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Supplement Rickart Modules

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ABSTRACT

This paper introduces a new generalization of Rickart modules, namely the Supplement-Rickart module. Let M be a module over a ring R with identity, and let $S = End_R(M)$ denotes the endomorphism ring of M. The module M is called Supplement-Rickart if for all $f \in S$, $Kerf$ is a supplement sub-module of M. Moreover, several remarks and examples are included, and various properties together with connections to other related notions are analyzed.

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1. Introduction

Idempotent elements $e \in R$ satisfying $e^2 = e$, are central to the structure of a ring R. Indeed, the presence of such elements allows the ring to be described as a direct sum of simpler parts, that is $R = eR \oplus (1-e)R$ [1]. Throughout this article, we assume that R is a ring with 1 and M is regarded as a left S-right R-bimodule, where $S = End_R(M)$. Moreover, for each $f \in S$, the right annihilator of f in M is given by $r_M(f) = r_M(Sf) = \{m \in M \mid f(m) = 0\} = Kerf$. The theory of Rickart structures has received considerable attention in the literature. In 1960, S. Maeda [2] introduced the notion of Rickart rings in a general framework. A ring is side to be right Rickart if the right annihilator of any single element is generated by an idempotent. A module M is called Baer if the right annihilator in M of any nonempty subset of $S = End_R(M)$ is generated by an idempotent of S (see [3]). G. Lee, S. T. Rizvi, and C. Roman in [1] studied the Rickart modules. M is a Rickart module if the right annihilator in M associated with each $e \in S$ is generated by an idempotent of S. In other words, an R-module M is side to be Rickart if for any $f \in S$, the kernel of f is a direct summand of M. Ali and Ghawi [4] studied the closed

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Rickart modules. Recall that a module M is said to be closed Rickart if the kernel of f is closed in M, for any $f \in S = End_R(M)$. The concept of coclosed Rickart modules was studied by Ghaleb Ahmed [5]. A module M is called co-closed Rickart provided, the kernel of f is a co-closed in M for each $f \in S$.

The following notions are also related to the sub-module structure: a sub-module $K \le M$ is small, denoted by ($K \ll M$), if whenever $M = K + N$, we have $M = N$. A sub-modules K, L of M. A sub-module K is a supplement of L in M, denoted by ($K \le_{sup} M$), if $M = K + L$ and $K \cap L$ is small in K (see[6]). Also, $K \le M$ is called closed, denoted by ($K \le_c M$), if it admits no proper essential extensions in M [7]. Moreover, K is said to be co-closed in M denoted by ($K \le_{co} M$), if for every sub-module $L \subseteq K$ when $\frac{K}{L} \ll \frac{M}{L}$ This leads inevitably to $K = L$ [8].

In this paper, introduces a new generalization of Rickart modules, namely the Supplement-Rickart module (or shortly, Sup-Rickart). M is called the Sup-Rickart module over a ring R, if for each $f \in S$, the kernel of f is a supplement sub-module of M.

This article is organized as follows. Section 2 introduces the concept of Supplement-Rickart module, together with several remarks, examples, and basic properties, and explores the relationships between Supplement-Rickart modules and other related concepts in module theory. In Section 3, we study direct sums of Supplement-Rickart modules and investigate related results.

2. Supplement Rickart Modules

In this section, we introduce a class of modules, called Supplement-Rickart modules, as a generalization of the Rickart module concept. We denoted by the Sup-Rickart. Several fundamental properties of this class of structures are examined. We start by recalling the next definition.

2.1.Definition

An R-module M is said to be Supplement-Rickart if for any $f \in S = End_R(M)$, The kernel of f is a supplement sub-module of M. Equivalently, for all $f \in S = End_R(M)$; $Kerf = \{m \in M \setminus f(m) = 0\} = r_M(Sf) = r_M(f)$ is a supplement sub-module in M. If $M = R$, then R is called a Sup-Rickart ring whenever R is a Sup-Rickart as an R-module. Equivalently, a ring R is Sup-Rickart if, for each element $a \in R$, the annihilator. $ann_R(a)$ for R is the supplement ideal.

2.2.Remarks

(1) Since every direct summand is a supplement sub-module, every Rickart module is a Sup-Rickart.

(2) It is well known that every Baer module is Rickart [1]; it follows that every Baer module is Sup-Rickart.

