Journal of Al-Qadisiyah for Computer Science and Mathematics Vol.18.(2) 2026,pp.Math 32–38

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Integration of Fractional Complex Transform with A-Transform and Adomian Decomposition Method for Solving Nonlinear Fractional Differential Equations

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ARTICLE INFO

Article history:

Received: 27 /02/2026

Rrevised form: 11 /04/2026

Accepted : 15 /04/2026

Available online: 30/06/2026

Keywords:

fractional differential equations(FDE), Fractional Complex Transform(FCT), A-transform, fractional Relaxation Equation and fractional Riccati Equation

ABSTRACT

In this study, we investigated the possibility of integrating the fractional complex transform (FCT) with the Aman transform (A). This methodology was applied to several nonlinear fractional-order differential equations The illustrative examples, including the fractional relaxation equation and the fractional Riccati equation using the Adomian Decomposition Method (ADM). The approach yielded accurate numerical solutions in a concise and efficient manner, highlighting the effectiveness of this integration in addressing complex mathematical models.

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https://doi.org/10.29304/jqcsm.2026.18.22687

1. Introduction

Fractional-order differential equations have received increasing attention in various fields such as science, physics, engineering, and applied mathematics, due to their notable ability to address complex problems. The origins of fractional calculus date back to 1695, when L'Hôpital sent a letter to Leibniz inquiring about the meaning of a derivative of order . Leibniz replied: "It will lead to a paradox, from which useful results will one day be derived." Since then, scientists have recognized the importance of this type of equations, and research in the field has expanded significantl.[1,2,3,4, 5,6] .

Several analytical methods have been developed to solve fractional differential equations, including the Laplace transform[7],the Sadik transform [8], and the Sumudu transform[9]. In this study, we examine the possibility of using the Fractional Complex Transform(FCT)[10] ,which converts fractional equations into their ordinary form, thereby eliminating the fractional property. The equations can then be solved using the A-transform[11] and its

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inverse, with the aid of the Adomian Decomposition Method (ADM)[12], in order to obtain exact solutions and avoid the complications associated with fractional orders.

We applied this methodology to two fundamental cases: the fractional Relaxation Equation[13,14] and nonlinear fractional Riccati differential equation[15,1] . The results demonstrated strong agreement with the physical reality of these models, as the solutions were represented through graphs and plots for different fractional values, confirming the effectiveness of the proposed approach.

1.2 Preliminaries on Fractional Calculus:

Definition 1: let $f(y), y > 0$ be in space $\Sigma_{\rho,}\rho \in R$ if there exists areal number $(n \geq \rho)$, such that $f(y) = y^n f_1(y)$where $f(y) \in \Sigma(0,\infty)$ and belong to space E^k if $f^k \in \Sigma_{\rho,}k \in N$ [11 , 16].

Definition 2: The Caputo's define in [11 , 16 , 17]of the fractional derivative of $f(y) \in \Sigma_{-1}^{\rho}$of order γ is :

$$ {}_0^cD_y^\gamma f(y) = \frac{1}{\Gamma(n-\gamma)}\int_0^y (y-r)^{n-\gamma-1} f^n(r)dr \ , \quad n-1 < \gamma \leq n, n \in N \qquad (i). $$

And the derivative

$$ {}_0^{RL}D_y^\gamma f(y) = \frac{1}{\Gamma(n-\gamma)}\frac{d^n}{dy^n}\left(\int_0^y (y-r)^{n-\gamma-1} f(r)dr\right), n-1 < \gamma \leq n, n \in N \quad (ii) $$

Is called Riemann-Liouvilles fractional derivative[11 , 16 , 17].

1.3 Fractional Complex Transform (FCT):

Transformations play a fundamental role in mathematics as powerful tools for tackling complex problems. In recent developments, the Fractional Complex Transform (FCT) [10] has emerged as an effective approach for converting fractional-order differential equations into integer-order differential equations. This transformation facilitates the use of advanced calculus techniques to derive exact or approximate solutions. Over the years, several notable transforms have been introduced in the literature, including the the Laplace transform[7], the A-transform[11], the Sadik transform [8], and the Sumudu transform[9].

