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Igi-induced Fuzzy Topology on Undirected Fuzzy Graphs

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A B S T R A C T

In this paper Associating a fuzzy tritopological space with an undirected fuzzy graph that is, three distinct fuzzy topologies derived from either three distinct fuzzy graphs or the same fuzzy graph is the goal of this article. The tritopologies have been proposed to link fuzzy topological spaces with fuzzy graphs

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1. Introduction

In 1965, Lotfi Zadeh proposed the idea of a fuzzy subset of a set to express uncertainty. His theories have piqued the interest of scholars throughout .[6] Azriel Rosenfeld was one such researcher. One of the pioneers of fuzzy graph theory was him. His creation of the fuzzy graph concept serves as the inspiration for this paper and the research it includes.

Fuzzy topological graph theory is one subfield of graph theory, which has a long history in mathematics. Researchers utilised the connection between fuzzy graph theory and fuzzy topological theory to infer a topology from a given fuzzy graph. Some of them create models based just on the graph's collection of vertices, while others do the same with the set of edges. They have been used in practically every scientific discipline and study fuzzy graphs as fuzzy topologies. The sources contain a wealth of great foundational information on fuzzy graph theory mathematics, fuzzy topological graph theory, and some applications.

Generally speaking, there are two kinds of graphs: directed and undirected. researchers associate fuzzy topological spaces with an undirected fuzzy graph. For example, A.M. Alzubaidi and M. Dammak. (2022) [9] defined a sub-basis family for a fuzzy graphic topology as a set of all vertices adjacent to the fuzzy vertex and associated a fuzzy graphic topology with the fuzzy vertex set of a locally finite fuzzy graph without isolated vertices.

Additionally, in 2025 [10], we linked a collection of fuzzy vertices to a fuzzy Incidence Topology for any simple fuzzy graph without isolated vertices. Wherein they defined a set of all fuzzy incident vertices with the fuzzy edge as a sub-basis family for a fuzzy incident topology

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In 2025 [11] We present a basic step towards studying some key properties of undirected fuzzy graphs by their corresponding fuzzy topologies by introducing a new Sub-basis family, which is defined as a set of all fuzzy vertices non-adjacent to the fuzzy vertex, to induce the new fuzzy topology (which we named Independent fuzzy topology),The idea of scientific research in tritopological spaces [5] has attracted the attention of many scientists, and most of these works have focused on generalizing the known concepts and ideas in topological spaces. The exploration of tritopological spaces began in 2000 with [Kover,2000], where he defined important separation axioms and clarified through various distinctive examples. These studies inspired numerous researchers to delve deeper into this field., a tritopological space is only a non-empty set associated with three arbitrary topologies [12]. Asmhan discussed it in depth and provided the definition of an open set in tritopological spaces in 2004 [14]. All of the topological structures of that open set, including the base, connectedness, compactness, lindelofness, countability, separability, product space, and quotient space, are used by the author (Asmhan F. H.) to establish the tritopological theory. and presented a few relationships, which the reader can locate in [14–24]. Additionally, the soft tritopological theory was first introduced by the same author in 2017 [23]. In 2020, we Associating a tritopological space with undirected graphs that is, three distinct topologies derived from a single graph or three distinct graphs [2]

Our motivation or target is to associate a tritopological space with undirected fuzzy graphs, i.e. three different fuzzy topologies induced from the one fuzzy graph or three different fuzzy graphs. These three different fuzzy topologies are the unique three proposed to associate fuzzy topological spaces with fuzzy graphs, the first [11] which we proposed recently (fuzzy Independent Topology) in 2025, the second is (fuzzy Incident Topology) we proposed recently in 2025 [10], and the third is (fuzzy Graphic Topology) proposed by A.M. Alzubaidi and M. Dammak. [9] in 2022. And giving a fundamental step toward studying some properties of undirected fuzzy graphs by their corresponding tritopological spaces

So, we have two goals for this work: Firstly, we introduce a definition of new tritopological space created from unique different three fuzzy topologies associated with fuzzy graphs. Secondly, we proposed a novel model of fuzzy topology induced from these three different fuzzy topologies associated with fuzzy graphs. In Section 2 of the article we give some fundamental definitions and preliminaries of fuzzy graph theory, fuzzy topology, tritopology and the three fuzzy topologies associated with fuzzy graphs. Section 3 is dedicated to main results of tritopological spaces on locally finite fuzzy graphs, we define our new - fuzzy induced topological space on locally finite fuzzy graphs. Section 4 is devoted to definition and some preliminaries results of Alexandroff fuzzy topology. In last Section, conclusions of this new fuzzy topology of tritopological spaces on undirected fuzzy graphs are presented.

