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Formulas of General Solution for linear System from Ordinary Differential Equations by Using Young transformation

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A B S T R A C T

In this paper we applied a new integral transformation called the Yang transformation to solve a system of linear differential equations , while homogeneous and non- homogeneous . We found general formula of the set solution of systems of first-order equations.

MSC..

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1-Introduction

Integral transformations are mathematical operations that transform complex functions and derivatives in to simpler algebraic equations .It facilitates the solution of differential equations, especially with the presence of boundary conditions equations. It is used in processing fixed linear systems and transforming them form the time domain to the frequency domain [8,11].

The most important of these are the Laplace and fourier transforms[3,7], and there are many new transforms that address some issues such as Sumudu , Elzaki , Mellin , GFI [4,2,5,1]

The Transform Temimi is a method of addressing differential equations, created by Ali Hassan and Athraa Neamah AL-Bukhuttar in 2008. [6].

In2019, Shehu Maitama and Weidong Zhan were able to discover a new integral transformations derived from the Laplace transformations Which is useful in solving ordinary partial , it is also used to solve the heat and trans port equations and called Shehu transform [9,12].

In 2016, in traduced anew integral transform which used to solve differential equations Called the Yang transform [10].

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In this paper , some formulas of general solution for system, whereas homogeneous or non homogeneous from using Yang transform

In section 2 , the defintions , properties and Yang transform for some fundamental functions

In section 3, we derive the general formula for asystem of first order, while homogeneous or non homogeneous by using Yang transform.

In last section , we used these formulas to solve some examples.

2. The fundamental definition and attributes of Young's transformation.

Definition [13]

If $q(\tau)$ is integrable function $\tau > 0$ The Yang transform of $q(\tau)$ defined by:

$$Y(q(\tau)) = \tau(v) = \int_0^\infty e^{\frac{-\tau}{v}}\, q(\tau)\, dt \quad , t > 0 \qquad (2.1)$$

provided *the integral* exists for some v, where $v \in (-\tau_1, \tau_2)$.

If we substitute $\frac{-\tau}{v} = \mu$then equation (2.1)becomes,

$$Y(q(\tau)) = \tau(v) = v \int_0^\infty e^{-\mu}\, q(v\mu)\, d\mu \quad , \quad \mu > 0 \qquad (2.2)$$

2-1Laplace- Yang duality Property [13] :

If the Laplace Transform of the function $q(\tau)$*is* $q(v)$,then

$$q(v) = l\{q(\tau)\} = \int_0^\infty e^{-v\tau}\, q(\tau) d\tau \qquad (2.3)$$

Substitute $\tau = \frac{\sigma}{v}$ in the integral hand side we get

$$q(v) = l\{q(\tau)\} = \frac{1}{v}\int_0^\infty e^{-\sigma}\, q\left(\frac{v}{v}\right) d\tau$$

Hence from equation(2.3):, get

$$q(v) = \tau\left(\frac{1}{v}\right) \qquad (2.4)$$

Also from equations ((2.3))and (2.4):, get

$$\tau(v) = q\left(\frac{1}{v}\right) \qquad (2.5)$$

Table1. the Yang transform for some function[10]

1. ID	**2. function $q(\tau)$**	**3. $Y(q(\tau))$**
4. 1	5. 1	6. v
7. 2	8. τ	9. v^2
10. 3	11. τ^η	12. $\eta!.v^{\eta+1}$
13. 4	14. $e^{\sigma\tau}$	15. $\frac{v}{1-v\sigma}$
16. 5	17. $\sin\sigma\tau$	18. $\frac{\sigma v^2}{1+\sigma^2 v^2}$
19. 6	20. $\cos\sigma\tau$	21. $\frac{v}{1+\sigma^2 v^2}$
22. 7	23. $\sinh\sigma\tau$	24. $\frac{\sigma v^2}{1-\sigma^2 v^2}$

25. 8	26. $\cosh \sigma\tau$	27. $\frac{\upsilon}{1+\sigma^2\upsilon^2}$

Theorem(2-2) [13] IF $\Upsilon_{\rfloor}(q(\tau)), \tau > 0, \upsilon > 0$, *then we have*:

$i. \Upsilon_{\rfloor}(q'(\tau)) = \frac{\tau(\upsilon)}{\upsilon} - q(0)$ (2.6)

ii. $\Upsilon_{\rfloor}(q''(\tau))=\frac{\tau(\upsilon)}{\upsilon^2} - \frac{q(0)}{\upsilon} - q'(0)$ (2.7)

iii. $\Upsilon_{\rfloor}(q'''(\tau))=\frac{\tau(\upsilon)}{\upsilon^3} - \frac{q(0)}{\upsilon^2} - -q''(0)$ (2.8)

iv. $\Upsilon_{\rfloor}(q^{\eta}(\tau))=\frac{\tau(\upsilon)}{\upsilon^{\eta}} \sum_{G=0}^{\eta-1} \frac{q^{\eta}(0)}{\upsilon^{\eta-G-1}}\ \forall\, \eta = 1,2\,,3,4,\ldots$ (2.9)

Theorem (2-3) [13] IF the Yang transform of the function $q(\tau)$ is given by $\Upsilon_{\rfloor}(q(\tau))$, then,

i. $\Upsilon_{\rfloor}(\tau q(\tau))= \upsilon^2\tau'(\upsilon)$ (2.10)

ii. $\Upsilon_{\rfloor}(\tau q(\tau))= \upsilon^4\tau''(\upsilon) + 2\upsilon^3\tau'(\upsilon)$ (2.11)

Theorem (2-4): Convolution Theorem

Let $q(\upsilon) and G(\upsilon)$ be functions having Yang transform $\tau_1(\upsilon), \tau_2(\upsilon)$ respectively

Then Yang transform of the convolution of q and G ,

$(q * G)= \int_0^{\tau} q(\tau)G(\tau-\varepsilon)d\tau$ is given by $\Upsilon[q * G] = \tau_1(\upsilon)\tau_2(\upsilon)$ (2.12)

3. General solution formula for a first-order system

The generic formula for a first-order system of any dimension η, whether or not it is uniform, is derived in this section..

3.1 General solution formula for homogeneous first-order system.

A first-order system is characterized by the expression $q' = \mathcal{C}q$

where $q' = \begin{pmatrix} \frac{dq_1}{d\tau} \\ \frac{dq_2}{d\tau} \\ \vdots \\ \frac{dq_{\eta}}{d\tau} \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \\ \vdots \\ \mathcal{C}_{\eta} \end{pmatrix}, \quad q = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{\eta} \end{pmatrix}$ so,

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{\eta} \end{pmatrix}' = \begin{pmatrix} \mathcal{C}_{11}\mathcal{C}_{12} & \cdots & \mathcal{C}_{1\eta} \\ \mathcal{C}_{21}\mathcal{C}_{22} & \cdots & \mathcal{C}_{2\eta} \\ \vdots \quad \vdots & \cdots & \vdots \\ \mathcal{C}_{m1}\mathcal{C}_{m2} & \cdots & \mathcal{C}_{m\eta} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{\eta} \end{pmatrix} \quad \ldots (3.1)$$

after taking Yang transformation for both sides, yields:

$$\frac{1}{\upsilon}\Upsilon(q_1) - q(0) = \mathcal{C}_{11}\Upsilon(q_1) + \mathcal{C}_{12}\Upsilon(q_2) + \cdots + \mathcal{C}_{1\eta}\Upsilon_{\rfloor}(q_{\eta})$$

$$\frac{1}{\upsilon}\Upsilon(q_2) - q(0) = \mathcal{C}_{21}\Upsilon(q_1) + \mathcal{C}_{22}\Upsilon(q_2) + \cdots + \mathcal{C}_{2\eta}\Upsilon_{\rfloor}(q_{\eta})$$