The general fractional defferential equation that follows:

$$ f\left(u, u_t^{(\alpha)}, u_x^{(\beta)}, u_y^{(\gamma)}, u_z^{(\lambda)}, u_t^{(2\alpha)}, u_x^{(2\beta)}, u_y^{(2\gamma)}, u_z^{(2\lambda)}, \dots\right) = 0 \qquad (1) $$

Where

$u_t^{(\alpha)} = \partial^\alpha u(x,y,z,t)/\partial^\alpha$ denotes the fractional derivative. $0 < \alpha \leq 1, 0 < \beta \leq 1, 0 < \gamma \leq 1, 0 < \lambda \leq 1$.

Presenting the further transformations:

$$ \mathrm{H} = \frac{qt^\alpha}{\Gamma(1+\alpha)}, \qquad (2) $$

$$ \mathrm{Z} = \frac{px^\beta}{\Gamma(1+\beta)}, \qquad (3) $$

$$ \mathrm{E} = \frac{ny^\gamma}{\Gamma(1+\gamma)}, \qquad (4) $$

$$ \Theta = \frac{mt^\lambda}{\Gamma(1+\lambda)}, \qquad (5) $$

where $p , q, n,$ *and* m are constants.

Fractional derivatives can be transformed into classical derivatives using the aforementioned transforms:

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = q\,\frac{\partial u}{\partial \mathrm{H}} \tag{6}$$

1.4 Integral Transformation or (A-Transform) [11]:

Let S is est of functions as:

$$s = \left(f(g) : \exists\, s^{*}, v_1, v_2 > 0, |f(g)| < s^{*} e^{\left(\frac{|g|}{v_i}\right)}, if, g \in (-1)^{i} \times [0,\infty) \right)$$

Then , there exists an A-transform of $f(g)$ specified throughout the collection of functions S

,having the following definition :

$$A[f(g)] = \bar{f}(v^2) = \frac{1}{v}\int_0^{\infty} f(vg) e^{\frac{-y}{v}}\, dg\,, v \in (-v_1, v_2). \tag{7}$$

The definition of inverse of A-transform is :

$$A^{-1}\left(\bar{f}(v^2)\right) = f(g), for\ g \geq 0 \text{ or } f(g) = A^{-1}\left(\bar{f}(v^2)\right) = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha+\infty} \frac{1}{u} e^{\frac{g}{u}} f(v^2)\, dv\,. \tag{8}$$

In complex plane , $(u = x + iy)$, this integral is taken along $v = \alpha^2$. The real α constant and the A-transform variables v^2 are used here .

Some basic function of A-transform[11]:

(1) $A[g^n] = n!\, v^{2n}$, $A^{-1}(v^n) = \frac{g^{n+1}}{(n+1)!}$, $A^{-1}(1) = g$ (9)

And $A^{-1}\left(\frac{1}{v}\right) = 1.$

(2) (i) $A[U'(g)] = \frac{1}{v}A(v) - \frac{U(0)}{v}$ (10)

(ii) $A[U''(g)] = \frac{1}{v^2}A(v) - \frac{U(0)}{v^2} - \frac{U'(0)}{v}$ (11)

(iii) $A[U'''(g)] = \frac{1}{v^3}A(v) - \frac{U(0)}{v^3} - \frac{U'(0)}{v^2} - \frac{U''(0)}{v}$ (12)

(3) $A[g^{\alpha}] = \Gamma(\alpha + 1) v^{2\alpha}, \alpha > -1.$ (13)

The Algorithm of solution : Let the fractional differential equation

$${}_0D_t^{\alpha} y(t) + a y(t) = f(t) \ \ ,\ \ 0 < \alpha \leq 0\ , y(0) = y_0 \tag{14}$$

By FCT method of (14) ,we take $\mathrm{H} = \frac{q t^{\alpha}}{\Gamma(1+\alpha)}$, $(q\ arbartiry\ constant = 1)$

We have $${}_0D_t^{\alpha} y(t) = \frac{dy}{dZ} \tag{15}$$

Eq(14) become $\frac{dy}{dZ} = a y(Z) - f(Z)$ (is ordinary equation) (16)