2.Preliminaries

We provide some basic definitions and overviews of, fuzzy graph theory and fuzzy topology in this section. Examples of these conventional definitions can be found in sources [1], [2], [9], [10], [11], [12], [13],In the fuzzy graph Ǥ =(V, σ, μ) Consider a non-empty set V and two mappings $\sigma: V \rightarrow [0,1]$
And $\mu: V \times V \rightarrow [0,1]$ If these mappings satisfy the subsequent condition For all $a, b \in V$, $\mu(a,b) \leq \min(\sigma(a), \sigma(b))$If the two nonempty vertex sets are partitioned, we regard Ǥ as a fuzzy bipartite graph.to the disjoint subsets .$\ddot{V}_1$and $\ddot{V}_2$ (i.e., $\ddot{V} = \ddot{V}_1 \cup \ddot{V}_2$ and $\ddot{V}_1 \cap \ddot{V}_2 = \emptyset$) such that for every $a \in \ddot{V}_1$ and every $b \in \ddot{V}_2$, the value of the edge membership function $\mu(a,b)$ is equal to 0, the fuzzy graph is said to be complete if $\mu(a,b) = min(\sigma(a), \sigma(b))$ for all $a, b \in \ddot{V}$,fuzzy topological spaces$(\mathcal{X}, \mathcal{T}_f)$ defines $\mathcal{B}$ bases as subfamily of $\mathcal{T}_f$, subbases as families of finite intersections, and topologies $\mathcal{T}_{f1}$ and $\mathcal{T}_{f2}$ on X, with $\mathcal{T}_{f1}$ being coarser or $\mathcal{T}_{f2}$ being finer if $\mathcal{T}_{f1} \subseteq \mathcal{T}_{f2}$.
A fuzzy topology $\mathcal{T}_f$ is generated by a subfamily $\mathcal{S}$ of fuzzy sets in $\mathcal{X}$, where each member is a union of finite intersections of $\mathcal{S}$, A non-empty set $\mathcal{X}$ that is connected to three arbitrary topologies is called a tritopological space.For any topological space, T_1, T_2 andT_3 on $\mathcal{X}$ as(X, T_1, T_2, T_3) can be tritopological. $(\mathcal{X}, \mathcal{T})$ is a tritopological space if $(\mathcal{X}, \mathcal{T})$ is a topological space.
Currently, fuzzy graphs are associated with only three fuzzy topological spaces, which are defined as fellows: fuzzy Independent Topology [10], fuzzy Graphic Topology [9], and fuzzy Incidence Topology [11].
[10]the fuzzy graph that is undirected and Let $S_{FI} = \{R(a): a \in V\}$ such that R(a)={(b, σ(b)) ∈ V(Ǥ) | μ(a,b)=0 } is the set of all vertices not adjacent to a, where Ǥ may contain one or more isolated vertices. As a result, S_{FI} forms a sub-basis for a topology $\mathcal{T}_{FI}$ on V and $\mathcal{T}_{FI}$ is known as fuzzy Independent Topology of Ǥ. [9] The locally finite fuzzy

graph, which is a fuzzy graph with a finite number of nearby vertices for each vertex (a basic fuzzy graph without solitary vertex).
Define $S_G = \{N_X : X \in V^*\}$ such that $N_X = \{Y \in V^*; \mu(X,Y) > 0\}$ where $V^* = \{X \in V: \sigma(X) > 0\}$ so that X is the set of all vertices adjacent to Y ,Since has no isolated vertex($i.e.\ N_X \neq \theta$) ,Hence S_G forms a sub-basis for a fuzzy topology $\mathcal{T}_{FG}$ on V , and $\mathcal{T}_{FG}$ called Graphic Topology on fuzzy graphs of G. [10] Let $R_c(E)$ be the incidence vertices with the edge $\forall\, e \in E$, Define S_{Fc}={R_c€ : (a,$\sigma(a)$) $\in \sigma(V)$} is a simple fuzzy graph without isolated vertex, $(i.e.\ e \in \grave{E})$ where $\grave{E} = \{e_i \in E : e_i = \mu(a,b) > 0\}$ for all $a, b \in V$ *such that* $R_c(E)$ ={(b,Ψ(b)) ∈ Ψ ($\grave{V}$) | $\mu(a,b) \in \grave{E}$}Hence S_{Fc} forms a sub basis for a topology $\mathcal{T}_{Fc}$ on V, and called fuzzy Incidence Topology of G. *we say u* an isolated vertex, if $R_{cu}(e) = \emptyset$. Where $R_{cu}(e)$ is the collection of all edges that come into incident with the vertex. u *where* $R_{cu}(e) \subset R_c(E)$

3. IGI-INDUCED FUZZY TOPOLOGY ON UNDIRECTED FUZZY GRAPHS

MAIN RESULTS

Only three fuzzy topological spaces, fuzzy Independent Topology [11], fuzzy Incidence Topology [10], and fuzzy Graphic Topology [9] are connected to fuzzy graphs. As you can see, the fuzzy Independent Topology is defined on an undirected fuzzy graph that may have one or more isolated vertices. The family of sets to each fuzzy vertex that is not next to that fuzzy vertex in the fuzzy graph is the sub-basis for a fuzzy topology. In contrast, A fuzzy topology's sub-basis is the family of sets to each incident vertex with that edge in the graph. The fuzzy incidence topology is defined on a locally finite fuzzy graph, which is a straightforward fuzzy graph without isolated vertices. The family of sets to each vertex that is next to that vertex in a fuzzy graph is the sub-basis for a fuzzy topology. Lastly, the fuzzy Graphic Topology is defined on a locally finite fuzzy graph, which is a straightforward fuzzy network without isolated vertices. In this work, every graph is a locally finite simple fuzzy graph.

Remark 3.1. [13] (1) It is possible for any topological space to be tritopological. Given a topological space (X,T),a tritopological space is (X,T,T,T).

(2) The fuzzy tritopological space $(V, \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI})$ is the result of the three fuzzy topologies $\mathcal{T}_{Fc}$, T_G *and* $\mathcal{T}_{FI}$ on V

Remark 3.2. [13] The set of fuzzy vertices will now be associated with the three fuzzy topologies mentioned above: $\mathcal{T}_{Fc}$, $\mathcal{T}_{FI}$ and $\mathcal{T}_{FG}$,This will make the set a fuzzy tritopological space, which may be expressed as $(V, \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI})$

Definition 3.3. Assuming that $(V, \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI})$is a fuzzy tritopological space, the IGI -induced fuzzy topological space on locally finite fuzzy graphs is $(V, \mathcal{T}_{IGI})$where $\mathcal{T}_{IGI} = \mathcal{T}_{Fc} \cap \mathcal{T}_{FG} \cap \mathcal{T}_{FI}$

Definition 3.4. Let $A \subset V$ and let $(V, \mathcal{T}_{Fc}, T_G, \mathcal{T}_{FI})$be a fuzzy tritopological space. If A is open in the IGI -induced fuzzy topology, then it is referred to as an IGI -tri-fuzzy open set in V (For example($i.e. A \in \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI}$)
Definition 3.5. Let $(V, \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI})$be a fuzzy tritopological space .Let $A \subset V$. A is called an IGI -tri- fuzzy closed set in V if A is closed in the IGI -induced fuzzy topology.

