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Using A Linear Programming Method for an Approximate Solution of Linear Fractional Volterra–Fredholm Integro-Differential Equations

Samah Mohammed Ali *

University of Baghdad, College of Science for Women, Department of Mathematics, Baghdad, Iraq. Email: samahma_math@csw.uobaghdad.edu.iq

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A B S T R A C T

This research presents a new algorithm for approximating the linear fractional Volterra-Fredholm integro-differential equation (LFVFIDE) of fractional order α, where (0 < α < 1). The strategy requires that the equation be transformed into a Linear Programming Problem (LPP), allowing the coefficients of the approximate solution to be obtained through an optimization process. The fractional order (α) is depicted in Caputo's sense. Some test examples with exact solutions are solved using the proposed approach, where the results demonstrate the high accuracy and low relative errors of the presented method. Furthermore, different values of α are assigned to each example to enhance the convergence of the proposed technique for fractional-order integro-differential systems.

1. Introduction

Fractional integro-differential equations can be employed to represent a variety of science and engineering problems, including those related to mechanics, biomedicine, and earthquakes [1]. Nevertheless, attaining the analytical solution to this type of equation is impossible. Consequently, using numerical or approximation methods to locate more accurate solutions can be helpful. Some numerical algorithms for solving a linear and nonlinear integro-differential equation of fractional order its best summarized like follows: cos and sin wavelet method [2], generalized hat function method [3], spectral Jacobi-collocation method [4], homotopy perturbation, and variational method[5], least-squares method and Bernstein polynomials [6], Sinc-collocation method [1], [7], modified Laplace decomposition method [8], shifted

∗Corresponding author

Email addresses: samahma_math@csw.uobaghdad.edu.iq

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Jacobi-spectral collocation method[9], Chebyshev wavelets collocation [10], modified Adomain decomposition method [11], Laguerre polynomials [12], Sumudu decomposition method [13], shifted Jacobi–Gauss-collocation[14], residual power series [15], reproducing-kernel method[16], block pulse functions[17], Haar wavelet [18], Lagrange polynomial [19].

In this paper, the solution of LFVFIDE of the form:

$$D^{\alpha}y(z) = q(z)y(z) + f(z) + \int_a^z k_1(z,t)y(t)dt + \int_a^b k_2(z,t)y(t)dt \quad ,z,t \in [a,b], 0 < \alpha \leq 1 \qquad (1)$$

including an initial condition:

$$y(a) = y_0 \qquad (2)$$

can be introduced as a polynomial of order n, defined as:

$$y_n(z) = \sum_{j=0}^{n} c_j z^j$$

Where:

$D^{\alpha}y(z)$ denotes the 'Caputo fractional derivative' of $y(z)$;$f(z), q(z), k_1(z,t)$ and $k_2(z,t)$ are continuous functions, z and t are real variables in [a,b], and $y(z)$ the undefined function to be evaluated. This paper provides the following contributions:

- Using Linear Fractional Volterra–Fredholm Integro–Differential Equations.
- The equation is transformed into a linear programming problem (LPP).
- The high accuracy of the procedure was shown by comparing the results with the actual solution.

The remainder of this article is organized as follows: Section 2 creates the new approach, and some preliminary steps are presented. Section 3 includes the derivation of the numerical approaches introduced in this article. In section 4, the algorithm for the suggested method is listed. In section 5, some numerical examples to show the effectiveness of the presented method through some comparisons with other existing methods are given. Finally, the conclusion and recommendations can be found in section 6.

2. Preliminaries

This portion will provide some introductions and notations about fractional calculus and linear programming topics. Many definitions for fractional-order derivatives can be found in the literature, for example, Grünwald-Letnikov, Riemann-Liouville, Riesz, and Caputo derivatives. Many researchers recommended using the Caputo definition to model real-life problems because the derivative of any constant is zero in the Caputo sense. [15].

Definition 1: The Caputo definition of the fractional differential operator is given by[19]:

$$D^{\alpha}u(z) = \begin{cases} \dfrac{d^m u(z)}{dx^m} & , \quad if\ \alpha = m \in \mathbb{Z}^+ \\ \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^x \dfrac{f^{(m)}(t)}{(z-t)^{\alpha+1-m}} dt & , \quad if\ 0 \leq m-1 < \alpha < m \end{cases}$$

The parameter α is the derivative order that may be a real or even complex number. In this paper, the real and positive α will be considered only, and $m \in \mathbb{Z}^+$ is the smallest integer greater than α.