BY take A-transform for Eq(16)

$$A\left[\frac{dy}{dZ} = a y(Z) - f(Z)\right]$$

$$A\left[\frac{dy}{dZ}\right] = aA[y(Z)] - A[f(Z)]$$

from (10) $$\left(\frac{1}{v}k(v) - \frac{y_0}{v}\right) + a k(v) - F(v)$$

$$\frac{1}{v}k(v) + ak(v) = F(v) + \frac{y_0}{v}$$

$$k(v)(\frac{1}{v} + a\ ) = F(v) + \frac{y_0}{v}$$

$$A^{-1}[k(v)] = A^{-1}\left[\frac{F(v) + \frac{y_0}{v}}{\left(\frac{1}{v} + a\ \right)}\right]$$

$$y(Z) = A^{-1}\left[\frac{F(v) + v\frac{y_0}{v}}{\left(\frac{1}{v} + a\ \right)}\right] \tag{17}$$

Applying the inverse of (17) based on the properties of the A-transform [11] yields the solution in terms of , after which we substitute its equivalent expression.

Some examples of applications in fractional differential equations

Avoid hyphenation at the end of a line. Symbols denoting vectors and matrices should be indicated in bold type. Scalar variable names should normally be expressed using italics. Weights and measures should be expressed in SI units. All non-standard abbreviations or symbols must be defined when first mentioned, or a glossary provided.

Example1: Consider the following fractional Relaxation Equation:

$${}_0D_t^{\gamma}u(t)\ = -\mu u(t) \quad with\ \ (0 < \gamma\ \leq 1)\ \ , u(0) = u_0, \mu = constant \tag{18}$$

By FCT in (6) , we have

$${}_0D_t^{\gamma}u(t)\ = \frac{dU(T)}{dT} \tag{19}$$

From Eq(14) and Eq(15) , we get

$$\frac{dY(T)}{dT} = -\mu U(T) \tag{20}$$

Now, Applying A-transform (7) to bouth sides of (20)

$$A\left[\frac{dY(T)}{dT}\right] = A[-\mu U(T)]$$

$$A(n) - nu_0 + n\mu A(n) = 0$$

$$A(n)(1 + \mu n) = nu_0$$

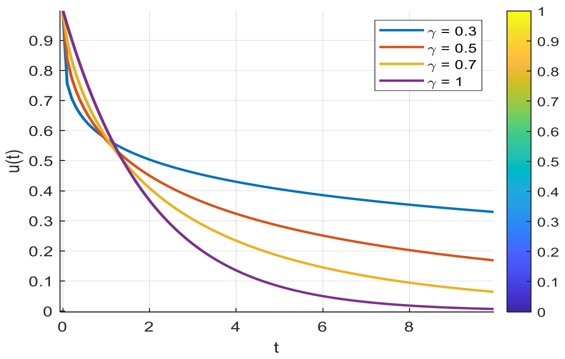
$$A(n) = \frac{nu_0}{(1 + \mu n)}$$

$$\mathrm{A(n)}{=}u_0\left(\frac{n}{(1+\mu n)}\right)$$

$$A^{-1}(A(n)){=}A^{-1}\left(u_0\left(\frac{n}{(1+\mu n)}\right)\right)$$

$$\mathrm{Y(T)} = \mathrm{u_0 e^{-\mu T}}$$

Then $$u(t) = \mathrm{u_0 e}^{-\mu\left(\frac{t^{\gamma}}{\Gamma(1+\gamma\ )}\right)}\ . \tag{21}$$



fiugre(1) solution of Eq(14)(where $t = [0, 1]$ $\mu = 0.5$, $u_{0=}1$

From the graph, we observe that the system is influenced by its previous states, as there is a noticeable delay in the decay process. The system remains affected by its initial condition for a longer period compared to systems governed by integer-order derivatives. Moreover, ensures that the physical system retains its inherent properties where we use of different values of the fractional order γ