Consider the fuzzy tritopological space $(V, \mathcal{T}_{Fc}, \mathcal{T}_{FG}, \mathcal{T}_{FI})$.Let $A \subset V$. A is closed in the IGI -induced fuzzy topology, then it is referred to as an IGI -tri-fuzzy closed set in V.

Example 3.6. As shown in Figure (1), let G =(V, σ, μ) be a simple fuzzy graph without isolated vertices so that, Then, V(G) ={(X_1,0.5),(X_2,1), (X_3 ,0.6), (X_4 ,0.3)}.
μ (E) ={(e_1, 0.03), (e_2, 0.05)}

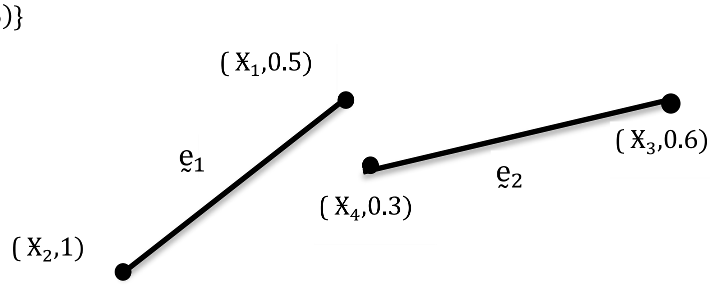


Figure .1 . simple fuzzy graph

The fuzzy Independent topology

$S_{FI} = \{\{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$
By taking finitely intersection the base obtained,
$B_{FI} = \{\emptyset, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FI} = \{\emptyset, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1), V(G)\}$

The fuzzy incident topology

$S_{Fc} = \{\{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$
By taking finitely intersection the base obtained,
$B_{Fc} = \{\emptyset, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{Fc} = \{\emptyset, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}, V(G)\}$

The fuzzy graphic topology

$S_{FG} = \{\{(Ӿ_3, 0.6)\}, \{(Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5)\}, \{(Ӿ_2, 1)\}\}$
By taking finitely intersection the base obtained,
$B_{FG} = \{\emptyset, \{(Ӿ_3, 0.6)\}, \{(Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5)\}, \{(Ӿ_2, 1)\}\}$
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FG} = \{\emptyset, \{(Ӿ_3, 0.6)\}, \{(Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5)\}, \{(Ӿ_2, 1)\}, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$
$\{(Ӿ_3, 0.6), (Ӿ_1, 0.5)\}, \{(Ӿ_1, 0.5), (Ӿ_4, 0.3)\}, \{(Ӿ_3, 0.6), (Ӿ_2, 1)\}, \{(Ӿ_4, 0.3), (Ӿ_2, 1)\},$
$\{(Ӿ_3, 0.6), (Ӿ_4, 0.3)(Ӿ_1, 0.5)\}, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)(Ӿ_2, 0.5)\}, \{(Ӿ_3, 0.6), (Ӿ_2, 1)(Ӿ_1, 0.5)\},$
$\{(Ӿ_2, 1), (Ӿ_4, 0.3)(Ӿ_1, 0.5)\}, V(G)\}$

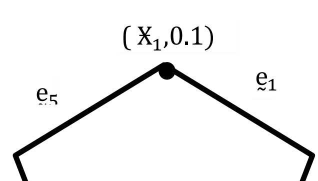
$\mathcal{T}_{\mathrm{IGI}} = \mathcal{T}_{\mathrm{Fc}} \cap \mathcal{T}_{\mathrm{FG}} \cap \mathcal{T}_{\mathrm{FI}}$, the IGI -induced fuzzy topology $\mathcal{T}_{\mathrm{IGI}} = \{\emptyset, \{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}$ $, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}, V(G)\}$, are IGI -tri- fuzzy open sets, and $\{(Ӿ_3, 0.6), (Ӿ_4, 0.3)\}, \{(Ӿ_1, 0.5), (Ӿ_2, 1)\}$ are IGI -tri- fuzzy closed sets.

Remark 3.7. the IGI-induced fuzzy topological space (V,$\mathcal{T}_{\mathrm{IGI}}$) of a fuzzy cycle graph C_n when $n \geq 5$ represent a discrete fuzzy topological space, because the fuzzy Graphic Topology is discrete fuzzy topology in $n > 4$ fuzzy Incidence Topology is discrete fuzzy topological space in $n \geq 3$ [9] and the fuzzy Independent Topology is discrete topological space in $n \geq 4$ [10].

Proof.
The fuzzy Independent Topology is discrete topological space when the fuzzy graph is a cycleC_n
And $n \geq 4$, let $x_1, x_2 \dots \dots x_i \in$ Vwhen $i = 1,2, \dots n$. $\mathcal{T}_{FI}$ is the discrete topology since
$R(x_i) = \{x_i\}$
The fuzzy Incidence Topology is discrete topological space when the fuzzy graph is a cycleC_n
And $n \geq 3$ if $d(v) \geq 2$ since every vertices in the cycle$C_n$has $d(v) \geq 2$ so
The fuzzy Graphic Topology is discrete topological space when the fuzzy graph is a cycleC_n

And The fuzzy Incidence Topology is discrete topological space when $n > 4$ Since G is a cycle let $x_1, x_2 \dots \dots x_i \in$ Vwhen $i = 1,2, \dots n$. $\mathcal{T}_{FG}$ is the discrete topology since $N(x_i) = \{x_i\}$, so the intersection of three fuzzy discrete topological space induced fuzzy discrete topological space for one fuzzy graph or three different fuzzy graph.