2.1 Properties of the operator D^{α}

The properties of the operator D^{α} are mentioned in many references. This paper will only mention the properties that need to find a solution for Eq.1.

- $D^{\alpha}c = 0, where\ \ c\ is\ constant$
- $D^{\alpha}[c\,u(z)] = cD^{\alpha}[u(z)]$
- $D^{\alpha}z^{c} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\alpha+1)} z^{\beta-\alpha} \quad ,\alpha > 0, \beta > -1, z > 0$

where c is constant.

Definition 2: The following definition provides the basic form for LPP [20] :

$$\left.\begin{array}{ll} \textit{minimize or maximize} & g = \sum_{j=1}^{n} e_j z_j \\ \textit{Subject to} & \\ \sum_{j=1}^{n} a_{ij} z_j \, (\le, =, \ge) \, c_i, & i = 1,2,3,\dots,m \\ z_j \ge 0, & j = 1,2,3,\dots,n \end{array}\right\}$$

Where:

e_j and c_i are real numbers termed cost coefficients and stipulations respectively, a_{ij} are constants referred to as structural coefficients and $z_j{}'s$ are named decision variables, for each ($i = 1,2,3,\dots,m$; $j = 1,2,3,\dots,n$).

3. Methodology

Consider Eq.1, where the solution to this problem can be assumed to be:

$$y_n(z) = \sum_{j=0}^{n} c_j z^j \qquad (3)$$

assuming that $c_0 = y_0$[the initial condition (Eq. 2)], and c_j , $j = 1,2,3,\dots,n$ are constant to be computed. When Eq. 3 is substituted into Eq. 1 yields:

$$D^\alpha\left(\sum_{j=0}^{n} c_j z^j\right) = q(z)\left(\sum_{j=0}^{n} c_j z^j\right) + f(z) + \int_a^z k_1(z,t)\left(\sum_{j=0}^{n} c_j t^j\right)dt + \int_a^b k_2(z,t)\left(\sum_{j=0}^{n} c_j t^j\right)dt \qquad (4)$$

Using the properties of the operator $\boldsymbol{D^\alpha}$ in Eq. 4, with some simplifications, the following equation yields:

$$\sum_{j=1}^{n} c_j \left[D^\alpha z^j - q(z) z^j - \int_a^x k_1(z,t) t^j dt - \int_a^b k_2(z,t) t^j dt\right] = f(z) + c_0\left[q(z) + \int_a^x k_1(z,t)dt + \int_a^b k_2(z,t)dt\right] \qquad (5)$$

with $c_0 = y_0$.

Therefore, assume that:

$$F(z) = f(z) + y_0\left[q(z) + \int_a^x k_1(z,t)dt + \int_a^b k_2(z,t)dt\right]$$

and

$$\varphi_j(z) = \left[D^\alpha z^j - q(z) z^j - \int_a^z k_1(z,t) t^j dt - \int_a^b k_2(z,t) t^j dt\right] \qquad (6)$$

and by dividing the range $[a, b]$ into m subranges to score a point $z_i = a + (i * h)$, $i = 0,1,2,\dots,m$ with $h = \frac{(b-a)}{m}$, $z_0 = a,\ and\ z_m = b$.

Therefore, the next substitution $z = z_i$, $for\ i = 0,1,2,\dots$,m in Eq. 6, the resulting formula is as follows:

$$\varphi_j(z_i) = \begin{cases} D^\alpha z_i^j - \int_a^b k_2(z_i,t) t^j dt, & if\ i = 0 \\ D^\alpha z_i^j - \int_a^{z_i} k_1(z_i,t) t^j dt - \int_a^b k_2(z_i,t) t^j dt, & if\ 1 \le i \le m \end{cases} \qquad (7)$$

Now assume:

$$\delta(z) = F(z) - \sum_{j=1}^{n} c_j \varphi_j(z) \qquad (8)$$

Moreover, substitute $z = z_i$ for $i = 0,1,\dots,m$ in Eq. 8 has become:

$$\delta(z_i) = F(z_i) - \sum_{j=1}^{n} c_j \varphi_j(z_i), \qquad i = 0,1,2,\dots,m \qquad (9)$$