(3) Both Q and Z, viewed as Z-modules, are Baer modules [1], hence they are a Sup-Rickart.

(4) Each semisimple M is a Sup-Rickart module. As a special case, if $M = Z_6$. But the converse fails; indeed, let $M = Z$, viewed as a Z-module, is Sup-Rickart but fails to be semisimple.

(5) Let $M = Z_4$ and $f: Z_4 \to Z_4$ defined by $f(\bar{a}) = 2\bar{a}$, $f \in End(M)$, for each $a \in Z_4$, then $Kerf = \{\bar{0}, \bar{2}\}$, but $Kerf$ is not a supplement sub-module in Z_4, then Z_4 is not a Sup-Rickart.

(6) If M is a Sup-Rickart module, it is not necessarily the case that the factor module M/N is Sup-Rickart for a sub-module $N \le M$. Indeed, Z as a Z-module is Sup-Rickart; however, $Z/4Z \cong Z_4$, is not a Sup-Rickart.

(7) It is well known that every supplement sub-module is co-closed [see 8], then every Sup-Rickart is a co-closed Rickart module.

(8) Whenever M is a simple R-module (i.e., only M and 0 are its supplement sub-modules in M [10]). Therefore, M does not have to be Sup-Rickart. As an example, Z_4, viewed as a Z-module, is a supplement simple R-module. But it is not a Sup-Rickart.

(9) If R integral domain, then it is a Sup-Rickart module.

Proof: Assume that R is an integral domain (ID). Then for all $x \in R$ and $f \in End(R)$ is isomorphic to R, and R is commutative. The endomorphism $f \in End(R)$ with $f(r) = rx$, for all $r \in R$. Consequently, $Ker\, f = \{0\}$, because R is (ID) and $ann_R(x) = \{r \in R |\, rx = 0\} = \{0\}$. Then, $Kerf \leq_{sup} R$, Hence R is a Sup-Rickart. But the converse fails, since Z_6 is a Sup-Rickart but it is not an (ID).

(10) The Sup-Rickart property is not, in general, inherited by sub-modules, as shown in the following example. Let $M = Q \oplus Z_2$ as a Z-module. Then M is a Sup-Rickart, since it is a Rickart module (see Example 2.5 in [1]). Now consider the sub-module $N = Z \oplus Z_2$ a sub-module of M. Claim that is not a Sup-Rickart. To show that, let f be a map $f : N \to N$ defined by $f\,(w,\bar{u}) = (0,\bar{w})$ where $w,\ u \in$ Z. Thus $Kerf = \{\,(w,\bar{u}) \in Z \oplus Z_2 \mid f\,(w,\bar{u}) = (0,\bar{0})\} = \{\,(w,\bar{u}) \mid \bar{w} = 0\,\} = 2Z \oplus Z_2$ is not a supplement in N, because there exists $L = Z \oplus \bar{0} \leq N$, but $Kerf \cap L$ is not a small sub-module in $Kerf$. Consequently, N fails to be Sup-Rickart.

(11) The Sup-Rickart property is not necessarily preserved under homomorphic images. Let $f : Z \longrightarrow Z_4$ be a natural epimorphism. Although Z is Sup-Rickart as a Z-module, its image $Im\, f \cong Z_4$ is not.

2.3.Proposition

The Sup-Rickart property is invariant under an isomorphism.