Example 3.8. Let G =(V, σ, μ) be a fuzzy cycle graph C_5 as in Figure (2) such that,
Then, V(G) ={$(Ӿ_1, 0.1)$,$(Ӿ_2, 0.2)$, $(Ӿ_3, 0.3)$, $(Ӿ_4, 0.4)$, $(Ӿ_5, 0.5)$}}
μ (E) ={$(e_1, 0.02)$, $(e_2, 0.03)$, $(e_3, 0.01)$, $(e_4, 0.05)$, $(e_5, 0.01)$ }



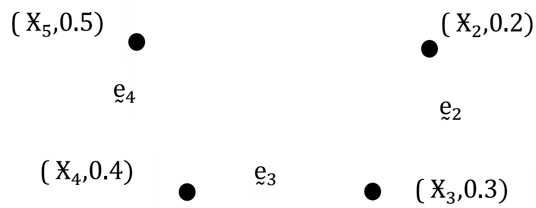


Figure.2. fuzzy cycle graph C_5

The fuzzy Independent topology

$S_{FI} = \{\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}\}$
By taking finitely intersection the base obtained,
$B_{FI} = \{\emptyset,\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\}$
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FI} = \{\emptyset,\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.6),(\text{Ӿ}_2,0.3)(\text{Ӿ}_4,0.5)\}$
$,\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5),(\text{Ӿ}_2,0.2)\},\{(\text{Ӿ}_5,0.5),(\text{Ӿ}_3,0.3)\}$
$,\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.5)\ (\text{Ӿ}_5,0.5)\},$
$,\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4t)\ (\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_5,0.5)\}V(\text{Ӷ})\}$

The fuzzy incident topology

$S_{Fc} = \{\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}\}$
By taking finitely intersection the base obtained,

$B_{Fc} = \{\emptyset,\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\}\}$

Then by taking all unions of the base then a fuzzy incident topology can be written as:
$\mathcal{T}_{Fc}=\{\emptyset,\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.6),(\text{Ӿ}_2,0.3)(\text{Ӿ}_4,0.5)\}$
$,\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5),(\text{Ӿ}_2,0.2)\},\{(\text{Ӿ}_5,0.5),(\text{Ӿ}_3,0.3)\}$
$,\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.5)\ (\text{Ӿ}_5,0.5)\},$
$,\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4t)\ (\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4),(\text{Ӿ}_3,0.3)\ (\text{Ӿ}_5,0.5)\}V(\text{Ӷ})\}$

The fuzzy graphic topology

$S_{FG} = \{\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\}$
$\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4)\}\}$
By taking finitely intersection the base obtained,

$B_{FG} = \{\emptyset,\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\}$
$\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4)\}\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\}\}$

Then by taking all unions of the base then a fuzzy graphic topology can be written as:
$\mathcal{T}_{FG} = \{\emptyset,\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2)\}$
$\{(\text{Ӿ}_2,0.2),(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_1,0.1)\},\{(\text{Ӿ}_2,2)\},\{(\text{Ӿ}_3,0.3)\},\{(\text{Ӿ}_4,0.4)\},\{(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.6),(\text{Ӿ}_2,0.3)(\text{Ӿ}_4,0.5)\}$
$,\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_2,0.2),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_3,0.3),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_3,0.3),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\},\{(\text{Ӿ}_1,0.1),(\text{Ӿ}_4,0.4),(\text{Ӿ}_5,0.5)\}$

$\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_4,0.4)\},\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_3,0.3),(Ӿ_4,0.4)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\}$
$\{(Ӿ_2,0.2),(Ӿ_4,0.4),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_4,0.4)\},\{(Ӿ_2,0.2),(Ӿ_4,0.4)\},\{(Ӿ_5,0.5),(Ӿ_2,0.2)\},\{(Ӿ_5,0.5),(Ӿ_3,0.3)\}$
$,\{(Ӿ_1,0.1),(Ӿ_3,0.3)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\ (Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_4,0.5)\ (Ӿ_5,0.5)\},$
$,\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_4,0.4t)\ (Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\ (Ӿ_4,0.4)\},\{(Ӿ_1,0.1),(Ӿ_4,0.4),(Ӿ_3,0.3)\ (Ӿ_5,0.5)\}V(G)\}$

$\mathcal{T}_{IGI} = \mathcal{T}_{Fc} \cap \mathcal{T}_{FG} \cap \mathcal{T}_{FI}$, the IGI -induced fuzzy topology
$\mathcal{T}_{IGI} = \{\emptyset,\{(Ӿ_3,0.3),(Ӿ_4,0.4)\},\{(Ӿ_4,0.4),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2)\}$
$\{(Ӿ_2,0.2),(Ӿ_3,0.3)\},\{(Ӿ_1,0.1)\},\{(Ӿ_2,2)\},\{(Ӿ_3,0.3)\},\{(Ӿ_4,0.4)\},\{(Ӿ_5,0.5)\},\{(Ӿ_1,0.6),(Ӿ_2,0.3)(Ӿ_4,0.5)\}$
$,\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_3,0.3),(Ӿ_5,0.5)\},\{(Ӿ_3,0.3),(Ӿ_4,0.4),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_4,0.4),(Ӿ_5,0.5)\}$
$\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_4,0.4)\},\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_3,0.3),(Ӿ_4,0.4)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\}$
$\{(Ӿ_2,0.2),(Ӿ_4,0.4),(Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_4,0.4)\},\{(Ӿ_2,0.2),(Ӿ_4,0.4)\},\{(Ӿ_5,0.5),(Ӿ_2,0.2)\},\{(Ӿ_5,0.5),(Ӿ_3,0.3)\}$
$,\{(Ӿ_1,0.1),(Ӿ_3,0.3)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\ (Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_4,0.5)\ (Ӿ_5,0.5)\},$
$,\{(Ӿ_2,0.2),(Ӿ_3,0.3),(Ӿ_4,0.4t)\ (Ӿ_5,0.5)\},\{(Ӿ_1,0.1),(Ӿ_2,0.2),(Ӿ_3,0.3)\ (Ӿ_4,0.4)\},\{(Ӿ_1,0.1),(Ӿ_4,0.4),(Ӿ_3,0.3)\ (Ӿ_5,0.5)\}V(G)\}$
,we note that the IGI-induced fuzzy topological
space (V,$\mathcal{T}_{IGI}$) of a cycle C_5 represent a discrete fuzzy topological space