Also, define γ as:

$$\gamma = \sum_{i=0}^{m} \delta(z_i) \qquad (10)$$

Thus, Eq. 1 can be expressed in the LPP form below:

$$
\left.\begin{aligned}
&\min \gamma = \sum_{i=0}^{m} \delta(z_i) \\
&\text{s.t} \\
&\qquad c_0 = y_0 \\
&\sum_{j=1}^{n} c_j \varphi_j(z_i) - \gamma \le f(z_i) \quad , \; i = 0,1,\dots,m \\
&\sum_{j=1}^{n} c_j \varphi_j(z_i) - \gamma \le -f(z_i) \quad , \; i = 0,1,\dots,m
\end{aligned}\right\} \qquad (11)
$$

Where c_j's are unlimited in sign, $\varphi_j(z_i)$'s could be properly assessed from Eq. 7, and the properties of the operator D^{α}. Hence, the solution of Eq. 1 is equivalent to the solution of Eq. 11 with the initial condition (Eq. 2).

4. General Algorithm for the Proposed Method

To find the numerical solution for **LFVFIDE** with fractional order $0 < \alpha < 1$ described in Caputo's sense, the following steps are suggested:

Step 1. Choose m and n, two positive integers (where m illustrates the number of times subintervals there are for the closed period [a, b], and n represents the total number of series terms in Eq. 3).

Step 2. Calculate $\varphi_j(z_i)$ in Eq. 7 for each $i = 1,2,3,\dots,m$; $j = 1,2,3,\dots,n$, using the exact value for the integral part and the definition of Caputo for the fractional part.

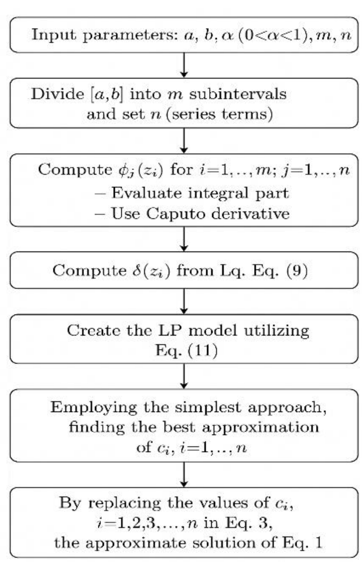
Step3. Compute $\delta(z_i)$ from Eq. 9.

Step 4. Create the LP model utilizing Eq. 11.

Step 5. Employing the simplest approach, finding the best approximation of c_i, $i = 1,2,3,\dots,n$.

Step 6. By replacing the values of $c_i, i = 1,2,3,\dots,n$ in Eq. 3, the approximate solution of Eq. 1 is obtained.

The visual representation of the proposed algorithm is indicated below.



The proposed approach is implemented in this paper using MATLAB R2018a

5. Numerical Applications

5.1 Test Example 1

Consider the FLVFIDE[21]:

$$D^{\alpha}u(z)+\frac{z^2e^z}{3}u(z)=\frac{z^{0.5}}{\Gamma(1.5)}-\frac{z^2}{2}-\frac{2z^3e^z}{3}+\int_0^z e^z t\,u(t)\,dt+\int_0^1 z^2\,u(t)\,dt\ ,0<z<1$$

With $u(0)=0$ is an initial condition, and the exact solution for $\alpha=0.5$ is $u(z)=z$.
Table 1 presents the estimated result for Test Example 1 achieved using the suggested approach using n(the degree of approximated polynomial) = 5,10,20 and m (the number of subintervals) = 5,10,20.

Table 1- The resulting approximate polynomial for a test example 1.

n	m	The approximated polynomial of test example 1 with exact solution $u(z)=z$ when $\alpha=0.5$
5	5	$y_5(z)=z$
	10	$y_5(z)=999.999999999998 * 10^{-3} * z$
	20	$y_5(z)=999.999999999999 * 10^{-3} * z$
10	5	$y_{10}(z)=z$
	10	$y_{10}(z)=999.999999999999 * 10^{-3} * z$
	20	$y_{10}(z)=z$
20	5	$y_{20}(z)=z$
	10	$y_{20}(z)=999.999999999998 * 10^{-3} * z$
	20	$y_{20}(z)=z$

To certify the convergence of the suggested method, different values of α are taken ($\alpha=0.2, 0.25, 0.3, 0.4$ *and* 0.5) as seen in Fig.1, where the true solution is given at $\alpha=0.5$.