Proof: Suppose that $M_1\ and\ M_2$ are R-modules, M_1 is Sup-Rickart, and $M_1 \cong M_2$. Let $\beta : M_1 \to M_2$ and $\alpha \in End(M_2)$, to demonstrate that $ker\alpha \leq_{sup} M_2$. Let $M_2 = ker\alpha + K$, such that $K \leq M_2$, to prove that $ker\alpha \cap K \ll ker\alpha$, then we get $M_1 = \beta^{-1}(ker\alpha) + \beta^{-1}(K)$. We claim that $\beta^{-1}(ker\alpha) = ker(\beta^{-1}\alpha\,\beta\,)$. Let $y \in \beta^{-1}(ker\alpha)$, $y = \beta^{-1}(x)$ and $x \in (ker\alpha)$, hence $\beta(y) = x$ and $\alpha(x) = 0$. then $\beta^{-1}\alpha\,\beta(y) = \beta^{-1}\alpha(x) = \beta^{-1}(0) = 0$, then $y \in ker\,(\beta^{-1}\alpha\,\beta\,)$. Conversely, let $x \in ker\,(\beta^{-1}\alpha\,\beta\,), \beta^{-1}\alpha\left(\beta(x)\right) = 0$, thus $\beta(x) \in ker\,(\beta^{-1}\alpha)$. But $ker\,(\beta^{-1}\alpha) = ker(\alpha)$ (clear), hence $\beta(x) \in ker(\alpha), x \in \beta^{-1}(ker\alpha)$. Thus $\beta^{-1}(ker\alpha) = ker(\beta^{-1}\alpha\,\beta)$. Since $\beta^{-1}\alpha\,\beta \in End(M_1)$, also M_1 is a Sup-Rickart, then $\beta^{-1}(ker\alpha) = ker\,(\beta^{-1}\alpha\,\beta\,) \leq_{sup} M_1$ and $\beta^{-1}(ker\alpha) \cap \beta^{-1}(K) \ll \beta^{-1}(ker\alpha)$. Therefore $ker\alpha \cap K \ll ker\alpha$ by [6]. Thus M_2 is a Sup-Rickart.

2.4.Proposition

Let M be a Sup-Rickart module. If $N \leq^{\oplus} M$, then N is the Sup-Rickart property.

Proof: Let M be a Sup-Rickart, and $M = N \oplus L$ where N, L are sub-modules of M and $f : N \to N$ be a homomorphism. Let $i : N \to M$ be the inclusion map, and $p : M \to N$ be the projection map. Consider the map $(i \circ f \circ p) : M \to M$. Hence $(i \circ f \circ p) \in End(M)$, and M is a Sup-Rickart, then $Ker(i \circ f \circ p) \leq_{sup} M$. But $Ker(i \circ f \circ p) = \{m \in M, (i \circ f \circ p)(m) = 0\}$

$$= \{n + l \in M, i(f(p(n + l)) = 0; n \in N, l \in L\}$$

$$= \{n + l \in M, f(p(x + y)) = 0;\ n \in N, l \in L\}$$
$$= \{n + l \in M, f(n) = 0;\ n \in N, l \in L\}$$
$$= Ker\, f \oplus L$$

Therefore $Ker\, f \oplus L \leq_{sup} M$, so $Kerf \leq^{\oplus} Ker\, f \oplus L$, then $Kerf \leq_{sup} M$. But $Kerf \leq N$, and N is also a supplement in M, then $Kerf \leq_{sup} N$, therefore N is a Sup-Rickart by [8].

The reverse implication of Proposition (2.4) is not valid in general. This can be seen from the following example. Let Z_{12} be viewed as a Z-module, and let $\varphi \in S = End(Z_{12})$ given by$\varphi(\bar{x}) = 6\bar{x}$ for all $\bar{x} \in Z_{12}$, then $Ker\varphi = \langle \bar{2} \rangle$, but this is not a supplement in Z_{12}, consequently $Z_{12}$is not Sup-Rickart. Now if, $N = \langle \bar{4} \rangle \leq^{\oplus} Z_{12}$, then $N \cong Z_3$. But Z_3 is a simple Z-module, then it is Sup-Rickart. So N is a Sup-Rickart.

2.5.Proposition

Let M be an R-module and f is split in M for any $f \in S$, then M is a Sup-Rickart.

Proof: Let $f : M \to M$ be an R-homomorphism. By our assumption f is split into M. Hence, the following short exact sequence is obtained.

$$0 \to kerf \overset{i}{\to} M \overset{f}{\to} f(M) \to 0$$

Is splits, where i is the inclusion map, then $kerf \leq^{\oplus} M$. Therefore $kerf \leq_{sup} M$. Thus M is a Sup-Rickart (see [10, Prop1.1.9]).

An R-module M has the supplement intersection property (briefly, SUIP) provided that the intersection of any two supplement sub-modules of M remains a supplement sub-module [11].

2.6.Theorem

The following conditions are equivalent for a module M with SUIP:

(a) M is a Sup-Rickart;

(b) For any finitely generated left ideal $I = \langle \varphi_1, \varphi_2, \dots, \varphi_n \rangle$ of $S = End(M)$, its right annihilator in M is a supplement sub-module of M.