Remark 3.9. The IGI -induced fuzzy topological space of a path is not discrete because, the fuzzy Graphic Topology of P_n is not discrete because the set contains , two vertices of degree one is not open [9]. The fuzzy Incidence Topology of the path P_n is not discrete because P_n contains two vertices incident with one edge is not open [10] . And fuzzy Independent Topology is not discrete because the set contains just two vertices of degree one is open [11].

Example 3.10. Let G =(V, 6, ɱ) be a fuzzy path p_4 as in Figure (3) such that,
Then, V(G) =$\{(Ӿ_1,0.1),(Ӿ_2,4),(Ӿ_3,0.6),(Ӿ_4,0.8)\}$
ɱ (E) =$\{(e_1,0.01),(e_2,0.02),(e_3,0.03)\}$

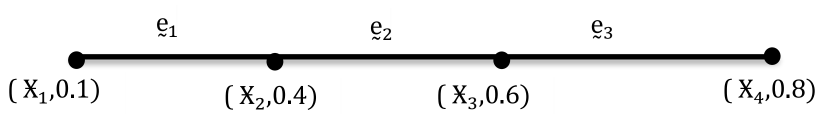


Figure .3 . fuzzy path p_4

The fuzzy Independent topology

$S_{FI} = \{\{(Ӿ_3,0.6),(Ӿ_4,0.3)\},\{(Ӿ_1,0.1)\},\{(Ӿ_4,0.8)\},\{(Ӿ_1,0.5),(Ӿ_2,1)\}$
By taking finitely intersection the base obtained,
$B_{FI} = \{\emptyset,\{(Ӿ_3,0.6),(Ӿ_4,0.3)\},\{(Ӿ_1,0.1)\},\{(Ӿ_4,0.8)\},\{(Ӿ_1,0.5),(Ӿ_2,1)\}$
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FI} = \{\emptyset,\{(Ӿ_3,0.6),(Ӿ_4,0.3)\},\{(Ӿ_1,0.1)\},\{(Ӿ_4,0.8)\},\{(Ӿ_1,0.5),(Ӿ_2,1)\}$
$\{(Ӿ_1,0.1)(Ӿ_3,0.6),(Ӿ_4,0.3)\},\{(Ӿ_1,0.5),(Ӿ_4,0.8)\},\{(Ӿ_1,0.1)(Ӿ_2,0.8),(Ӿ_4,0.3)\},V(G)\}$
The fuzzy incident topology

$S_{Fc} = \{\{(Ӿ_1,0.1),(Ӿ_2,0.4)\},\{(Ӿ_2,0.4),(Ӿ_3,0.6)\}\{(Ӿ_3,0.6),(Ӿ_4,0.8)\}\}$
By taking finitely intersection the base obtained,
$B_{Fc} = \{\emptyset,\{(Ӿ_1,0.1),(Ӿ_2,0.4)\},\{(Ӿ_2,0.4),(Ӿ_3,0.6)\}\{(Ӿ_3,0.6),(Ӿ_4,0.8)\}$
$\{(Ӿ_2,0.4)\},\{(Ӿ_3,0.6)\}\}$
Then by taking all unions of the base then a fuzzy incident topology can be written as:
$\mathcal{T}_{Fc} = \{\emptyset,\{(Ӿ_1,0.1),(Ӿ_2,0.4)\},\{(Ӿ_2,0.4),(Ӿ_3,0.6)\}\{(Ӿ_3,0.6),(Ӿ_4,0.8)\},\{(Ӿ_2,0.4)\},\{(Ӿ_3,0.6)\}$
$,\{(Ӿ_1,0.1),(Ӿ_2,0.4),(Ӿ_3,0.6)\}\{(Ӿ_4,0.8),(Ӿ_2,0.4),(Ӿ_3,0.6)\},V(G)\}$
The fuzzy graphic topology
$S_{FG} = \{\{(Ӿ_2,0.4)\},\{(Ӿ_3,0.6)\},\{(Ӿ_2,0.4),(Ӿ_4,0.8)\},\{(Ӿ_1,0.1),(Ӿ_3,0.6)\}$
By taking finitely intersection the base obtained,
$B_{FG} = \{\emptyset,\{(Ӿ_2,0.4)\},\{(Ӿ_3,0.6)\},\{(Ӿ_2,0.4),(Ӿ_4,0.8)\},\{(Ӿ_1,0.1),(Ӿ_3,0.6)\}$
Then by taking all unions of the base then a fuzzy graphic topology can be written as:
$\mathcal{T}_{FG} = \{\emptyset,\{(Ӿ_2,0.4)\},\{(Ӿ_3,0.6)\},\{(Ӿ_2,0.4),(Ӿ_4,0.8)\},\{(Ӿ_1,0.1),(Ӿ_3,0.6)\}$
$\{(Ӿ_2,0.8)(Ӿ_3,0.6),(Ӿ_4,0.3)\},\{(Ӿ_2,0.4),(Ӿ_3,0.6)\},\{(Ӿ_1,0.1)(Ӿ_2,0.8),(Ӿ_3,0.6)\},V(G)\}$

$\mathcal{T}_{IGI} = \mathcal{T}_{Fc} \cap \mathcal{T}_{FG} \cap \mathcal{T}_{FI}$, the IGI -induced topology $\mathcal{T}_{IGI} = \{\emptyset, V(G)\}$
We note that only when $n = 4$. The IGI -induced fuzzy topological space of a path p_4
is indiscrete

Remark 3.11. A fuzzy complete bipartite' s IGI -induced fuzzy topological space is not discrete. Since the fuzzy Independent Topology and the fuzzy Incident Topology are discrete fuzzy topologies, the fuzzy Graphic Topology is not.