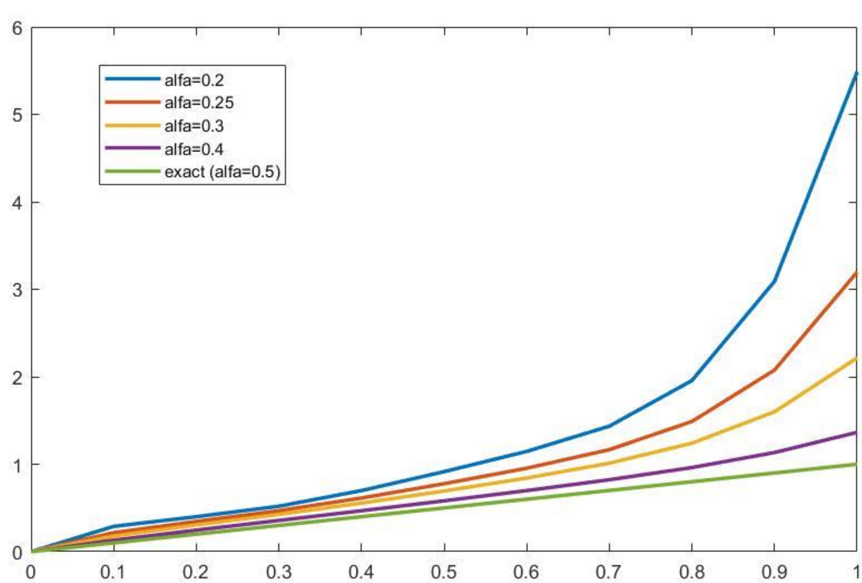


Fig. 1- The solution of test example 1 for different values of α.

The content of Fig. 1 proves that when the value of α converges to the exact one, the approximate solution converges to the exact solution.

5.2 Test Example 2

Consider the LFVIDE [18]:

$$D^{\alpha}u(z) = u(z)(\cos z - \sin z) + f(z) + \int_0^z z \sin t \ u(t)dt \ ,0 < z < 1$$

With $u(0) = 0$ the initial condition, $f(z) = \frac{2z^{1.5}}{\Gamma(2.5)} + z(2 - 3\cos z - z\sin z + z^2 \cos z) + \frac{z^{0.5}}{\Gamma(1.5)}$, and the true solution for $\alpha = 0.5$ is $u(z) = z^2 + z$.

Table 2 shows the approximate solution obtained using the suggested method for test example 2 using n (the degree of approximated polynomial) = 5,10, and 20 and m (the number of subintervals) = 5, 10, 20.

Table 2- The resulting approximate polynomial for a test example 2.

n	m	The approximated polynomial of test example 2 with exact solution $u(z) = z^2 + z$ when $\alpha = 0.5$
5	5	$y_5(z) = 1.00000000000007 \ * z^2 + 999.999999999982 \ * 10^{-3} * z$
	10	$y_5(z) = 999.999999999962 \ * 10^{-3} \ * z^2 + 1.00000000000001 * z$
	20	$y_5(z) = 999.999999999958 \ * 10^{-3} * z^2 + 1.00000000000001 * z$
10	5	$y_{10}(z) = z^2 + 999.999999999999 \ * 10^{-3} * z$
	10	$y_{10}(z) = 1.00000000000001 * z^2 + 999.999999999994 \ * 10^{-3} * z$
	20	$y_{10}(z) = 999.999999999975 * 10^{-3} * z^2 + 1.00000000000002 * z$
20	5	$y_{20}(z) = 1.00000000000002 * z^2 + 999.999999999991 * 10^{-3} * z$
	10	$y_{20}(z) = 999.999999999958 \ * 10^{-3} * z^2 + 1.00000000000001 * z$
	20	$y_{20}(z) = z^2 + 999.999999999998 \ * 10^{-3} * z$

To certify the convergence of the presented method, different values of α as seen in Fig. 2 are taken ($\alpha = 0.125, 0.2, 0.25, 0.3, 0.4$ *and* 0.5), where the true solution is given at $\alpha = 0.5$.