$\vdots$

$$\frac{1}{\upsilon}\Upsilon(q_\eta) - q_\eta(0) = c_{\eta 1}\Upsilon(q_1) + c_{\eta 2}\Upsilon(q_2) + \cdots + c_{m\eta}\Upsilon(q_n)$$

where $q_1(0)$, $q_2(0), \cdots, q_\eta(0)$ are initial conditions,

$$(\frac{1}{\upsilon} - \mathcal{C}_{11})\Upsilon(q_1) - \mathcal{C}_{12}\Upsilon(q_2) - \cdots - \mathcal{C}_{1\eta}\Upsilon(q_\eta) = q_1(0)$$

$$(\frac{1}{\upsilon} - \mathcal{C}_{22})\Upsilon(q_2) - \mathcal{C}_{21}\Upsilon(q_1) - \cdots - \mathcal{C}_{2\eta}\Upsilon(q_\eta) = q_2(0)$$

$$\vdots$$

$$(\frac{1}{\upsilon} - \mathcal{C}_{m\eta})\Upsilon(q_\eta) - \mathcal{C}_{m1}\Upsilon(q_1) - \cdots - \mathcal{C}_{m2}\Upsilon(q_\eta) = q_\eta(0)$$

Moreover, simple calculation to obtain $\Upsilon(q_1), \cdots, \Upsilon(q_\eta)$,

$$\Delta = \begin{vmatrix} \left(\frac{1}{\upsilon} - \mathcal{C}_{11}\right) & -\mathcal{C}_{12} & \cdots & -\mathcal{C}_{1\eta} \\ -\mathcal{C}_{21} & \left(\frac{1}{\upsilon} - \mathcal{C}_{22}\right) & \cdots & -\mathcal{C}_{2\eta} \\ \vdots & \vdots & \cdots & \vdots \\ -\mathcal{C}_{m1} & -\mathcal{C}_{m2} & \cdots & (\frac{1}{\upsilon} - \mathcal{C}_{m\eta}) \end{vmatrix}$$

Also,

$$\Upsilon(q_1) = \frac{1}{\Delta}\begin{vmatrix} q_1(0) & -\mathcal{C}_{12} & \cdots & -\mathcal{C}_{1\eta} \\ q_2(0) & \left(\frac{1}{\upsilon} - \mathcal{C}_{22}\right) & \cdots & -\mathcal{C}_{2\eta} \\ \vdots & \vdots & \cdots & \vdots \\ q_\eta(0) & -\mathcal{C}_{m2} & \cdots & \left(\frac{1}{\upsilon} - \mathcal{C}_{m\eta}\right) \end{vmatrix}$$

$$\Upsilon(q_2) = \frac{1}{\Delta}\begin{vmatrix} \left(\frac{1}{\upsilon} - \mathcal{C}_{11}\right) & q_1(0) & \cdots & -\mathcal{C}_{1\eta} \\ -\mathcal{C}_{21} & q_2(0) & \cdots & -\mathcal{C}_{2\eta} \\ \vdots & \vdots & \cdots & \vdots \\ -\mathcal{C}_{m1} & q_\eta(0) & \cdots & \left(\frac{1}{\upsilon} - \mathcal{C}_{m\eta}\right) \end{vmatrix}$$

$$\vdots$$

$$\Upsilon(q_\eta) = \frac{1}{\Delta}\begin{vmatrix} (\frac{1}{\upsilon} - \mathcal{C}_{11}) & -\mathcal{C}_{12} & \cdots & q_1(0) \\ -\mathcal{C}_{21} & \left(\frac{1}{\upsilon} - \mathcal{C}_{22}\right) & \cdots & q_2(0) \\ \vdots & \vdots & \cdots & \vdots \\ -\mathcal{C}_{m1} & -\mathcal{C}_{m2} & \cdots & q_\eta(0) \end{vmatrix}$$

The solution set of the system (3.1) is composed of the inverse Young transform of the vector $\Upsilon(q_i)$, $i = 1,2,3, \cdots, \eta$

3.2.TheFormulaGeneralSolution of Non-Homogeneous System of Order one

Systems that are not homogenous have the composition $q' = \mathcal{C}q + G$

$$q' = \begin{pmatrix} \frac{dq_1}{d\tau} \\ \frac{dq_2}{d\tau} \\ \vdots \\ \frac{dq_\eta}{d\tau} \end{pmatrix}, \quad \mathcal{C} = \begin{pmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \\ \vdots \\ \mathcal{C}_\eta \end{pmatrix}, \quad q = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_\eta \end{pmatrix}, G = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{pmatrix} \text{ so,}$$