Example 3.12. As shown in Figure (4), let G =(V, σ, μ) be a fuzzy full bipartite such that,
Then, V(G) ={($Ӿ_1$,1),($Ӿ_2$,0.1), ($Ӿ_3$,0.2), ($Ӿ_4$,0.3), ($Ӿ_5$,0.4)}}
μ (E) ={(e_1, 0.02), (e_2, 0.03), (e_3, 0.01), (e_4, 0.05), (e_5, 0.01), (e_6, 0.04)}

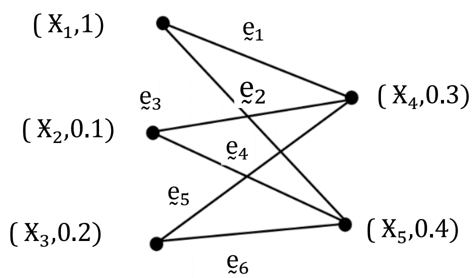


Figure.4. fuzzy complete bipartite

The fuzzy Independent topology

S_{FI} = {{($Ӿ_2$,0.1), ($Ӿ_3$,0.2)}, {($Ӿ_1$, 1), ($Ӿ_3$, 0.2)}, {($Ӿ_1$,1), ($Ӿ_2$,0.1)}, {($Ӿ_4$, 0.3)}, {($Ӿ_5$, 0.4)}
By taking finitely intersection the base obtained,
B_{FI} = {∅, {($Ӿ_2$,0.1), ($Ӿ_3$,0.2)}, {($Ӿ_1$, 1), ($Ӿ_3$, 0.2)}, {($Ӿ_1$,1), ($Ӿ_2$,0.1)}, {($Ӿ_4$, 0.3)},
{($Ӿ_5$, 0.4)}, {($Ӿ_3$,0.2)}, {($Ӿ_2$,0.1)}, {($Ӿ_1$, 1)}}
Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FI}$ = {∅, {($Ӿ_3$,0.2), ($Ӿ_4$,0.3)}, {($Ӿ_4$, 0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_5$,0.4)}, {($Ӿ_1$, 1), ($Ӿ_2$, 0.1)}
{($Ӿ_2$,0.1), ($Ӿ_3$,0.2)}, {($Ӿ_1$, 1)}, {($Ӿ_2$, 0.1)}, {($Ӿ_3$,0.2)}, {($Ӿ_4$,0.3)}, {($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$,0.2)($Ӿ_4$, 0.3)}
, {($Ӿ_1$ 1), ($Ӿ_2$,0.1), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_3$,0.2), ($Ӿ_5$, 0.4)}, {($Ӿ_3$,0.2), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}
{($Ӿ_2$,0.1), ($Ӿ_3$,0.2), ($Ӿ_4$, 0.3)}, {($Ӿ_2$,0.1), ($Ӿ_3$,0.2), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_3$,0.2), ($Ӿ_4$, 0.3)}, {($Ӿ_1$,1), ($Ӿ_2$,0.1), ($Ӿ_3$, 0.2)}
{($Ӿ_2$,0.1), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_4$,0.3)}, {($Ӿ_2$,0.1), ($Ӿ_4$,0.3)}, {($Ӿ_5$,0.4), ($Ӿ_2$,0.1)}, {($Ӿ_5$,0.4), ($Ӿ_3$,0.2)}
, {($Ӿ_1$,1), ($Ӿ_3$,0.2)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_3$, 0.2) ($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_4$, 0.3) ($Ӿ_5$,0.4)},
, {($Ӿ_2$,0.1), ($Ӿ_3$, 0.2), ($Ӿ_4$, 0.3) ($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_3$, 0.2) , ($Ӿ_4$,0.3)}, {($Ӿ_1$,1), ($Ӿ_4$, 0.4), ($Ӿ_3$, 0.2) ($Ӿ_5$,0.4)}, V(G)}

