 5.3

Let D_3 *be a Dihedral group of order 6* from example(3.1)we can see.

$X = \{f2, f4\}$, $\mu_X^{\tau*}(f1) = 0$, $\mu_X^{\tau*}(f2) = 1/2$, $\mu_X^{\tau*}(f3) = 1/2$, $\mu_X^{\tau*}(f4) = 1/3$, $\mu_X^{\tau*}(f6) = 1/3$,

Then $X = \{f2, f4\} \notin \tau*$

Lemma5.2

$(U,*\underline{R},\tau*)$ is a topological approximation space induce by finite group , $X \subseteq U$.

the value all elements of the rough membership function be, $0 \le \mu_X^{\tau*} \le 1$

Then X is rough set in $\tau*$

Proposition 5.3

Let $(U,*R,\tau*)$ is a topological approximation space induce by finite group.

If $\mu_X^{\tau*}(a) = 0$ then $a \in \underline{*R}(X^c)$

proof:

Let a ∈ U and X ⊆ U

Since $\mu_X^{\tau*}(a) = 0$, then x ∈ U-$*\bar{R}(X)$ by proposition (2.4.1)

Hence a ∈ U⋂ $(*\bar{R}(X))^c$ → a ∈ U & a ∈ $(*\bar{R}(X))^c$

Since $(\bar{R}(X))^c = \underline{*R}(X^c)$ by proposition (2.3.1). Therefor a ∈ $\underline{*R}(X^c)$

6. CONCLUSION

The aim of this study is to suggest new type of topological approximate space induce by finite groups $(U, *R, \tau*)$, With conjugacy relation and introduced some Remark, proposition, and Example about those connotations, also rough set in topological approximation induce by finite group was studied in this paper and rough membership function induced by τ∗ with some results about this concept.

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