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_\eta \end{pmatrix}' = \begin{pmatrix} \mathcal{C}_{11}\mathcal{C}_{12} & \cdots & \mathcal{C}_{1\eta} \\ \mathcal{C}_{21}\mathcal{C}_{22} & \cdots & \mathcal{C}_{2\eta} \\ \vdots \quad \vdots & \cdots & \vdots \\ \mathcal{C}_{m1}\mathcal{C}_{m2} & \cdots & \mathcal{C}_{m\eta} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_\eta \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{pmatrix} \qquad \ldots (3.2)$$

after taking Yang transformation for both sides, yields:

$$\frac{1}{v}Y(q_1) - q(0) = \mathcal{C}_{11}Y(q_1) + \mathcal{C}_{12}Y(q_2) + \cdots + \mathcal{C}_{1\eta}Y(q_\eta) + Y(G_1)$$

$$\frac{1}{v}Y(q_2) - q(0) = \mathcal{C}_{21}Y(q_1) + \mathcal{C}_{22}Y(q_2) + \cdots + \mathcal{C}_{2\eta}Y(q_\eta) + Y(G_2)$$

$$\vdots$$

$$\frac{1}{v}Y(q_\eta) - q_\eta(0) = c_{\eta1}Y(q_1) + c_{\eta2}Y(q_2) + \cdots + c_{m\eta}Y(q_n) + Y(G_\eta)$$

where $q_1(0)$, $q_2(0), \cdots, q_\eta(0)$ are initial conditions,

$$(\frac{1}{v} - \mathcal{C}_{11})Y(q_1) - \mathcal{C}_{12}Y(q_2) - \cdots - \mathcal{C}_{1\eta}Y(q_\eta) = q_1(0) + Y(G_1)$$

$$(\frac{1}{v} - \mathcal{C}_{22})Y(q_2) - \mathcal{C}_{21}Y(q_1) - \cdots - \mathcal{C}_{2\eta}Y(q_\eta) = q_2(0) + Y(G_2)$$

$$\vdots$$

$$(\frac{1}{v} - \mathcal{C}_{m\eta})Y(q_\eta) - \mathcal{C}_{m1}Y(q_1) - \cdots - \mathcal{C}_{m2}Y(q_\eta) = q_\eta(0) + Y(G_\eta)$$

Moreover, simple calculation to obtain $Y(q_1), \cdots, Y(q_\eta)$,

$$\Delta = \begin{vmatrix} \left(\frac{1}{v} - \mathcal{C}_{11}\right) & -\mathcal{C}_{12} & \ldots & -\mathcal{C}_{1\eta} \\ -\mathcal{C}_{21} & \left(\frac{1}{v} - \mathcal{C}_{22}\right) & \ldots & -\mathcal{C}_{2\eta} \\ \vdots & \vdots & \ldots & \vdots \\ -\mathcal{C}_{m1} & -\mathcal{C}_{m2} & \ldots & (\frac{1}{v} - \mathcal{C}_{m\eta}) \end{vmatrix}$$

$$Y(q_1) = \frac{1}{\Delta}\begin{vmatrix} q_1(0) + Y(G_1) & -\mathcal{C}_{12} & \ldots & -\mathcal{C}_{1\eta} \\ q_2(0) + Y(G_2) & \left(\frac{1}{v} - \mathcal{C}_{22}\right) & \ldots & -\mathcal{C}_{2\eta} \\ \vdots & \vdots & \ldots & \vdots \\ q_\eta(0) + Y(G_\eta) & -\mathcal{C}_{m2} & \ldots & \left(\frac{1}{v} - \mathcal{C}_{m\eta}\right) \end{vmatrix}$$

$$\vdots$$

$$Y(q_\eta) = \frac{1}{\Delta}\begin{vmatrix} (\frac{1}{v} - \mathcal{C}_{11}) & -\mathcal{C}_{12} & \ldots & q_1(0) + Y(G_1) \\ -\mathcal{C}_{21} & \left(\frac{1}{v} - \mathcal{C}_{22}\right) & \ldots & q_2(0) + Y(G_2) \\ \vdots & \vdots & \ldots & \vdots \\ -\mathcal{C}_{m1} & -\mathcal{C}_{m2} & \ldots & q_\eta(0) + Y(G_\eta) \end{vmatrix}$$