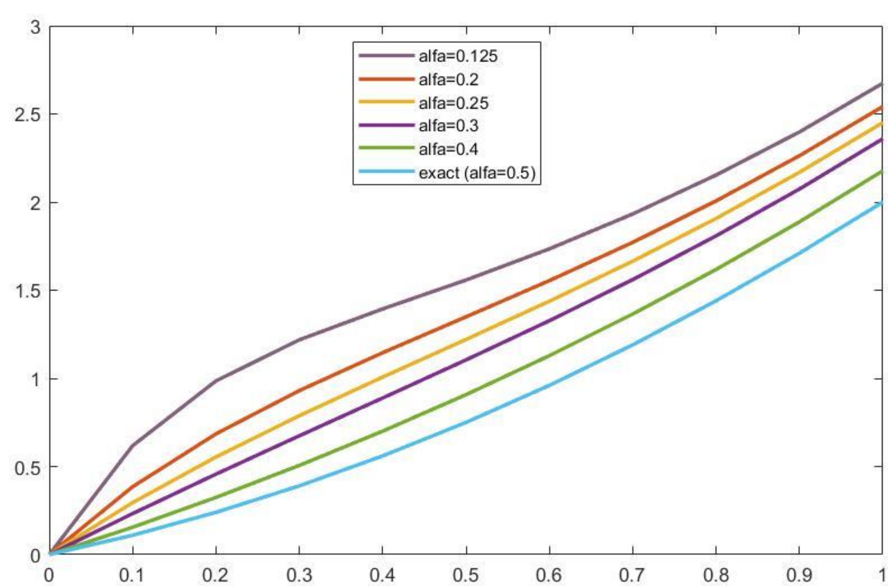


Fig. 2 - The solution of test example 2 for different values of α.

Fig. 2 proves that when the value of α converges to the exact one, the approximate solution converges to the exact solution.

5.3 Test Example 3

Consider the LFFIDE [6], [12]:

$$D^{\alpha}u(z) = f(z) + \int_0^1 ze^t\, u(t)dt \ ,0 < z < 1$$

Given the initial condition $u(0) = 0, f(z) = -\frac{3}{91}\frac{z^{\frac{1}{6}}\Gamma\left(\frac{5}{6}\right)(-91+216\ \text{س4ض}z^2)}{\pi} + (5 - 2e^1)z$, where the precise solution $u(z) = z - z^3$ for $\alpha = 5/6$.

Table 3 shows the approximate solution obtained using the suggested method for test example 3 using n (the degree of approximate polynomial) =5,10,20 and m (the number of subintervals) =5,10,20.

Table 3 - The resulting approximate polynomial for a test example 3.

n	m	The approximated polynomial of test example 3 with exact solution $u(z) = z - z^3$ when $\alpha = \frac{5}{6}$
5	5	$y_5(z) = z - z^3$
	10	$y_5(z) = z - z^3$
	20	$y_5(z) = z - z^3$
10	5	$y_{10}(z) = 999.999999999999 * 10^{-3} * z - 999.999999999997 * 10^{-3} * z^3$
	10	$y_{10}(z) = z - z^3$
	20	$y_{10}(z) = 999.999999999994 * 10^{-3} * z - 1.00000000000031 * z^3 + 2.817e - 13 * z^5$
20	5	$y_{20}(z) = z - z^3$
	10	$y_{20}(z) = z - 999.999999999999 * 10^{-3} * z^3$
	20	$y_{20}(z) = 999.999999999999 * 10^{-3} * z - 1.00000000000001 * z^3$

To approve the convergence of the proposed method, different values of α are taken ($\alpha = 0.2, 0.4, 0.5, \frac{4}{6}, 0.75\ and\ \frac{5}{6}$) as seen in Fig. 3, where $\alpha = \frac{5}{6}$ is the exact solution.

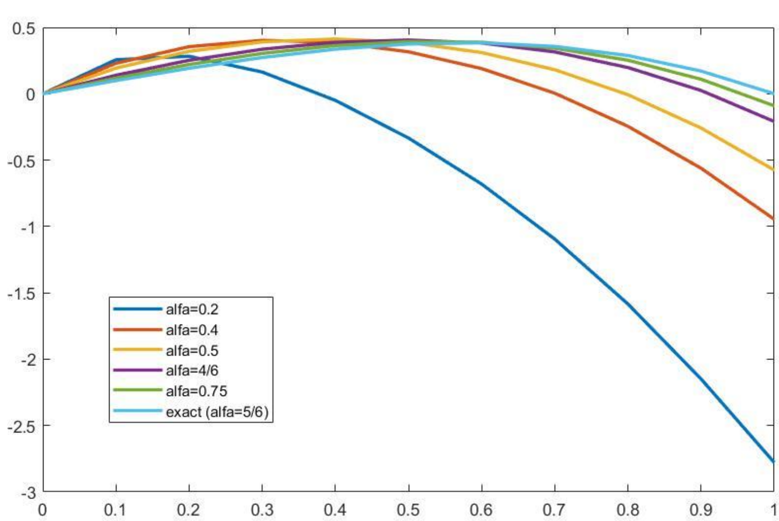


Fig. 3 - A comparison between the solutions of test example 3 for different values of α.