Example2: Consider the following fractional Riccati Equation:

$$D_t^{\gamma} u(t) + u^2 = 1 \quad , \qquad (0 < \gamma \ \leq 1) \ \ , u(0) = 0 \tag{22}$$

Applying FCT (6) ,we get

$$\frac{du(\theta)}{d\theta} + u^2(\theta) = 1 \tag{23}$$

$$\text{where} \quad \theta = \frac{p\, t^{\gamma}}{\Gamma(1+t)} \quad ,\text{p=1}$$

Now, Applying A-transform (7) to bouth sides of (23)

$$\mathrm{A}\left[\frac{du(\theta)}{d\theta}\right] + A[u^2(\theta)] = A[1] \tag{24}$$

$$\mathrm{A}\left[\frac{du(\theta)}{d\theta}\right] = \frac{1}{v} A(v) - \frac{U(0)}{v} \quad \text{and} \qquad A[1] = \frac{1}{v} \tag{25}$$

$$\frac{1}{v} A(v) - \frac{U(0)}{v} + \ A[u^2(\theta)] = \frac{1}{v}$$

$$A(v) - U(0) + v\, A[u^2(\theta)] = 1$$

$$A(v) = 1 - v\, A[u^2(\theta)]$$

We used (ADM or Adomian method [9]) to obtain the $A[u^2(\theta)]$

$$u(\theta) = \textstyle\sum_{n=0}^{\infty} u_n(\theta) \tag{26}$$

$$A(u_0) = 1 \quad u_0(\theta) = \theta$$

$$A(u_1) = -vA[u_0^2] = -vA[\theta^2]$$

$$\text{Since } A[\theta^2] = 2!\, v^{2-1} = 2v$$

$$\text{Then } \ A(u_1) = -2v^2$$

By A^{-1} to bouth sides

$$\text{We have} \qquad u_1(\theta) = -2\frac{\theta^3}{3!}$$

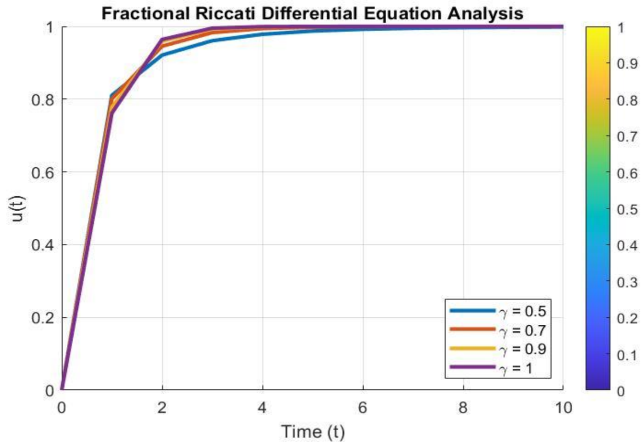
$$= -\frac{\theta^3}{3}$$

Equation (26) become :

$$u(\theta) = \theta - \frac{\theta^3}{3} + \frac{2}{15}\theta^3$$
$$= tanh(\theta)$$

Then

$$u(t) = tanh\left(\frac{t^{\gamma}}{\Gamma(1+t)}\right). \tag{27}$$



fiugre(2) solution of Eq(18)(wher $t = [0,1]$*and defrent value of* γ

We observe a change in the shape of the curve when varying the value of γ. As γ decreases (for instance, γ = 0.5), the curve rises more sharply at the beginning, which indicates a faster system response due to past effects. This behavior cannot be observed in the classical derivative when γ = 1.

Conclusion

We presented a methodological study employing a hybrid approach that combines the fractional complex transform (FCT) with the Aman transform (A). Through this technique, the fractional-order differential equation was converted into an integer-order form using (FCT), and subsequently solved via the Aman transform. By applying the inverse, we obtained solutions for the fractional Riccati and fractional Relaxation Equation with less computational effort compared to other transformations. The resulting solutions are characterized by their simplicity, avoidance of dealing directly with complex fractional orders, and accuracy consistent with the physical behavior of quantum systems. This approach paves the way for future applications in solving other equations that are difficult to address using conventional methods, particularly in the fields of physics and engineering.

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