Proof: $(a) \Rightarrow (b)$ Let M be a Sup-Rickart, and let I be a nonzero finitely generated left ideal of S, generated by $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$. Since $r_M(I) = \bigcap_{i=1}^{n} r_M(\varphi_i) \leq_{sp} M$ for $1 \leq i \leq n$, as M has the SUIP. Hence $r_M(I) \leq_{sup} M$.

$(b) \Rightarrow (a)$ Let $\varphi \in S$, then $\langle \varphi \rangle$ is a singly generated left ideal of S, then $r_M(I) \leq_{sup} M$. Therefore, M is Sup-Rickart.

A ring R is von-Neumann regular provided that each element $a \in R$ can be written in the form $a = aba$ for some $b \in R$ [1].

2.7.Proposition

If the endomorphism ring $S = End(M)$ of an R-module M is von-Neumann regular, then M is Sup-Rickart.

Proof: If S is a von-Neumann regular ring, then for any $\beta \in S$, both the kernel and the image of β are direct summands of M (see [1]). Therefore, M is a Sup-Rickart. However, the converse fails in general. Indeed, Z is Sup-Rickart as a Z-module, while its endomorphism ring. $End_Z(Z) \cong Z$ is not regular.

The following definition is recalled. A module M is said to be a supplement extending provided that all closed submodules of M are supplements in M [12].

2.8.Proposition

Every closed-Rickart supplement extending an R-module is Sup-Rickart.

Proof: This can be easily verified.

2.9.Proposition

Let M be an R-module, with Rad $(Ker\varphi) = 0$, for every $\varphi \in S = End(M)$. Then M is closed-Rickart if M Sup-Rickart.

Proof: Take M is Sup-Rickart, then $M = Ker\varphi + L$ and $Ker\varphi \cap L \ll Ker\varphi$, but Rad$(Ker\varphi) = 0$ (then no nonzero small sub module in $Ker\varphi$, hence $Ker\varphi \cap L = 0$. Thus $M = Ker\varphi \oplus L$. implies $Ker\varphi \leq^{\oplus} M$, then M Rickart module. Therefore, M is a closed-Rickart module.

We now introduce the next definitions. M is a nonsingular R-module provided that, if, for all $m \in M$ with $r_R(m) \leq_e R$ implies $m = 0$. Moreover, M is called polyform if, for any $K \leq M$, and for all $\varphi \in Hom(K, M), Ker\varphi = 0$, (resp. $Ker\varphi$ is closed in K). It has been established that every nonsingular module is polyform; see ([13], [14], [15]).

2.10.Proposition

Let M be a polyform and supplement extending R-module, then M is Sup-Rickart.

Proof: Let M be a polyform module; then M is closed-Rickart, by [4]. But M is a supplement extending module, then M is Sup-Rickart.

2.11.Corollary

M is Sup-Rickart if M is a supplement extending R-module with
a nonsingular

Proof: The proof is straightforward.

2.12.Remark

Every quasi-Dedekind R-module satisfies the Sup-Rickart property. By definition, M is a quasi-Dedekind R-module if $Ker\, f = 0$ for all $f \in End(M)$ with $f \neq 0$ [16]. Nevertheless, the converse implication does not hold in general. Indeed, Z_6, viewed as a Z-module, is Sup-Rickart, but not quasi-Dedekind.

2.13.Proposition

For a supplement simple module M. The next statements are equivalent:

(a) M is a quasi-Dedekind;

(b) M is a Sup-Rickart.

Proof: (a) $\Longrightarrow$ (b) Obvious.

(b) $\Longrightarrow$ (a) Let $\varphi \neq 0 \in End(M)$. Since M is Sup-Rickart and M is a supplement simple, it follows that $Ker\varphi = 0$, Accordingly, the homomorphism φ is injective, which implies that M is a quasi-Dedekind.

Notice: The hypothesis that M is supplement simple in Proposition (2.13) cannot be omitted. This is demonstrated by the following example. Let $M = Z_2 \oplus Z_2$ be a Z-module. Consequently, M is semisimple, hence M is Sup-Rickart. The following short exact sequence is considered:

$M \overset{\rho}{\rightarrow} Z_2 \oplus 0 \overset{i}{\rightarrow} M$, where ρ is a projection map, and i is the inclusion map. Then $h = i \circ \rho \in End(M)$ and $h \neq 0$, but $Ker = (0) \oplus Z_2$, and hence h fails to be injective. Consequently, M is not a quasi-Dedekind Z-module.