Fig. 3 above demonstrates how the approximate solution converges to the precise solution when the value approaches its exact value.

5.4 Test Example 4

Consider the following LVFIDE of 1st order:

$$u'(z) = -u(z) + f(z) + \int_0^z (zt - t)\, u(t)\, dt + \int_0^1 e^{z-t}\, u(t)\, dt$$

by the initial condition $u(0) = 0$, $f(z) = z^4 + 4z^3 - z^2 - 2z - \frac{1}{12}[z^4(2z^2 - 3)(z - 1)] - 2e^{1+z}(11e^1 - 30)$ the exact solution: $u(z) = z^4 - z^2$.

Table 4 shows the approximate solution obtained using the suggested method for test example 4 using n (the degree of approximated polynomial) =5,10,20 and m (the number of subintervals) =5,10,20.

Table 4 - The resulting approximate polynomial for a test example 4.

n	**m**	**The approximated polynomial of test example 4 with exact solution $u(z) = z^4 - z^2$ when $\alpha = 1$**
5	5	$y_5 = z^4 - 1.00000000000001 * z^2$
	10	$y_5 = 1.00000000000001 * z^4 - 1.00000000000001 * z^2$
	20	$y_5 = 999.999999999978 * 10^{-3} * z^4 - 999.999999999997 * 10^{-3} * z^2$
10	5	$y_{10} = 1.00000000000001 * z^4 - 1.00000000000002 * z^2$
	10	$y_{10} = 1.00000000000015 * z^4 - 1.00000000000017 * z^2$
	20	$y_{10} = 1.00000000000002 * z^4 - 1.00000000000003 * z^2$
20	5	$y_{20} = z^4 - z^2$
	10	$y_{20} = 1.00000000000001 * z^4 - 1.00000000000002 * z^2$
	20	$y_{20} = z^4 - 1.00000000000001 * z^2$

To approve the convergence of the proposed method, different values of α are taken ($\alpha = \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ *and* 1) where $\alpha = 1$ is the precise solution, as seen in Fig. 4.

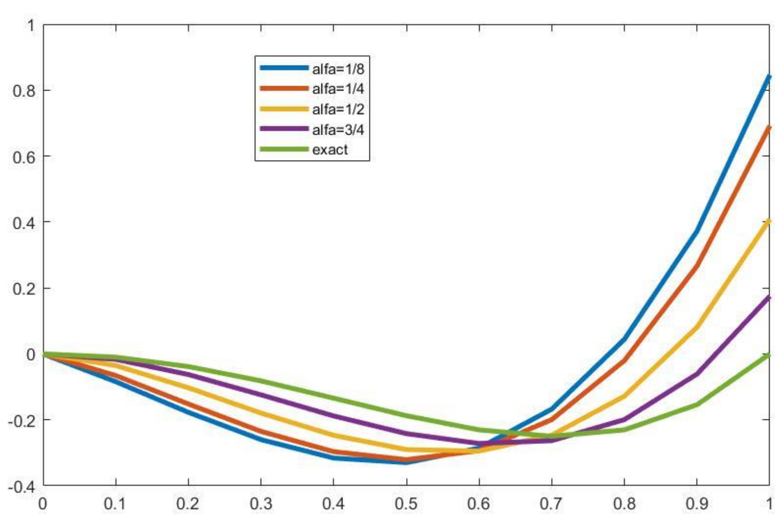


Fig. 4 - A comparison between the solution of test example 4 for different values of α.

Fig. 4 above proves that when the value of α converges to the exact one, the approximate solution also converges toward the exact (precise) solution.

6. Conclusion

The study presents a method for estimating the solutions to the fractional order α; $0 < \alpha < 1$ linear Volterra-Fredholm integro-differential problem employing the linear programming technique. The polynomial of degree (n) is utilized to transform the LVFIE of the 2^{nd} type into LPP. An exact solution for an integral part is proposed. Different test examples were used, and the results were compared to the actual solution to show the procedure's accuracy. It was hypothesized that the optimal solution could be achieved by raising the number of basis functions (n) and the number of limitations (m) together. For prospective investigations, the submitted method will be applied to solve delay integro-differential equations by choosing an appropriate approximation for the derivative component.

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