The fuzzy incident topology

S_{Fc} = {{($Ӿ_2$,0.1), ($Ӿ_4$,0.3)}, {($Ӿ_1$, 1), ($Ӿ_4$, 0.3)}, {($Ӿ_1$,1), ($Ӿ_5$,0.4)}, {($Ӿ_2$, 0.1)}, {($Ӿ_5$, 0.4)}
{($Ӿ_3$, 1), ($Ӿ_4$, 0.3)}, {($Ӿ_3$,0.2), ($Ӿ_5$,0.4)}}
By taking finitely intersection the base obtained,
B_{Fc} = {∅, {($Ӿ_2$,0.1), ($Ӿ_4$,0.3)}, {($Ӿ_1$, 1), ($Ӿ_4$, 0.3)}, {($Ӿ_1$,1), ($Ӿ_5$,0.4)}, {($Ӿ_2$, 0.1)}, {($Ӿ_5$, 0.4)}
{($Ӿ_3$, 1), ($Ӿ_4$, 0.3)}, {($Ӿ_3$,0.2), ($Ӿ_5$,0.4)}, {($Ӿ_4$, 0.3)}, {($Ӿ_5$, 0.4)}{($Ӿ_1$, 1)},{($Ӿ_3$,0.2)}, {($Ӿ_2$,0.2)}
Then by taking all unions of the base then a fuzzy incident topology can be written as:
$\mathcal{T}_{Fc}$ = {∅, {($Ӿ_3$,0.2), ($Ӿ_4$,0.3)}, {($Ӿ_4$, 0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_5$,0.4)}, {($Ӿ_1$, 1), ($Ӿ_2$, 0.1)}
{($Ӿ_2$,0.1), ($Ӿ_3$,0.2)}, {($Ӿ_1$, 1)}, {($Ӿ_2$, 0.1)}, {($Ӿ_3$,0.2)}, {($Ӿ_4$,0.3)}, {($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$,0.2)($Ӿ_4$, 0.3)}
, {($Ӿ_1$ 1), ($Ӿ_2$,0.1), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_3$,0.2), ($Ӿ_5$, 0.4)}, {($Ӿ_3$,0.2), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}
{($Ӿ_2$,0.1), ($Ӿ_3$,0.2), ($Ӿ_4$, 0.3)}, {($Ӿ_2$,0.1), ($Ӿ_3$,0.2), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_3$,0.2), ($Ӿ_4$, 0.3)}, {($Ӿ_1$,1), ($Ӿ_2$,0.1), ($Ӿ_3$, 0.2)}
{($Ӿ_2$,0.1), ($Ӿ_4$,0.3), ($Ӿ_5$, 0.4)}, {($Ӿ_1$,1), ($Ӿ_4$,0.3)}, {($Ӿ_2$,0.1), ($Ӿ_4$,0.3)}, {($Ӿ_5$,0.4), ($Ӿ_2$,0.1)}, {($Ӿ_5$,0.4), ($Ӿ_3$,0.2)}
, {($Ӿ_1$,1), ($Ӿ_3$,0.2)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_3$, 0.2) ($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_4$, 0.3) ($Ӿ_5$,0.4)},
, {($Ӿ_2$,0.1), ($Ӿ_3$, 0.2), ($Ӿ_4$, 0.3) ($Ӿ_5$,0.4)}, {($Ӿ_1$,1), ($Ӿ_2$, 0.1), ($Ӿ_3$, 0.2) , ($Ӿ_4$,0.3)}, {($Ӿ_1$,1), ($Ӿ_4$, 0.4), ($Ӿ_3$, 0.2) ($Ӿ_5$,0.4)}, V(G)}
The fuzzy graphic topology
S_G = {{($Ӿ_4$,0.2), ($Ӿ_5$,0.5)}, {($Ӿ_1$, 0.1), ($Ӿ_2$,0.2), ($Ӿ_3$, 0.3)}}
By taking finitely intersection the base obtained,
B_G = {∅, {($Ӿ_4$,0.2), ($Ӿ_5$,0.5)}, {($Ӿ_1$, 0.1), ($Ӿ_2$,0.2), ($Ӿ_3$, 0.3)}}

Then by taking all unions of the base then a fuzzy graphic topology can be written as:
$\mathcal{T}_G=\{\emptyset,\{(Ҳ_4,0.2),(Ҳ_5,0.5)\},\{(Ҳ_1,0.1),(Ҳ_2,0.2),(Ҳ_3,0.3)\}\},V(G)\}$

$\mathcal{T}_{IGI}=\mathcal{T}_{Fc}\cap\mathcal{T}_{FG}\cap\mathcal{T}_{FI}$, the IGI -induced fuzzy topology
$\mathcal{T}_{IGI}=\{\emptyset,\{(Ҳ_4,0.2),(Ҳ_5,0.5)\},\{(Ҳ_1,0.1),(Ҳ_2,0.2),(Ҳ_3,0.3)\}\},V(G)\}$ we note that the IGI -induced fuzzy topological space $(V,\mathcal{T}_{IGI})$ of fuzzy complete bipartite is not discrete fuzzy topological space

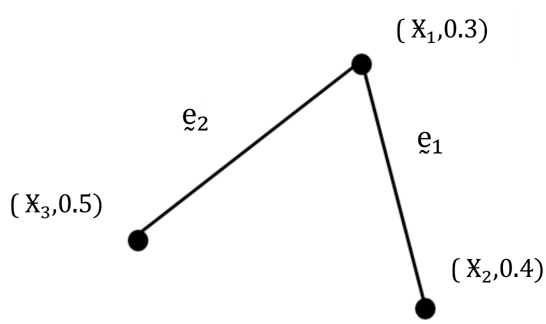
Remark 3.13. We can obtain the IGI -Induced fuzzy Topology From Three Different fuzzy Graphs On The Same Set Of Vertices, as in the following example

Example 3.14. Let $G=(V,\sigma,\mu)$ be three different fuzzy graph in Figure (5) (6) (7) such that,
in Figure (5)
Then, $V(G)=\{(Ҳ_1,0.2),(Ҳ_2,0.1),(Ҳ_3,0.4)\}$
$\mu(E)=\{(e_1,0),(e_2,0),(e_3,0)\}$

Figure (5). null fuzzy graph

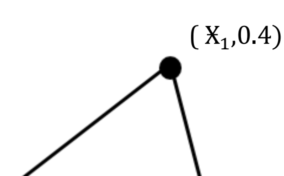
$S_{FI}=\{\{(Ҳ_2,0.1),(Ҳ_3,0.4)\},\{(Ҳ_1,0.2),(Ҳ_3,0.3)\},\{(Ҳ_1,0.2),(Ҳ_2,0.1)\}\}$
By taking finitely intersection the base obtained,
$B_{FI}=\{\emptyset,\{(Ҳ_2,0.1),(Ҳ_3,0.4)\},\{(Ҳ_1,0.2),(Ҳ_3,0.3)\},\{(Ҳ_1,0.2),(Ҳ_2,0.1)\},\{(Ҳ_1,0.2)\},\{(Ҳ_2,0.1)\},\{(Ҳ_3,0.4)\}\}$