Recall that M is called hopfian an R-module if for every $\varphi \in End_R(M)$, φ is subjective implies φ is injective ($Ker\varphi = 0$). Also, M is called a generalized Hopfian module if for any $\varphi \in End_R(M)$, and φ is surjective implies $Ker\varphi \ll M$ see ([17], [18].

2.14.Corollary

If a module M is supplement simple and Sup-Rickart, then M is hopfian or (generalized hopfian).

Proof: Let $\varphi: M \rightarrow M$; $\varphi \neq 0$. Because M is Sup-Rickart, $Ker\varphi \leq_{sup} M$. But M is supplement simple and $\varphi \neq 0$, then $Ker\varphi = 0$, hence φ is injective. Therefore, M is a Hopfian or (generalized Hopfian) module.

3. Direct Sums and the Supplement Rickart Property

The behavior of Sup-Rickart modules under direct sums is investigated in this section.

3.1.Remarks

(i) In general, the class of Sup-Rickart is not closed under direct sums. For example, Z and Z_2 are Sup-Rickart as Z-modules; however, their direct sum $Z \oplus Z_2$, viewed as a Z-module, is not Sup-Rickart (see Remark 10).

(ii) It is known that if $M \oplus M$ has (SIP, i.e., intersection of any two direct summands is a direct summand) then M is a Rickart module. So it's Sup-Rickart (see [1]).

3.2.Proposition

Let $M = \bigoplus_{i\in\Lambda} M_i$ be a duo R-module. M is a Sup-Rickart precisely when each M_i is a Sup-Rickart for every $i \in \Lambda$.

Proof: One direction follows immediately from proposition (2.4). Conversely, let $\beta = (\beta_{ij}) \in End(M)$ where $\beta_{ij} \in Hom(M_i, M_j)$. Since M_i is fully invariant in $M = \bigoplus_{i\in\Lambda} M_i$, it follows that $Hom(M_i, M_j) = 0$ for each $i \neq j$ by [19]. Consequently, $\beta(M_i) \subseteq M_i, \forall\ i \in \Lambda$. Let $\mu = \sum_{i\in\Lambda} \mu_i \in M$. Then $\beta(\mu) = \sum_{i\in\Lambda} \beta_{ii}(\mu_i)$. Therefore, $\beta(\mu) = 0$ if and only if $\beta_{ii}(\mu_i) = 0, \forall\ i \in \Lambda$, which implies that $\mu_i \in Ker\beta_{ii}$, and hence $Ker\beta = \bigoplus_{i\in\Lambda} Ker\beta_{ii}$. We claim that $Ker\beta \leq_{sup} M$. Indeed, since $Ker\beta_{ii}$ is a supplement sub-module of L_{ii}, where $L_{ii} \leq M_i$, $\forall\ i \in \Lambda$, $Ker\beta = \bigoplus_{i\in\Lambda} Ker\beta_{ii} \leq_{sup} M = \bigoplus_{i\in\Lambda} M_i$ [20]. Therefore, M is a Sup-Rickart.

3.3.Proposition

A supplement simple Sup-Rickart module M with a nonzero maximal sub-module K does not preserve the Sup-Rickart property under the direct sum $M \oplus (M / K)$.

Proof: Assume that $M \oplus (M/K)$ is Sup-Rickart, and $\varphi \in End\big(M \oplus (M/K)\big)$ specified by $\varphi(m, \bar{\omega}) = (0, \bar{m})$, $\forall\ m \in M$, $\bar{\omega} \in M/K$. Then $Ker\varphi = \{(m, \bar{\omega}) \in M \oplus (M/K) \mid \varphi(m, \bar{\omega}) = (0, \bar{0})\} = \{(m, \bar{\omega}) \mid m + K = K\} = \{(m, \bar{\omega}) | m \in K\} =$

$K\oplus(M/K)$. Then $Ker\varphi = K\oplus(M/K) \leq_{Sup} M\oplus(M/K)$. But $K \leq^{\oplus} K\oplus(M/K)\ and\ K \leq_{sup} N\oplus(M/K)$, then by [14, p.238], $K \leq_{sup} M\oplus(M/K)$. Also by result [21], $K \leq_{sup} M$. This leads to a contradiction, since M is supplement simple. Yield $M\oplus(M/K)$ is not Sup-Rickart.