After taking of Yang transform to$\mathcal{Y}(q_1)$, $\cdots$, $\mathcal{Y}(q_\eta)$, obtaining the solution of the system(3.2).

4-Illustrative Examples

The following example demonstrates the practicality and value of Young's transform in solving a system of linear equations.

Example(1):

Solve the system of two dimensional first-order differential equations: $q' = \mathcal{A}q$ where $\mathcal{A}=\begin{pmatrix}5 & -4\\ 3 & -2\end{pmatrix}$, $q(0) = \begin{pmatrix}3\\ 2\end{pmatrix}$ ...(4.1)

Solution: Using Yang transformation and apply formula (3.1), yield :

$$\mathcal{Y}(q_1) = \frac{v^2}{(2v-1)(v-1)}\begin{vmatrix}3 & 4\\ 2 & \left(\frac{1}{v}+2\right)\end{vmatrix}$$

After simple calculation using partition fraction:

$$\mathcal{Y}(q_1(\tau)) = \frac{4v^2}{(1-2v)} - \frac{v^2}{(1-v)}$$

Employing the Young's transform to both sides of the equation above, we have:

$q_1(\tau) = 4e^{2\tau} - e^{\tau}$,

In similar way, $\mathcal{Y}(q_2)$can be obtained by :

$$\mathcal{Y}(q_2) = \frac{v^2}{(2v-1)(v-1)}\begin{vmatrix}\left(\frac{1}{v}-5\right) & 3\\ -3 & 2\end{vmatrix}$$

$$\mathcal{Y}(q_2(\tau)) = \frac{-3v^2}{(1-2v)} - \frac{v^2}{(1-v)}$$

$q_2(\tau) = 3e^{2\tau} - e^{\tau}$,

Example (2):

Discover the general solution to the system of equations $q' = \mathcal{A}q + G$.

where $\mathcal{A}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $q(0) = \begin{pmatrix}2\\ 0\end{pmatrix}$, $G=\begin{pmatrix}\sin(\tau)\\ \cos(\tau)\end{pmatrix}$ ...(4.2)

Solution: Using formula (3.2), yield :

$$\mathcal{Y}(q_1) = \frac{v^2}{(1-v^2)}\begin{vmatrix}2+\frac{v^2}{(1+v^2)} & 1\\ 0+\frac{v}{(1+v^2)} & \left(\frac{1}{v}-0\right)\end{vmatrix} = \frac{v^2}{(1-v^2)}\left(\frac{2v^2+2}{(1+v^2)}\right) = \frac{2v}{1-v^2}$$

simple fiction and taking inverse Yang transform of the above equation,

$q_1(\tau) = 2\cosh\tau$

Similarly, the value of $\mathcal{Y}(q_2)$is obtained as follows:

$$\mathcal{Y}(q_2) = \frac{v^2}{(1-v^2)}\begin{vmatrix}\left(\frac{1}{v}-0\right) & 2+\frac{v^2}{(1+v^2)}\\ 1 & 0+\frac{v}{(1+v^2)}\end{vmatrix} = \frac{v^2}{(1-v^2)}\left(\frac{v(v+3v^2)}{v(1+v^2)}\right)$$

Additionally, the inverse of Yang's transform from the above equations.:

$$q_2(\tau) = -2\sin\tau + \sinh\tau$$

Conclusion

Applying Yang's transformation for some mathematical systems with constant coefficients, we concluded the following:

1) General formulas were obtained for solving mathematical systems with constant coefficients subject to initial conditions.

2)The application of the general formula was obtained for solving mathematical systems with constant coefficient subjecting to known initial conditions.

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