Then by taking all unions of the base then a fuzzy Independent topology can be written as:
$\mathcal{T}_{FI}=\{\emptyset,\{(Ҳ_2,0.1),(Ҳ_3,0.4)\},\{(Ҳ_1,0.2),(Ҳ_3,0.3)\},\{(Ҳ_1,0.2),(Ҳ_2,0.1)\},\{(Ҳ_1,0.2)\},\{(Ҳ_2,0.1)\},\{(Ҳ_3,0.4)\},V(G)\}$
in Figure (6)
Then, $V(G)=\{(Ҳ_1,0.3),(Ҳ_2,0.4),(Ҳ_3,0.5)\}$
$\mu(E)=\{(e_1,0.1),(e_2,0.2)\}$



The fuzzy incident topology

$S_{Fc}=\{\{(Ҳ_1,0.3),(Ҳ_2,0.4)\},\{(Ҳ_1,0.3),(Ҳ_3,0.5)\}\}$
By taking finitely intersection the base obtained,
$B_{Fc}=\{\emptyset,\{(Ҳ_1,0.3),(Ҳ_2,0.4)\},\{(Ҳ_1,0.3),(Ҳ_3,0.5)\},\{(Ҳ_1,0.3)\}\}$
Then by taking all unions of the base then a fuzzy incident topology can be written as:
$\mathcal{T}_{Fc}=\{\emptyset,\{(Ҳ_1,0.3),(Ҳ_2,0.4)\},\{(Ҳ_1,0.3),(Ҳ_3,0.5)\},\{(Ҳ_1,0.3)\},V(G)\}$
in Figure (7)
Then, $V(G)=\{(Ҳ_1,0.4),(Ҳ_2,0.7),(Ҳ_3,0.8)\}$
$\mu(E)=\{(e_1,0.02),(e_2,0.004),(e_3,0.05)\}$



e_2

e_1

$(X_3, 0.8)$

$(X_2, 0.7)$

e_3

Figure.7. complete fuzzy graph

The fuzzy graphic topology

$S_{FG} = \{\{(X_1, 0.4), (X_2, 0.7)\}, \{(X_2, 0.7), (X_3, 0.8)\}, \{(X_1, 0.4), (X_3, 0.8)\}\}$
By taking finitely intersection the base obtained,
$B_{FG} = \{\emptyset, \{(X_1, 0.4), (X_2, 0.7)\}, \{(X_2, 0.7), (X_3, 0.8)\}, \{(X_1, 0.4), (X_3, 0.8)\}, \{(X_1, 0.1)\} \{(X_2, 0.7)\}, \{(X_3, 0.8)\}\}$
Then by taking all unions of the base then a fuzzy graphic topology can be written as:
$\mathcal{T}_{FG} = \{\emptyset, \{(X_1, 0.4), (X_2, 0.7)\}, \{(X_2, 0.7), (X_3, 0.8)\}, \{(X_1, 0.4), (X_3, 0.8)\}, \{(X_1, 0.4)\} \{(X_2, 0.7)\}, \{(X_3, 0.8)\}, V(G)\}$

$\mathcal{T}_{\mathrm{IGI}} = \mathcal{T}_{\mathrm{Fc}} \cap \mathcal{T}_{FG} \cap \mathcal{T}_{\mathrm{FI}}$, the IGI -induced topology
$\mathcal{T}_{\mathrm{IGI}} = \{\emptyset, \{(X_1, 0.2), (X_2, 0.1)\}, \{(X_1, 0.2), (X_3, 0.3)\}, \{(X_1, 0.2)\}, V(G)\}$ we note that the IGI-induced fuzzy topological space $(V, \mathcal{T}_{\mathrm{IGI}})$ of fuzzy complete bipartite is not discrete fuzzy topological space

Proposition 3.15. Let $G = (V, \sigma, \mu)$ be a simple graph. And $(V, \mathcal{T}_{\mathrm{Fc}}, \mathcal{T}_{FG}, \mathcal{T}_{\mathrm{FI}})$be a fuzzy tritopological space, then $(V, \mathcal{T}_{\mathrm{IGI}})$is an Alexandroff topological space, where is the IGI –induced fuzzy topology

Proof. Since The fuzzy incident topology $(V, \mathcal{T}_{\mathrm{Fc}})$,The fuzzy graphic topology$(V, \mathcal{T}_{FG})$ and The fuzzy Independent topology $(V, \mathcal{T}_{\mathrm{FI}})$ are a Alexandroff topological space[10],[9],[11] then by definition of Alexandroff topological space each intersection of elements in $\mathcal{T}_{\mathrm{Fc}}$, T_{G} and $\mathcal{T}_{\mathrm{FI}}$ is an element in $\mathcal{T}_{\mathrm{Fc}}$, T_{FG} and $\mathcal{T}_{\mathrm{FI}}$, $\mathcal{T}_{\mathrm{IGI}} = \mathcal{T}_{Fc} \cap \mathcal{T}_{FG} \cap \mathcal{T}_{FI}$ so the intersection of elements in$\mathcal{T}_{\mathrm{IGI}}$ is an element in $\mathcal{T}_{\mathrm{IGI}}$

4. Conclusions. This study presents a synthesis of fuzzy tritopological theory with fuzzy graph theory. Each fuzzy locally finite fuzzy graph has been linked to a fuzzy tritopological space. After that, a few characteristics of this fuzzy tritopological space were thoroughly examined. Additionally, a first step towards examining several fuzzy locally finite graph properties by their corresponding fuzzy tritopological spaces has been demonstrated.

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