An R-module M satisfies the D_2 condition provided that every sub-module $L \leq M$ whose factor $M/L \cong K \leq^{\oplus} M$, then $L \leq^{\oplus} M$ [1].

3.4.Proposition

Let M be an R-module satisfying the D_2 condition. If $Im\varphi \cong A$ with $A \leq^{\oplus} M$ for all $\varphi \in End(M)$, then M is a Sup-Rickart .

Proof: Let $Im\varphi \cong A \leq^{\oplus} M, \forall\, \varphi \in End(M)$. But $Im\varphi \cong M/Ker\varphi$, hence $M/Ker\varphi \cong A \leq^{\oplus} M$, then $Ker\varphi \leq^{\oplus} M$, by (D_2). It implies $Ker\varphi \leq_{sup} M$. Thus, M is a Sup-Rickart.

References

[1] Lee, Gangyong, S. Tariq Rizvi, and Cosmin S. Roman. "Rickart modules." Communications in Algebra 38.11 2010: 4005-4027.
[2] S. Maeda. “On a ring whose principal right ideals generated by idempotents form a lattice”. J. Sci. Hiroshima Univ. Ser. A, 24:509–525, 1960.
[3] S.T. Rizvi and C.S. Roman. “Baer and quasi-Baer modules”. Comm. Algebra, 32(1):103–123, Jan. 2004.
[4] Kareem, Mohammed A., Inaam Mohammed Ali, and Tha'ar Younis Ghawi. "On closed Rickart modules." Iraqi Journal of Science 2016: 2746-2753.
[5] Ahmed, Ghaleb. "Coclosed Rickart Modules." Ibn AL-Haitham Journal For Pure and Applied Science 2018: 452-462.
[6] Wisbauer, R. Foundations of Module and Ring Theory, Reading, Gordon and Breach Science Publishers, 1991.
[7] Goodearl, K.R. Ring Theory, Nonsingular Rings and Modules, Marcel Dekker, Newyork, 1976.
[8] J. Clark; CH. Lomp; N. Vanaja; R. Wisbauer, Lifting modules, supplements and projectivity in module theory. Birkhauser Verlag, Basel, Boston, Berlin, 2006.
[9] N. Orhan. D.KTutuncu and R.Tribak, "On Hollow-lifting Modules," Taiwanese J. Math, vol. 11, no. 2, pp. 545-568, 2007.
[10] M. Q. Rahman. Some Generalizations of the Continuous and Discrete Modules, M. Sc. Thesis, University of Baghdad, Iraq, 2020.
[11] M. A. Sagban, Supplement-Small Sub-modules and Related Concepts, Ph. D. Thesis, University of Baghdad, Iraq, 2025.
[12] Yaseen, Sahira M., and Mustafa Mamoon Tawfeek. "Supplement extending modules." Iraqi Journal of Science 56.3B 2015: 2341-2345.
[13] Goodearl, K.R. Ring Theory, Nonsingular Rings and Modules, Marcel Dekker, Newyork, 1976.
[14] Zelmanowitz, J. M. "Representation of rings with faithful polyform modules." Communications in algebra 14.6 1986: 1141-1169.
[15] Dung, N.V., Huyn, D.V., Smith, P.F., and Wisbauer, R. Extending Modules, Pitman Research notes in math. Series, 313, Longman Scientific and Technical: Harlow, 1994.
[16] A.S.Mijbass, Quasi-Dedekind Modules, Ph.D. Thesis, University of Baghdad, (1997).
[17] V. A. Hiremath, Hopfian Rings and Hopfian Modules, Indian J. pure appl.Math. 17(7), 895-900(1986).
[18] A.Ghorbani and A.Haghany, "Generalized Hopfian Modules," J.Algebra, pp. 324-341, 2002.
[19] Özcan, A. C., Ayla Harmanci, and P. F. Smith. "Duo modules." Glasgow Mathematical Journal 48.3 (2006): 533-545.
[20] M.A.Inaam, Module Theory, Iraq, Baghdad, 2021.
[21] J. Clark, C. Lamp, N. Vanaja, and R. Wisbauer, Lifting Modules, Supplements and Projectivity in Module Theory, 2006.